

Uncertainty Quantification Of Fuel Assembly Bow In Pressurized Water Reactor Through a Thermomechanical Simulation

Ali Abboud^{1,2,*}, Josselin Garnier², Bertrand Leturcq¹, Julien Pacull³, Olivier Fandeur¹, Stanislas de Lambert¹

¹Université Paris-Saclay, CEA, Service d'Études Mécaniques et Thermiques, 91191, Gif-sur-Yvette, France;

²CMAF, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France;

³Framatome, 2 rue du Professeur Jean-Bernard, 69007 Lyon, France

ABSTRACT

In the core of nuclear reactors, fluid-structure interaction and intense irradiation lead to progressive deformation of fuel assemblies. Detrimental deformation can lead to additional costs, constrained core operation due to longer fuel unloading and reloading operations. Therefore, it is preferable to adopt a fuel management that avoids excessive deformations and interactions between fuel assemblies. However, the prediction of deformation and interactions between fuel assemblies is complex. Uncertainties affect neutronics, thermohydraulics and thermomechanics parameters. These uncertainties are propagated through the coupling of non-linear, nested and multidimensional thermal hydraulic and thermomechanical simulations. This article studies a structural model of the fuel assembly and quantifies its uncertainties. The objective is achieved through a multi-stage approach, beginning with an initial sensitivity analysis to identify the most influential parameters in the mechanical model. The sensitivity analysis aims to build an accurate and suitable surrogate model that represents the mechanical behavior of fuel assembly bowing within the reactor core. This surrogate model is then integrated with the hydraulic model developed in [A. Abboud et al., BEPU 2024, 272] to quantify the final uncertainties in simulating fuel assemblies deformation within a pressurized water reactor.

Keywords: Uncertainty Quantification, Surrogate Modeling, Thermomechanics, Thermal Hydraulics, Assembly bow

1. INTRODUCTION

The rapid insertion of control rod clusters is vital for reactor safety, enabling quick reduction or immediate shutdown of reactor power to manage reactivity. Since the 1990s, several incidents of incomplete control rod insertion have posed risks to this safety function. Notably, in 1994, a control rod failed to fully insert during an emergency shutdown at the Ringhals 4 [1] reactor due to increased friction between the control rod and the assembly's guide thimbles, caused by deformation of the guide tubes. Similar issues, including increased control rod insertion times and significant deformation of fuel assemblies, have been observed in other reactors, with deformations taking characteristic *C*, *S*, or *W* shapes. The fuel assemblies bow, by altering the size of the surrounding water gaps, directly impacts moderation and consequently neutronics, resulting in local variations in flux and power [2].

*ali.abboud@cea.fr

1.1. Literature review on uncertainty quantification in the context of fuel assembly deformation and motivation for our work.

The primary factors contributing to fuel assembly deformation include hold down forces, structural growth, structural creep, grid spring relaxation, fast neutron irradiation, and lateral hydraulic forces [3] [4] [5]. Existing studies on uncertainty quantification predominantly focus on the phenomena contributing to or influencing fuel assembly deformation rather than the deformation itself. For instance, Wanninger's 2018 [6] sensitivity and uncertainty analysis assessed the model's behavior to various influencing mechanisms. Despite recognizing the importance of uncertainties in numerical simulations of assembly deformations, previous research needs to be further developed. Our work aims at addressing these limitations by developing a methodology to quantify uncertainties in nonlinear thermal hydraulic and thermomechanical coupling, providing a comprehensive framework adaptable to the multidimensional and multiphysics nature of these simulations.

1.2. Summary of the objectives of this article.

This article studies a structural mechanical model and quantifies its uncertainties. To gain a comprehensive understanding of the mechanical behavior of the fuel assembly and accurately quantify the sources of uncertainty in mechanical calculations, the sensitivity analysis approach is decomposed into several stages prior to constructing an efficient metamodel. First, the creep behavior of the assembly is investigated by considering a wide variety of hydraulic forces and exploring the full range of grid clamping values, including the uncertainties in the creep and growth laws. Subsequently, a focused study is conducted on a specific scenario during full power operation to elucidate the effects of creep and growth under constant or variable clamping conditions. Following this, an irradiation cycle is analyzed, concentrating on the total displacement variation on each grid of the fuel assembly, which holds significant industrial relevance as explained previously. This analysis takes into account the initial deformations of the assembly and narrows down the range of hydraulic forces considered.

2. METHODOLOGY : THE THERMOMECHANICAL REFERENCE MODEL

To simulate the mechanical deformation of the assembly, the calculation code of CEA, DACC, is employed. A light thermomechanical model represents a fuel assembly (FA), based and validated on a more detailed model. Extensive simulations reveal that over 90% of the 264 rods in a fuel assembly consistently exhibit similar deformations and sliding behavior through the grids at any given time. The remaining 10% of the rods behave comparably, though they typically exhibit slightly greater sliding. These findings justify modeling all the rods as a single beam with aggregated stiffness for tension and bending, while employing Huygens' formula to account for torsion. A particular attention is given to the rod-to-grid linkage. Six connection elements are used to represent contact and Coulomb friction between the equivalent rod and each grid. Obviously, each one of those connectors embed the 3 local stiffnesses which account for the local deformations of dimples and springs subjected to rod insertion and movements (see Fig.1). The skeleton in this model uses 3 beams and spring elements per grid to cope with the 6 translational and rotational stiffness linking 2 successive grids as well as the creep that affects the guide thimbles and their connection to the grids. The hold-down spring system smoothes the variations of the axial forces. It is characterized by plasticity and friction, represented here through a non-linear behavior applied on a single axial connector (see Fig. 2). Lastly, reversible and irreversible curvatures of rods and guide thimbles, related to thermal expansion gradients and axial irradiation growth gradients, are evaluated and integrated as free deformations within the pipe elements, primarily for outer assemblies.

The material laws used with the DACC code consider the following phenomena:

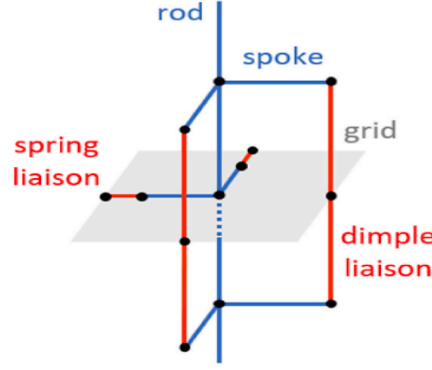


Figure 1. Rod-to-grid link using 6 connection elements (in red) between the grid mid plane (in grey) and the rod surface (from [5]).

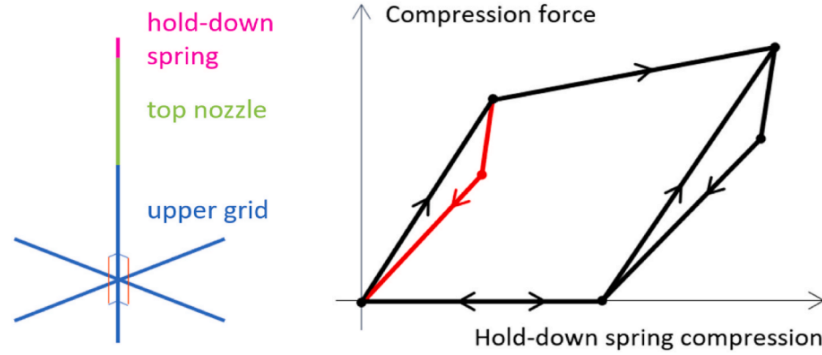


Figure 2. On the left, mesh of the upper part of the simplified fuel assembly model. On the right, cyclic axial load curve accounting for plasticity and friction of the hold-down system (supported by a connection element, in pink) (from [5]).

Neutron-Induced Growth of Guide Tubes: the growth is given by $\varepsilon_{\text{growth}} = C_{\text{growth}} G_t \cdot \phi^{g_t}$ where ϕ is the fast neutron fluence ($E > 1$ MeV), g_t and G_t are positive constants, $\varepsilon_{\text{growth}}$ is the axial deformation, and C_{growth} is the uncertain multiplicative constant for the growth law. The nominal parameters are $g_t = 0.5$, $G_t = 10^{-16} (n/m^2)^{-g_t}$, and the ϕ -domain goes from 0 to $10^{26} n/m^2$. Temperature sensitivity is neglected, although it exists to a minor extent within the useful range [7].

Fuel Rod Growth: it is the result of a number of complex phenomena and is practically represented by the average elongation curve given by industrial feedback [8]. It is described by $\varepsilon_{\text{growth}} = C_{\text{growth}} G_r \cdot \phi^{g_r}$, where ϕ is the fast neutron fluence and $\varepsilon_{\text{growth}}$ is the axial deformation. The nominal parameters are $g_r = 0.5$, $G_r = 5.7 \times 10^{-16} (n/m^2)^{-g_r}$.

Thermal Creep: this is neglected as it constitutes less than 10% of the total deformation from irradiation creep [7].

Irradiation Creep: for fuel rods and guide tubes, irradiation creep follows the same law used in the FRAPCON-4.0 code for RXA material [7]: $\dot{\varepsilon}_{\text{creep}} = C_{\text{creep}} \cdot \varphi^{C_1} \cdot \sigma_{\text{eff}}^{C_2} \cdot f(T)$ where φ is the fast neutron flux ($n/m^2.s$), σ_{eff} is the effective stress (Mpa), T is the temperature (K) and C_{creep} is the uncertain multiplicative constant for the creep law and $\dot{\varepsilon}_{\text{creep}}$ is the creep rate.

Elasticity: Elasticity is assumed to be linearly dependent on temperature with a negligible uncertainty.

3. SENSITIVITY ANALYSIS OF THE MECHANICAL MODEL DURING A ONE MONTH POWER STAGE

First, the study focuses on the non-linear material behavior, starting from an undeformed initial state with constant hydraulic force over a short duration (1 month). The loading procedure follows these steps: the first seven days of the simulation are dedicated to setting the loads to the mechanical model. The sequence consists of first tightening the rods, applying the weight and Archimede's load. Then the top plate boundary conditions are set and the hydraulic loads are applied to the mechanical model, the neutron flux and temperature are activated on Days 7-37, and deactivated on Days 38-40, the top plate is lifted on Day 41. The grid clamping force are modeled by an exponential distribution between 1 and 40 N with rapid decay to favor lower clamping forces (see Figure 3). The output of interest is the grid velocity of the grid G_z during irradiation $V_{G_z} = (U_{G_z,37} - U_{G_z,7})/30$ where $U_{G_z,j}$ is the displacement of the grid G_z at day j .

3.1. Uncertain parameters of the mechanical model in the first study

Table I lists the uncertain input parameters for the mechanical model, summarized as follows: the inlet core temperature, the fast neutron flux, the grid clamping force, the multiplicative constants for the creep and growth laws (C_{creep} and C_{growth} , respectively), the forces of the holding system (MSI), and finally, the hydraulic forces. Initially, hydraulic forces across all grids are considered. Based on prior hydraulic calculations detailed in [9] a Gaussian distribution for the hydraulic forces was admitted, with the hyperparameters estimated using the maximum likelihood method [10] (see Figure 4).

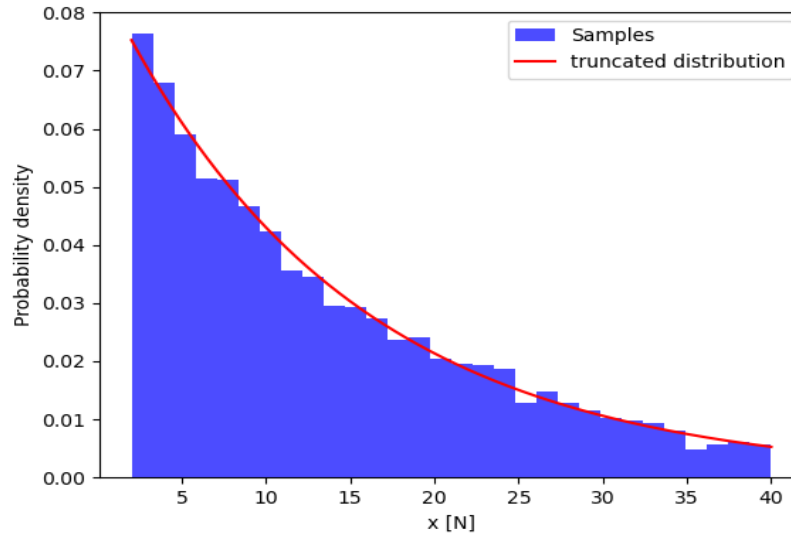


Figure 3. Clamping force presented by a truncated exponential distribution between 1 and 40 N with parameter $\lambda = 0.07 \text{ N}^{-1}$.

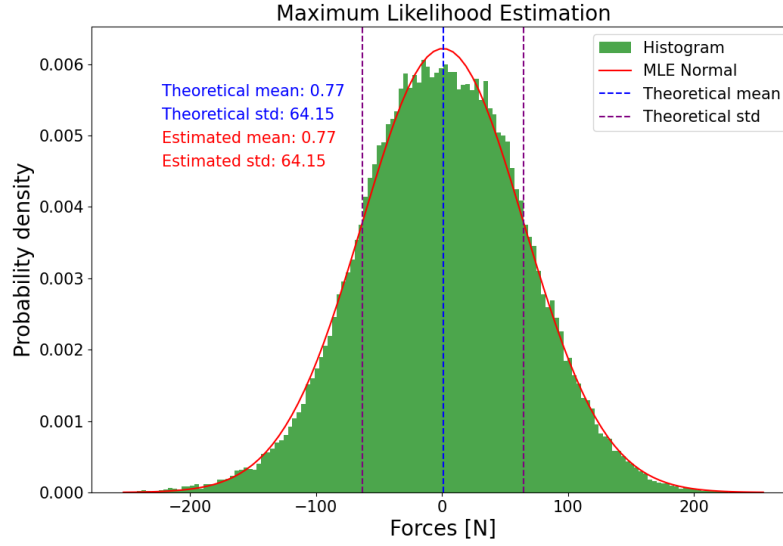


Figure 4. Distribution of the lateral hydraulic force based on the prior information from [9].

Parameter	Probability Distribution	References
$\vec{F}$ (N)	$\mathcal{N}(\mathbf{0}, \sigma_F^2 \mathbf{Id}_{11}), \sigma_F = 62$	Figure 4
T_{in} (K)	$\mathcal{N}(\mu = 553.13, \sigma = 3.33)$	[11]
φ (n/m ² .s)	$\mathcal{N}(\mu = 0.8 \times 10^{18}, \sigma = 0.03\mu)$	[12]
<i>grid_clamping</i> (N)	$exp(1, 40, \lambda = 0.07)$	Figure 3
<i>MSI</i> (N)	$\mathcal{U}(4000, 10000)$	-
C_{creep}	$\mathcal{N}(\mu = 1, \sigma = 0.3)$	-
C_{growth}	$\mathcal{N}(\mu = 1, \sigma = 0.3)$	-

Table I. Table of uncertain input parameters with their distributions and the references on which these values are based. $\mathcal{N}(\mu, \sigma)$ stands for the normal distribution with mean μ and standard deviation σ , $exp(a, b, \lambda)$ is the exponential distribution between a and $b \in \mathbb{R}$ with parameter λ and $\mathcal{U}(c, d)$ is the uniform distribution between c and $d \in \mathbb{R}$.

3.2. Results

Figure 5 illustrates the spatial distribution of the normalised first order Sobol indices (see appendix A) for grid displacement rate, presented in a pie chart. This visual representation allows for a detailed analysis of the most influential parameters within each grid of the assembly, thereby providing a physical understanding to the study. According to the results, the Sobol indices highlight the significant influence of hydraulic forces denoted as F_{G_z} , where $z = 2, \dots, 9$, on lateral deformations of the assemblies. Additionally, they reveal that grid clamping has a secondary influence following hydraulic forces. On the other hand, the effects of creep and growth are less pronounced but more significant in the middle grids.

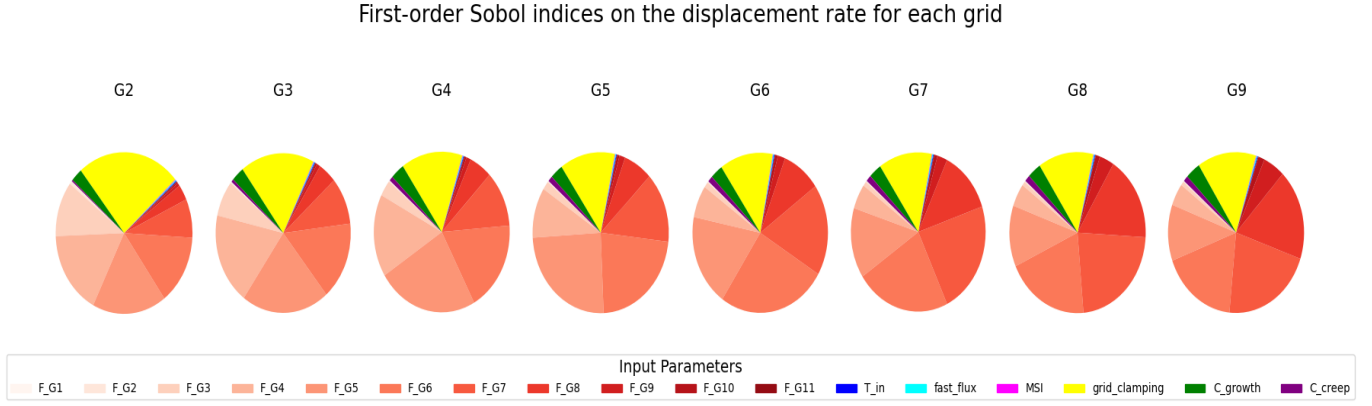


Figure 5. Sobol indices on the displacement rate for each grid.

4. A SPECIFIC STUDY TO HIGHLIGHT THE CREEP AND GROWTH PHENOMENA

The study conducted in Section 3 is resumed using the same uncertain parameters (seen in Table I) and loading sequence. However, the hydraulic forces parameter is updated, reducing it to an uncertain multiplicative constant ($F_h \sim \mathcal{N}(\mu = 1, \sigma = 0.2)$) for the hydraulic force profile seen in Figure 7. The quantity of interest remains the grid velocities, and the analysis focuses on the output for grids 2 to 9.

4.1. Results

According to the results in Figure 6, the variability in grid displacement rate, hydraulic forces remain the predominant parameter, contributing to fifty percents of the variance, with grid clamping, creep, and growth collectively accounting for the remaining variance. Notably, grid clamping exerts a more pronounced effect on the lower and upper grids compared to the central grids, while creep significantly impacts the central grids, in contrast to growth, which has a lesser impact. The holddown force (MSI) influence in this context is minimal.

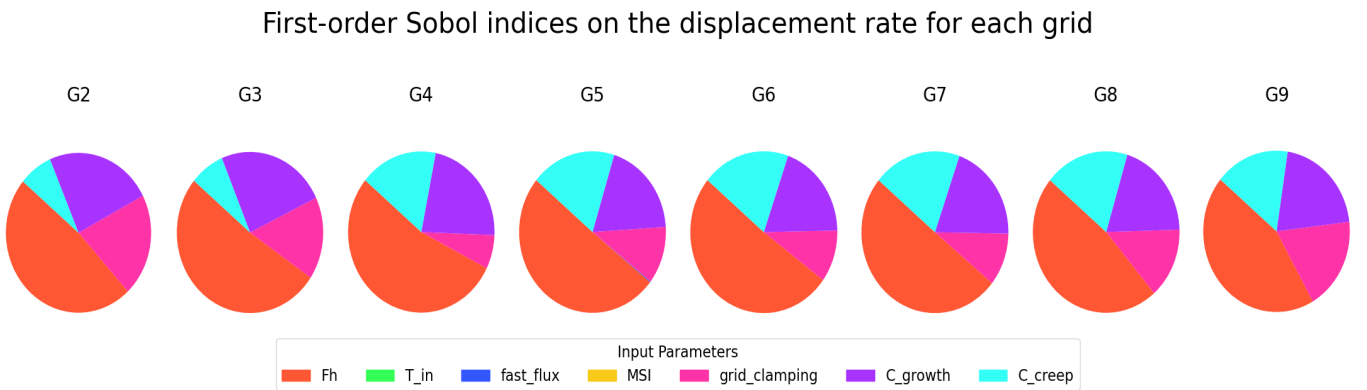


Figure 6. Sobol indices on the displacement rate for each grid with constant grid clamping.

4.2. A specific study to highlight the effects of creep and growth

The second study will delve deeper into a specific case during full operation, with a specified hydraulic force pattern with uncertain intensity, to highlight the effects of creep and growth. To achieve this, preliminary mechanical calculations were performed with constant hydraulic forces (see Figure 7). In the first calculation, creep was activated and growth was deactivated. In the second calculation, growth was activated and creep was deactivated. The third calculation considered both phenomena simultaneously. The output of interest is the displacement of grid 6 over 41 days (including one month of irradiation) with constant grid clamping in Figure 8, and variable grid clamping in Figure 9.

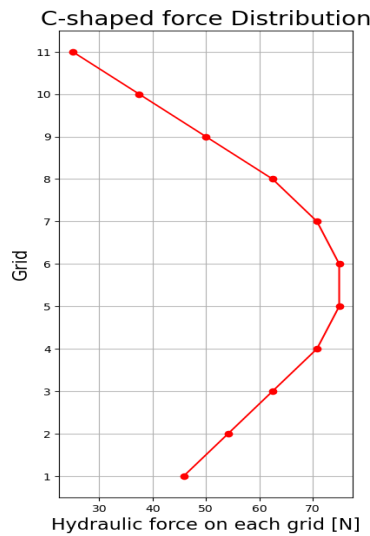


Figure 7. Chosen lateral hydraulic force on grids.

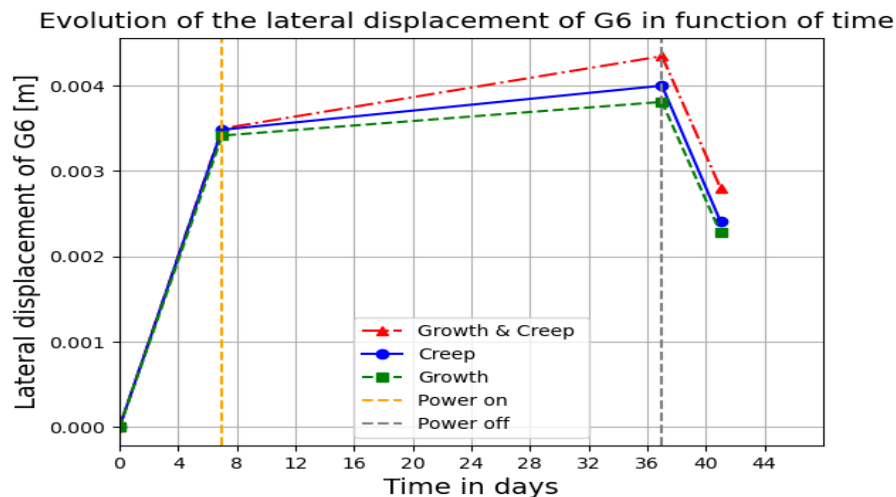


Figure 8. Evolution of the lateral displacement of grid 6 with constant grid clamping.

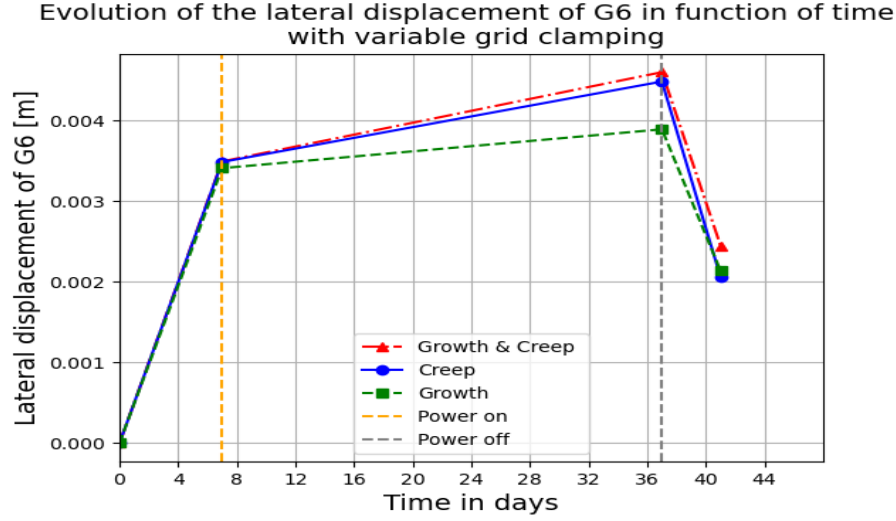


Figure 9. Evolution of the lateral displacement of grid 6 with decreasing grid clamping.

Given the short duration of the calculations (1 month under constant hydraulic loading) compared to that of an actual irradiation cycle (12 to 24 months), the contribution of creep should be analysed in terms of displacement rate. It means that the final deformation amplitudes remain low, largely reflecting the initial elastic deformations that are essentially locked in, rather than being significantly amplified. At constant grid clamping, differential growth has a significant contribution on lateral displacement over time : axial sliding maintains the majority of the contact zones on the edge of the Coulomb cone. By activating the relaxation of grid clamping (decreasing clamping), this effect continuously lowers the friction limit and dominates the effect of the differential growth. Moreover, the competition between these two phenomena, as shown in Figures 8 and 9, highlights the importance of sensitivity analysis for this particular phenomenon.

5. ON THE TOTAL DISPLACEMENT VARIATION ON EACH GRID AFTER ONE CYCLE INCLUDING INITIAL DEFORMATION

In this section, the analysis centers on a factor of significant industrial relevance, the total displacement variation on each grid ($U_{G_z,341} - U_{G_z,0}$) of the fuel assembly, after 11 months irradiation cycle. The focus is on a fuel assembly initially in a deformed state. A sensitivity analysis, employing new input parameters, is conducted, and these parameters are presented in the subsequent section.

In this study, as previously mentioned, the initial deformations of the assembly have been considered. When an assembly deforms, it generally exhibits three deformation modes, denoted as C , S , and W , as described in the literature [1] and roughly correspond to the natural modes of a clamped-clamped beam. An orthogonal basis of these three modes is used to represent the initial deformations of the assembly. The displacements are then projected onto this three-mode basis, yielding an expression for the grid displacements as follows: $U_{G_z} = \sum_{i=1}^3 Def_i U_i(z)$ (m), where z is the grid level and i is the index of the C , S , and W modes with profiles U_C , U_S , U_W , with Def_i being their modal coefficients denoted by Def_C , Def_S , and Def_W in Table II.

5.1. Dimension reduction applied to the description of hydraulic forces

Based on the hydraulic calculations conducted in [9], and to align with future work coupling hydraulics and structures, dimensionality reduction was applied to the hydraulic forces using Principal Component Analysis (PCA) [10].

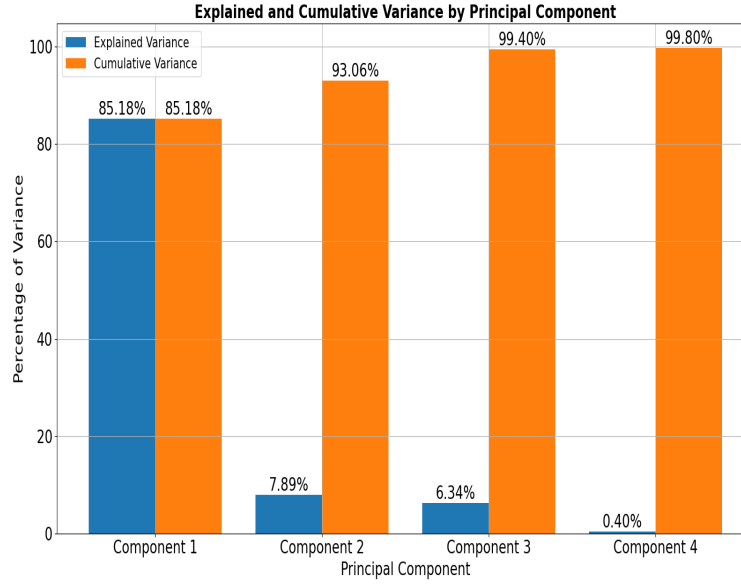


Figure 10. Explained and cumulative variance by the principal component analysis of hydraulic forces.

According to the results in Figure 10, the hydraulic forces can be decomposed into a basis of four orthogonal modes, which exhibit the spatial forms shown in Figure 11. This decomposition allows the hydraulic force field to be expressed as a linear combination of these four modes: $F_{G_z} = \sum_{i=1}^4 F_i S_i(z)$ (N), where z is the grid level, $i = 1, 2, 3, 4$ is the mode index (mode $i = 1$ corresponds to the redistributions induced by the core inlet/outlet profiles, $i = 2, 3, 4$ correspond to C , S , and W modes largely generated by the deformations of the assemblies themselves), $S_i(z)$ is the z -profile of mode i , and F_i is the amplitude of mode i .

5.2. Results

Based on the sensitivity analysis of the total displacement variation on each grid ($U_{G_z,341} - U_{G_z,0}$) of the fuel assembly (see Figure 12), the significant impact of hydraulic forces on the deformation of the assembly is highlighted. It is evident that the F_1 mode of hydraulic forces predominantly influences the lateral deformation of the assemblies. The major impact of hydraulic forces is accompanied by the effects of creep and the grids clamping, with a slight advantage to the former. Conversely, the effect of initial deformations is minimal, showing a slight predominance of mode S on the lower grids and a slight predominance of mode C on the middle grids. It means that the initial deformation of the assembly is completely transferred into the final deformation since the study is focused on the displacement instead of the position.

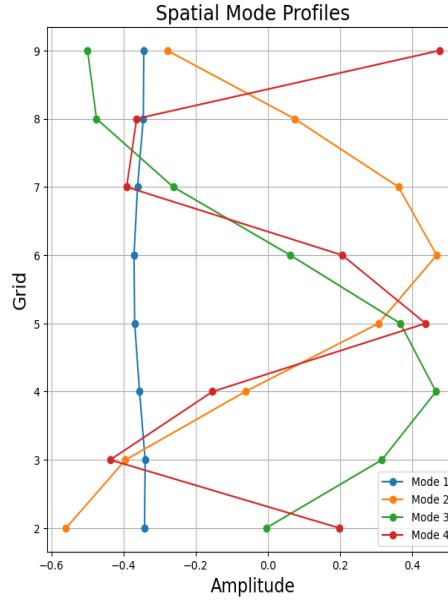


Figure 11. Spatial distribution of the 4 principal components of hydraulic forces along a fuel assembly. The distribution are normalized so that $\sum_{i=1}^9 U_i(z)^2 = 1$.

Parameter	Probability distribution	References
Def_C	$\mathcal{N}(\mu = 0, \sigma = 0.002)$	-
Def_S	$\mathcal{N}(\mu = 0, \sigma = 0.002)$	-
Def_W	$\mathcal{N}(\mu = 0, \sigma = 0.001)$	-
F_1 (Mode 1)	$\mathcal{N}(\mu = 0, \sigma = 100)$	PCA results
F_C (Mode C)	$\mathcal{N}(\mu = 0, \sigma = 80)$	PCA results
F_S (Mode S)	$\mathcal{N}(\mu = 0, \sigma = 60)$	PCA results
F_W (Mode W)	$\mathcal{N}(\mu = 0, \sigma = 30)$	PCA results
C_{creep}	$\mathcal{N}(\mu = 1, \sigma = 0.3)$	-
C_{growth}	$\mathcal{N}(\mu = 1, \sigma = 0.3)$	-
T_{in} (K)	$\mathcal{N}(\mu = 553.13, \sigma = 3.33)$	[11]
φ ($n/m^2.s$)	$\mathcal{N}(\mu = 0.8 \times 10^{18}, \sigma = 0.03\mu)$	[12]
$grid_clamping$ (N)	$exp(1, 40, \lambda = 0.07)$	-
MSI (N)	$\mathcal{U}(4000, 10000)$	-

Table II. Updated table of uncertain input parameters including initial deformations and modal coefficients of hydraulic forces.

6. PHYSICAL INTERPRETATION AND DISCUSSION

The initial thermomechanical calculations performed on an assembly over a short period confirm that many factors influence the deformation of an assembly during a power plateau. The analysis becomes easier by rationalizing the initial deformations and hydraulic forces using a PCA (Principal Component Analysis). Over the long term of a complete 12-month power cycle and under the highly simplifying assumption of a

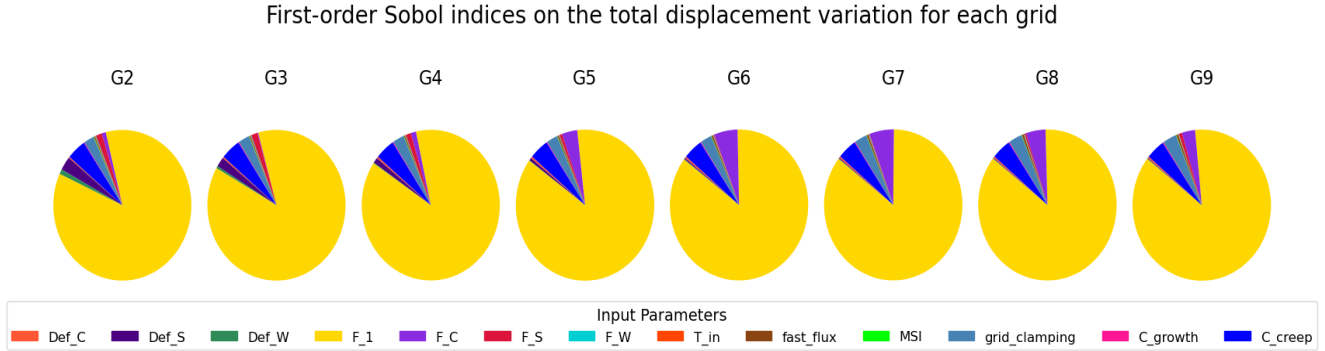


Figure 12. Sobol indices on the total displacement variation for each grid after one year with decreasing grid clamping.

constant hydraulic force field, the most important parameter by far becomes the intensity of the first mode of lateral hydraulic forces. This indicates the major and almost constant effect during irradiation of the axial speed profiles at the lower and upper core plates, which thus largely determine the final result. Additionally, the disturbance of the hydraulic force field in modes *C* and *S* plays a significant role, whereas the forces in mode *W* have a negligible contribution. This confirms the necessity of subsequently performing a sensitivity analysis of the fluid-structure coupling that balances the real-time deformations of the assembly and the coolant flows. The other external force, namely the clamping force of the hold-down force, contributes very little to the variance in grid displacements.

Regarding the assembly resistance to these external forces, the important variability of the irradiation creep parameter significantly contributes to the variability in grid displacement evolution. The temperature dependence of this parameter only moderately affects the result variability. Next in terms of contribution are two competing phenomena that facilitate gradual slips between the rods and grids, thereby promoting lateral deformations of the assembly during irradiation under mechanical load: the elongation of the rods relative to the skeleton, and the relaxation of the grid springs. These two phenomena affect the friction state at the contact points between each rod and each grid in distinct ways. The axial elongation of the rods shifts the local balance of friction forces on the friction cone, making potential tangential slippage of the rod easier (non-elastic rotation of the rod in the grid requires tangential slips over some dimples). Meanwhile the second phenomenon, grid spring relaxation, leads to a gradual reduction in the friction criterion with the same effect. This second phenomenon is far more significant than the first, at least in the first power cycle, which explains why rod growth had never been clearly identified before. Finally, it is noteworthy that initial deformations generate little variances in relative displacements, they contribute almost one-to-one to the final positions of the grids in this type of analysis under a constant hydraulic force field.

7. PROOF OF CONCEPT FOR A THERMOMECHANICAL SURROGATE MODEL

This section presents a methodology for metamodeling the variations in grid displacements using three Gaussian processes (GP) [10], applied to an isolated fuel assembly. These Gaussian processes serve to model the evolution of deformation by leveraging the dataset provided in Table II, which contains key modal information. The objective of this approach is to predict the modal amplitudes associated with the three primary deformation modes—*C*, *S*, and *W*—as described in Section 5. The modeling is performed for two distinct phases: first, the pre-irradiation stage (Day 7) and post-irradiation stage (Day 41), and second, the irradiation stage itself, which spans from Day 7 to Day 41. Specifically, the first and third Gaussian processes

estimate the modal amplitudes at these fixed time points, while the second Gaussian process is designed to predict the modal velocity, providing insights into the temporal evolution of deformations during irradiation.

The effectiveness of this approach is illustrated in Figures 13, 14 and 15, which highlights the accuracy of GP predictions for the modal amplitudes of deformation mode C at Day 7 (C_7) and Day 41 (C_{41}), as well as the corresponding modal velocity (V_C) during irradiation. The high correlation between predicted and actual values confirms the robustness of the GP-based metamodeling technique.

The prediction quality, quantified through correlation metrics, consistently exceeds 0.9 across all three cases for mode C , demonstrating the reliability of the approach. A similar level of accuracy is achieved for deformation modes S and W , where the predicted modal amplitudes at Day 7 (S_7 , W_7) and Day 41 (S_{41} , W_{41}), as well as their corresponding modal velocities (V_S , V_W) during irradiation, exhibit the same high precision. However, to avoid redundancy, only the results pertaining to mode C are presented in detail, while the findings for modes S and W follow the same pattern.

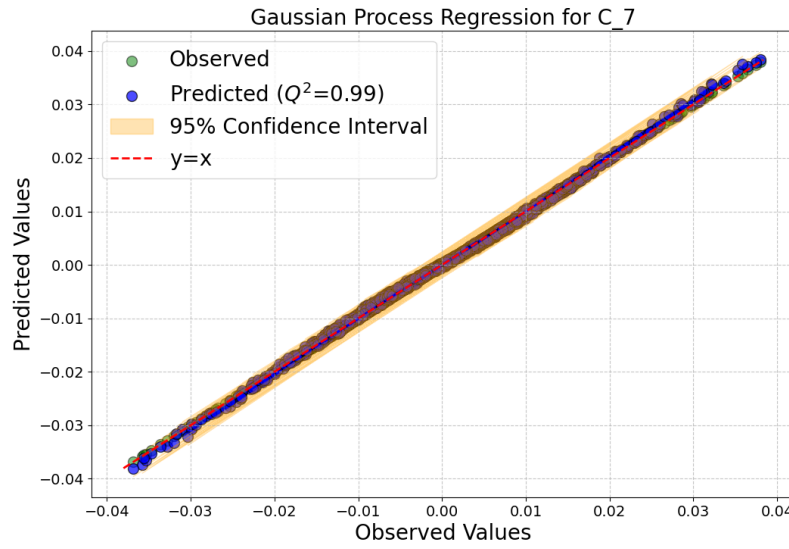


Figure 13. Gaussian Process prediction on the modal amplitude C_7

8. CONCLUSIONS

This study focuses on sensitivity analysis of the thermomechanical response of a single fuel assembly modeled using finite elements method. It follows a previous study [9] that detailed the topology and variance of flows within the core. This analysis is a step toward implementing a future sensitivity analysis of fluid-structure coupling at the scale of an assembly line, or even a complete core. In this study, the variance of grid displacements was quantified using the Sobol method. The parameter that most predominantly influences the variation in assembly bow calculations is the first mode of hydraulic forces, strongly controlled by the hydraulic calculation boundary conditions and, therefore, underlines the great importance of improving them in hydraulic calculations. Three parameters then have equal influence: the creep parameter, the C-shape hydraulic force (related to fluid-structure interactions) and the rod clamping force. The last significant one appears to be the S-shaped forces. It was highlighted that other parameters are significantly less influential (holddown force) or less uncertain (flux, temperature). Furthermore, it was observed that the effect of the differential elongation of rods relatively to the skeleton competes with the effect of spring relaxation during

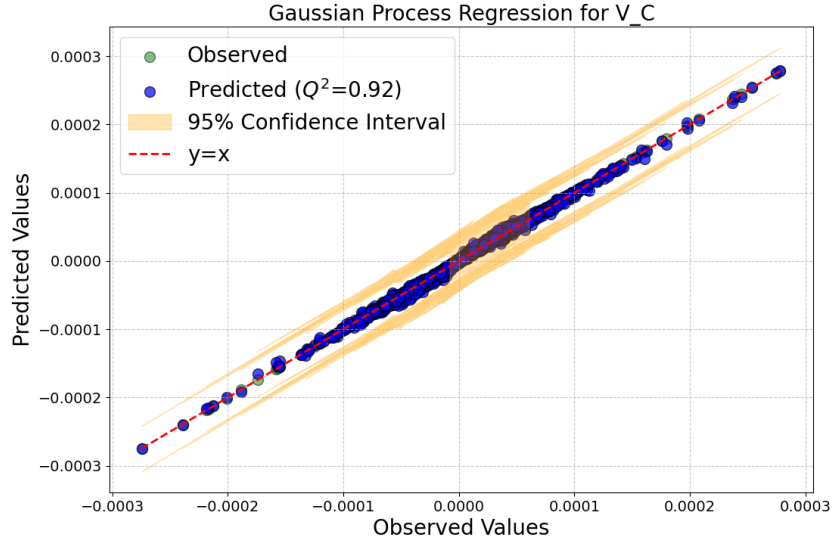


Figure 14. Gaussian Process prediction on the modal velocity V_C

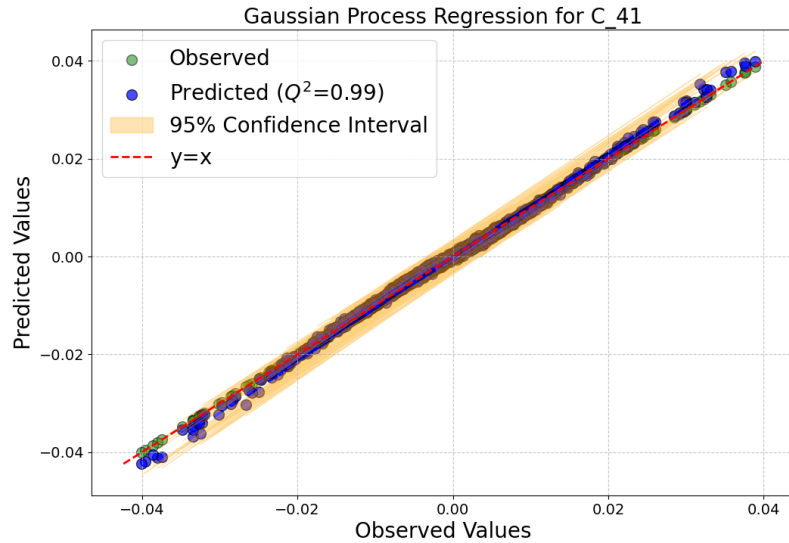


Figure 15. Gaussian Process prediction on the modal amplitude C_{41}

the irradiation phase, meaning they are not cumulative. Thus, for the upcoming sensitivity analysis of fluid-structure coupling on an assembly line, particular attention must be given to the choice of hydraulic boundary conditions, ideally considering the coupling effect of plenums in the calculations. Additionally, the initial deformation state of assemblies, which tends to carry over one-to-one into the residual deformation after a cycle, should be considered over multiple irradiation cycles, meaning accounting for the repositioning strategy of assemblies. Finally, the possibility of metamodeling the evolution of fuel assembly deformations during different phases of a cycle using Gaussian processes with satisfactory precision has been highlighted.

This could be useful in future studies of FSI coupling to save on resolution time and/or extend design of experiment (DoE) and number of simulations size.

REFERENCES

- [1] T. Andersson, J. Almberger, and L. Björnkvist. ““A Decade of Assembly Bow Management at Ringhals”.” In *IAEA-TECDOC-1454 Structural Behaviour of Fuel Assemblies for Water Cooled Reactors*, pp. 129–136. IAEA, Vienna, Austria (2005).
- [2] S. de Lambert, G. Campioni, V. Faucher, B. Leturcq, and J. Cardolaccia. “Modeling the consequences of fuel assembly bowing on PWR core neutronics using a Monte-Carlo code.” *Annals of Nuclear Energy*, **volume 134**, pp. 330–341 (2019).
- [3] B. Leturcq. *Réductions de modèles thermomécaniques non-linéaires pour l'évaluation des déformations d'un cœur de réacteur nucléaire*. thesis, Institut Polytechnique de Paris (2022).
- [4] “Coulomb Friction.” <https://www.sciencedirect.com/topics/engineering/coulomb-friction>. Accessed: 2024-07-18.
- [5] S. de Lambert, J. Cardolaccia, V. Faucher, B. Leturcq, and G. Campioni. “Semi-analytical fluid-structure model for the analysis of fuel assembly bow in full PWR cores.” *Progress in Nuclear Energy*, **volume 160**, p. 104668 (2023).
- [6] A. Wanninger, M. Seidl, and R. Macián-Juan. ““Mechanical analysis of the bow deformation of a row of fuel assemblies in a PWR core”.” *Nuclear Engineering and Technology*, **volume 50**, pp. 297–305 (2018).
- [7] K. Geelhood, W. Luscher, P. Raynaud, and I. Porter. “FRAPCON-4.0: A Computer Code for the Calculation of Steady-State, Thermal-Mechanical Behavior of Oxide Fuel Rods for High Burnup.” *Pacific Northwest, National Laboratory Richland, Washington* (2015).
- [8] J. P. Mardon, P. B. Hoffmann, and G. L. Garner. “High burnup behavior and licensing of alloy M5™.” In *Proceedings of the 2005 Water Reactor Fuel Performance Meeting*, p. 1236. Atomic Energy Society of Japan, Japan (2005).
- [9] A. Abboud, J. Garnier, B. Leturcq, S. de Lambert, and N. , Lamorte. “Sensitivity analysis of a flow redistribution model for a multidimensional and multifidelity simulation of fuel assembly bow in a pressurized water reactor.” In *Best Estimate Plus Uncertainty (BEPU): Status and perspectives*, (Nuclear Engineering and Design, p. 272. Lucca, Italy (2024).
- [10] C. E. Rasmussen and C. K. I. Williams. “*Gaussian Processes for Machine Learning*”, volume 2. MIT Press, Cambridge, MA (2006).
- [11] J. Jang, C. Kong, B. Ebiwonjumi, A. Cherezov, Y. Jo, and D. Lee. ““Uncertainty quantification in decay heat calculation of spent nuclear fuel by STREAM/RAST-K”.” *Nuclear Engineering and Technology*, **volume 53**, pp. 2803–2815 (2021).
- [12] L. Clouvel. “*Uncertainty quantification of the fast flux calculation for a PWR vessel*”. thesis, Université Paris Saclay (2019).
- [13] A. Saltelli, S. Tarantola, F. Campolongo, and M. Ratto. “*Sensitivity Analysis in Practice: A Guide to Assessing Scientific Models*”. Wiley, New York (2004).

APPENDIX A. THE SOBOL METHOD

The goal is to analyze how uncertainty in the model output is distributed among the various sources of uncertainty in the input parameters [13]. This is done using Sobol sensitivity indices, which quantify the contribution of each input to the output variance.

Given a vector of independent input parameters $\mathbf{X} = (X^{(i)})_{i=1}^d$ and a scalar output Y , the first-order and total Sobol indices are defined as:

$$S_i = \frac{\text{Var}(\mathbb{E}[Y|X^{(i)}])}{V}, \quad T_i = 1 - \frac{\text{Var}(\mathbb{E}[Y|\mathbf{X}^{(-i)}])}{V}, \quad (1)$$

where $V = \text{Var}(Y)$ and $\mathbf{X}^{(-i)}$ represents all input variables except $X^{(i)}$. The first-order index S_i measures the contribution of input $X^{(i)}$ to the variance of Y , considering only its direct effect. The total index T_i captures both the direct effect of $X^{(i)}$ and all interactions with other inputs.

Since higher-order interactions contribute to the variance, the sum of first-order indices is generally less than one:

$$1 = \sum_{i=1}^d S_i + \sum_{1 \leq i < j \leq d} S_{ij} + \dots + S_{1,\dots,d}. \quad (2)$$

To represent S_i in a pie chart, it is normalized by dividing by the total sum of the first-order indices:

$$S_{\text{sum}} = \sum_{i=1}^d S_i, \quad S_i^{\text{norm}} = \frac{S_i}{S_{\text{sum}}}. \quad (3)$$

This ensures that the normalized indices sum to one, making them suitable for visual representation. S_{sum} is very close to 1 in Figures 5, 6 and 12, which means that the interaction effects between the input parameters are negligible.