Irreversible Thermalization vs Reversible Dynamics Mediated by Anomalous Correlators: Wave Turbulence Theory and Experiments in Optical Fibers

T. Torres,[1] J. Garnier,[2] L. Zanaglia,[3] M. Ferraro,[1,4] C. Michel,[3] V. Doya,[3] J. Fatome,[1] B. Kibler,[1] S. Wabnitz,[5] A. Picozzi,[1] and G. Millot[1]

[1]*Université Bourgogne Europe, CNRS, Laboratoire Interdisciplinaire Carnot de Bourgogne ICB UMR 6303, 21000 Dijon, France*
[2]*CMAP, CNRS, Ecole polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France*
[3]*Université Côte d'Azur, CNRS, Institut de Physique de Nice, Nice, France*
[4]*Department of Physics, University of Calabria, Via Pietro Bucci, Rende 87036, (CS), Italy*
[5]*Department of Information Engineering, Electronics, and Telecommunications, Sapienza University of Rome, Via Eudossiana 18, Rome 00184, Italy*

(Received 19 December 2025; revised 13 February 2026; accepted 27 March 2026; published 29 April 2026)

We theoretically and experimentally investigate spontaneous self-organization in a conservative (Hamiltonian) turbulent wave system, operating far from thermodynamic equilibrium. Our system is governed by two coherently coupled nonlinear Schrödinger equations, describing the polarization evolution of light in a dispersive nonlinear optical fiber. The analysis reveals the emergence of two fundamentally distinct turbulent regimes. In a first regime, the waves undergo a slow, irreversible thermalization process, which is accurately described by the wave turbulence kinetic equation and the associated H theorem of entropy growth. In stark contrast with this expected irreversible process, we identify a second different regime, where strong phase correlations spontaneously emerge, giving rise to a fast reversible oscillatory dynamics of the normal correlator and anomalous phase correlator. Experimental observations confirm the occurrence of both irreversible thermalization and reversible dynamics mediated by the anomalous correlated fluctuations.

DOI: 10.1103/tgtw-qnyq

Introduction—Nonintegrable Hamiltonian systems of random waves generally undergo thermalization—an irreversible evolution toward thermodynamic equilibrium, characterized by a state of maximum entropy. In the weakly nonlinear regime, this process is accurately described by the wave turbulence (WT) theory [1–6], which has been successfully applied to various physical systems [7–12], including optical waves [13–18]. The WT kinetic equation (KE) describes the actual irreversible evolution to the Rayleigh-Jeans equilibrium distribution, which is expressed by an H theorem of entropy growth. Interestingly, light thermalization has recently been studied theoretically [19–21] and experimentally [21–24] in multimode fibers, thus spurring the emerging field of optical thermodynamics [19,25–35].

Although rigorous derivations of the WT KE have been accomplished for certain partial differential equations [36–38], the broader validity of the WT theory across a wide range of nonlinear wave-interacting partial differential equations remains unclear. A central requirement in deriving the KE is the demonstration that *phase correlations* among the random waves can be neglected.

Phase correlations are known to underlie large-scale coherent structures, such as solitons or condensates, that emerge from strongly nonlinear turbulent regimes [2–8,13,39–43]. These coherent structures are, by their nature, phase-correlated entities. In contrast, we will focus here on phase correlations in the *weakly nonlinear regime* of random dispersive waves.

Phase correlations—also referred to as "anomalous correlations"—were central to the original Bardeen-Cooper-Schrieffer theory of superconductivity [44]. They have since been extensively studied within the framework of S theory [45–47], where anomalous correlations arise from an external forcing (pumping) that parametrically excites a dissipative magnetic system [47]. Anomalous correlators connected to optical polarization were originally introduced in Ref. [48] to study turbulence of electromagnetic waves in isotropic plasma. From a different perspective, recent theoretical advances revealed the presence of phase correlations in conservative (unforced) wave systems. Anomalous phase correlations have been identified in Fermi-Pasta-Ulam-Tsingou chains [49], or in the coupled nonlinear Schrödinger equation (NLSE), arising from strong convection between wave components [50]. In *scalar* wave systems, the mechanism underlying the emergence of phase correlations between different frequencies has been recently elucidated by showing that they may only arise from Hamiltonian terms that break phase invariance, whereas systems that preserve phase invariance preclude such anomalous correlations [51].

In this Letter, we investigate a *vector* system of coherently coupled NLSEs, which describe the polarization evolution of temporally incoherent light waves in optical fibers. We identify two fundamentally different regimes: an irreversible thermalization, and a reversible turbulent dynamics driven by phase correlations. Unlike recent 2D spatial studies in multimode optical fibers [21–24], we examine here the 1D temporal dynamics of incoherent waves propagating in a single-mode fiber [14–18]. By using the standard WT theory (neglecting phase correlations), we describe nonequilibrium thermalization accurately. In contrast to this expected irreversible process, we show that phase correlations can grow exponentially from fully uncorrelated initial random waves, leading to a reversible oscillatory turbulent dynamics. These anomalous phase correlations emerge among different wave components in a phase-invariant vector system, hence they are of different nature from those investigated in Ref. [51]. Experimental results provide evidence of both irreversible thermalization and reversible phase-correlated dynamics. From a broader perspective, this Letter advances our understanding of far-from-equilibrium self-organization processes in closed (Hamiltonian) turbulent wave systems.

Model—We consider a generic model for vector phenomena based on coherently coupled NLSEs. It describes spinor Bose-Einstein condensates with field-induced spin-flip coupling [52], allowing studies of quantum phase transitions [53], or bubble nucleation in ferromagnetic superfluids [54]. The coherently coupled NLSEs also describe the polarization dynamics of an optical wave propagating in a weakly birefringent fiber [55,56],

$$i\partial_z \boldsymbol{u} = -\beta\partial_{tt}\boldsymbol{u} + \alpha\boldsymbol{\sigma}\boldsymbol{u} + \gamma(\kappa\boldsymbol{u}^\dagger\boldsymbol{u}\boldsymbol{u} + \rho\boldsymbol{u}^T\boldsymbol{u}\boldsymbol{u}^*), \quad (1)$$

where $\boldsymbol{u}(t,z) = (u_x, u_y)^T$ is the vector field in the linear polarization basis, and the superscripts $(T, \dagger)$ stand for the transpose and conjugate transpose operations. As usual in optics, the distance z of propagation plays the role of an evolution "time" variable, while t denotes the retarded time in a reference frame moving with the waves. γ is the nonlinear parameter, β is the group-velocity dispersion, while κ and ρ are dimensionless interaction coefficients; we will consider the case $\kappa = 2\rho$ relevant to our experiments. The coherent coupling parameter α originates from the weak birefringence of the optical fiber, with the matrix $\boldsymbol{\sigma} = \mathrm{diag}(+1,-1)$. We assume $\alpha > 0$ without loss of generality. The vector NLSE (1) conserves the power (particle number) $N = \sum_\mu N_\mu$, where $N_\mu(z) = (1/T)\int_0^T |u_\mu|^2 dt$, with T the size of the numerical window, and $\mu = x,\ y$. It also conserves the Hamiltonian $H = E + U$, with a linear E, and a nonlinear U, contribution [57].

Let us introduce the usual normal correlators $n_\mu(\omega,t,z) = \int \langle u_\mu(t+\tau/2,z)u_\mu^*(t-\tau/2,z)\rangle \exp(-i\omega\tau)d\tau \quad (\mu = x,\ y)$. The dependence of n_μ on the ("spatial") variable t accounts for possible statistical inhomogeneities in the random waves, while the angle brackets denote an average over the random initial conditions. Furthermore, the theorem in Ref. [51] for phase-invariant scalar systems can be extended to vector systems such as the vector NLSE (1), which implies the absence of anomalous correlations among distinct frequencies. However, as we show below, phase correlations may emerge between different wave (i.e., polarization) components at the same frequency: these will be characterized by the anomalous correlator $m(\omega,t,z) = \int \langle u_y(t+\tau/2,z) u_x^*(t-\tau/2,z)\rangle \exp(-i\omega\tau)d\tau$.

Standard WT theory: Irreversible thermalization—We consider the weakly nonlinear regime, where linear effects dominate over nonlinear effects $|E/U| \gg 1$. Using the standard WT theory [1–6,13,18], we derive the KE under the usual assumptions: (i) statistical homogeneity, so that n_μ are t-independent, and (ii) absence of phase correlations between u_x and u_y, i.e., the anomalous correlator is zero at any "time" z, $m(\omega,t,z) = 0$. The KE governing the evolution of the averaged spectra $\boldsymbol{n}(\omega,z) = (n_x,n_y)^T$ then takes the usual form,

$$\partial_z \boldsymbol{n}(\omega,z) = \kappa^2 \mathcal{C}oll_i[\boldsymbol{n}] + \rho^2 \mathcal{C}oll_c[\boldsymbol{n}]. \quad (2)$$

The collision terms are rather cumbersome; see Ref. [57]. They are cubic nonlinear terms $\mathcal{C}oll_{i,c}[\boldsymbol{n}] \sim n^3$, describing the four-wave interaction as a collisional gas of "particles." The KE (2) conserves $N = \sum_\mu N_\mu$, with $N_\mu(z) = [1/(2\pi)]\int n_\mu(\omega,z)d\omega \ (\mu = x,\ y)$, and the linear energy E [57]. At variance with the NLSE (1), the KE (2) is irreversible, as expressed by a H-theorem of entropy growth, $d\mathcal{S}/dz \geq 0$, where $\mathcal{S} = \sum_\mu \mathcal{S}_\mu$, and $\mathcal{S}_\mu(z) = [1/(2\pi)]\int \log(n_\mu(\omega,z))d\omega$ is the nonequilibrium entropy of the μth component ($\mu = x,\ y$).

We have performed numerical simulations of NLSE (1). As initial condition, we consider two *uncorrelated* random waves $u_{x,y}(t, z=0)$ with zero mean, Gaussian statistics, and Gaussian-shaped initial spectrum $n_x^0(\omega) = n_y^0(\omega)$, i.e., $N_x^0 = N_y^0$, with $n_\mu^0(\omega) = \sqrt{2\pi}N_\mu^0 \exp(-\omega^2/(2\sigma^2))/\sigma$. Let us introduce the degree of polarization of the optical wave [65],

$$\mathcal{P}(z) = \sqrt{\Delta N(z)^2 + 4|M(z)|^2}/N, \quad (3)$$

where $M(z) = [1/(2\pi)]\int m(\omega,z)d\omega = \langle u_y(t,z)u_x^*(t,z)\rangle$ denotes the integrated anomalous correlator, and $\Delta N(z) = N_y(z) - N_x(z)$ the unbalanced power distribution. Note that $\mathcal{P}$ is bounded by 0 and 1, which correspond to unpolarized and fully polarized waves, respectively.

We have also performed numerical simulations of the WT KE (2): as shown in Fig. 1, a good agreement (without adjustable parameters) is obtained with direct simulations of the NLSE (1). Recalling that the KE neglects phase correlations, such an agreement means that anomalous correlations do not grow, i.e., $M(z) \simeq 0$ and $\mathcal{P}(z) \simeq \Delta N(z)/N$. Accordingly, the thermalization process is

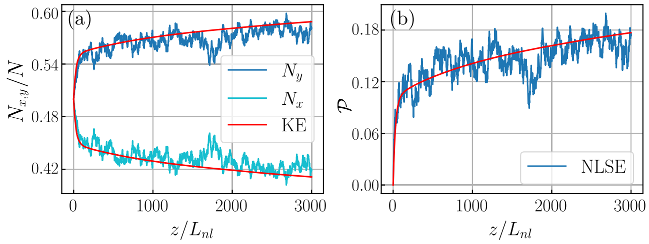

FIG. 1. Irreversible thermalization. Evolution during the propagation of the power fraction $N_{x,y}(z)/N$ (a), and corresponding degree of polarization $\mathcal{P}(z)$ [Eq. (3)] (b), obtained from the simulation of the NLSE (1) (5 realizations, dark and light blue lines), and the simulation of the WT KE (2) (red lines). Parameters: $\alpha L_{nl} = 1, \Delta N^0 = 0, \sigma\tau_0 = 0.8\pi, \kappa = 2/3$, with $L_{\rm nl} = 1/(\gamma N)$ the nonlinear length and $\tau_0 = \sqrt{|\beta| L_{\rm nl}}$ the "healing time" [$|E/U| \simeq (\sigma\tau_0)^2$].

characterized by an irreversible transfer of power from N_x to N_y, thus leading to the emergence of a nonvanishing degree of polarization [57].

Reversible dynamics mediated by phase correlations—We now show that the thermalization process can be inhibited by the spontaneous emergence of phase correlations. To clarify this regime in a general framework, we do not implicitly assume the waves obey homogeneous statistics, i.e., we leave $n_\mu(\omega, t, z)$ and $m(\omega, t, z)$ to depend on a slow nonhomogeneous variation in the t variable. Starting from the NLSE (1), we derive in Supplemental Material the equations governing the coupled evolution of the normal and anomalous correlators [57],

$$\partial_z n_x(\omega, t) = -2\beta\omega\partial_t n_x + \gamma\partial_t(2N_x(t) + \kappa N_y(t))\partial_\omega n_x + 2\gamma\kappa\partial_t M^r(t)\partial_\omega m^r + 4\gamma\kappa M^r(t) m^i, \quad (4)$$

$$\partial_z m(\omega, t) = -2\beta\omega\partial_t m + \gamma(1+\kappa/2)\partial_t(N_x(t) + N_y(t))\partial_\omega m + \gamma\kappa\partial_t M^r(t)\partial_\omega(n_x + n_y) - im(\gamma(2-\kappa)(N_y(t) - N_x(t)) - 2\alpha) - 2i\gamma\kappa(n_x - n_y)M^r(t), \quad (5)$$

with $M(t, z) = [1/(2\pi)]\int m(\omega, t, z)d\omega$, $N_\mu(t,z) = [1/(2\pi)]\int n_\mu(\omega,t,z)d\omega$, while the superscripts $m^{r,i}$ ($M^{r,i}$) denote the real and imaginary parts of m (M). The equation for n_y follows from Eq. (4) by exchanging $x \leftrightarrow y$ and reversing the sign of the last term.

The first two terms on the right-hand side of Eq. (4) correspond to the well-established Vlasov coupling between two *uncorrelated* random waves. Here, the generalized Vlasov-like Eqs. (4) and (5) provide an original extension accounting for phase correlations, $m(\omega, t)$. In contrast to the KE (2), the system [(4) and (5)] is formally reversible in 'time' z [57]. To avoid confusion with the usual KE (2), we shall refer to Eqs. (4) and (5) [and Eq. (8) below] as the anomalous-correlator kinetic equation (AC-KE).

We consider an initial condition of two uncorrelated random waves, of zero mean, with homogeneous statistics and spectra $n_\mu(\omega, z=0) = n_\mu^0(\omega)$, so that the initial anomalous correlator is zero, $m(\omega, t, z=0) = 0$. We carry out a linear stability analysis of Eqs. (4) and (5) around this state. Using the Laplace-Fourier transform $\hat{m}(\omega, \Omega, \lambda) = \int_0^z dz \int m(\omega, t, z)\exp(-\lambda z - i\Omega t)dt$, we obtain the dispersion relation $\lambda(\Omega)$ of the anomalous correlator [57],

$$\frac{2\pi}{\gamma\kappa} = \sum_s \int \frac{n_x^0(\omega) - n_y^0(\omega) - s\Omega\partial_\omega(n_x^0(\omega) + n_y^0(\omega))/2}{is\lambda - 2s\beta\omega\Omega - \gamma(2-\kappa)\Delta N^0 + 2\alpha} d\omega,$$

where $\Delta N^0 = N_y^0 - N_x^0$, and $s = \pm 1$. Assuming the initial spectra Gaussian-shaped, we compute the corresponding growth rate $\mathrm{Re}[\lambda(\Omega)]$. The analysis reveals that, in general, the homogeneous mode with $\Omega = 0$ is the most unstable, with the maximum growth rate of the anomalous correlator,

$$\lambda(\Omega = 0) = 2\sqrt{\alpha(2\gamma\Delta N^0/3 - \alpha)}, \quad (6)$$

where we have considered the case $\kappa = 2/3$ relevant to our experiments. The instability criterion then reads

$$\alpha L_{\rm nl} < (2/3)\Delta N^0/N, \quad (7)$$

where $L_{\rm nl} = 1/(\gamma N)$ denotes the nonlinear length. Simulations of NLSE [Eq. (1)] confirm the theoretical prediction [Eq. (6)], as illustrated in Fig. 2(a). The figure shows 100 independent realizations of the initial uncorrelated random waves $u_{x,y}(t, z=0)$ (black lines), whose ensemble average (green line) agrees with the instability growth rate (6) (red line).

The fact that the homogeneous mode $\Omega = 0$ is the most unstable, indicates that the subsequent nonlinear dynamics described by Eqs. (4) and (5) preserves the homogeneous statistics, i.e., $n_\mu(\omega, t)$ and $m(\omega, t)$ are t-independent. In this limit, the dynamics for the normal and anomalous correlators (4) and (5) can be recast in the form of a Stokes vector $\boldsymbol{S}(z) = (\Delta N, 2M^r, 2M^i)^T$ that rotates on the surface of the Poincaré sphere [57],

$$d_z\boldsymbol{S}(z) = \boldsymbol{R}(z) \times \boldsymbol{S}(z), \quad (8)$$

with $\boldsymbol{R}(z) = (2\alpha - \gamma(2-\kappa)S_1(z), -2\gamma\kappa S_2(z), 0)^T$. This formalism differs substantially from that commonly used to describe polarization effects for fully coherent stationary waves [56,66]. Conversely, a related kinetic approach for random waves accounting for anomalous correlations was originally developed in the context of isotropic plasma turbulence, where nonlinear interactions due to stimulated scattering drive the electromagnetic field toward complete polarization [48] (we recall that classical Thomson scattering likewise produces fully polarized light at a scattering angle $\pi/2$ [67]). Note, however, that the increase of $\mathcal{P}(z)$

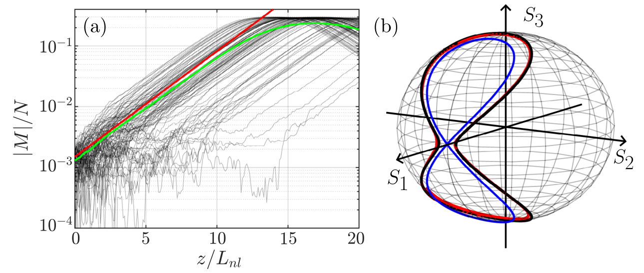

FIG. 2. Reversible turbulent dynamics. (a) Exponential growth of the anomalous correlator starting from initially uncorrelated random waves $|M(z=0)| \simeq 0$: simulations of the NLSE (1) (100 realizations, black lines), and corresponding average (green line). The red line reports the theoretical prediction [Eq. (6)]. (b) Nonlinear dynamics on the Poincaré sphere: simulation of the NLSE (1) (red line), and corresponding theoretical prediction from the simulation of AC-KE (8) (black line), showing the periodic exchange among the normal and anomalous correlators. The blue line reports the homoclinic orbit emanating from the unstable fixed point $S_1 = +S_0$, i.e., $|M| = 0$. Parameters: $\alpha L_{\rm nl} = 0.2$, $\sigma\tau_0 = 4\pi$, $\Delta N^0/N = 0.6$, $\kappa = 2/3$, $L_{\rm nl} = 1/(\gamma N)$, $\tau_0 = \sqrt{|\beta| L_{\rm nl}}$ ($|E/U| \simeq (\sigma\tau_0)^2$).

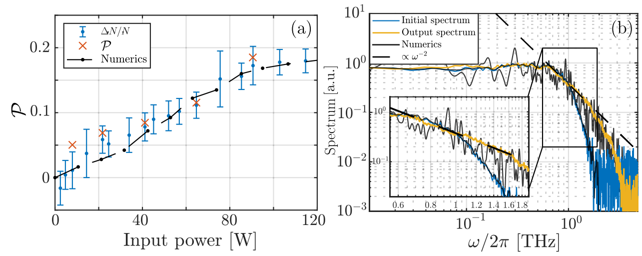

FIG. 3. Observation of optical repolarization. (a) Experimental measurements of the normalized power difference $\Delta N/N$ (blue dots), and degree of polarization $\mathcal{P}$ (red crosses), vs input power: In the thermalization regime, the anomalous correlator M is negligible, and $\mathcal{P} \simeq \Delta N/N$; see Eq. (3). The measured repolarization is in good agreement with the simulations of NLSE (1) (dashed black lines). (b) Measured power spectrum at the input (blue line), and output (orange line), of the fiber. The experimental spectra agree well with the simulation of NLSE (1) (black lines), and exhibit a power-law signature of light thermalization $\sim\omega^{-2}$ in the spectral tail.

discussed above through Fig. 1 is of different nature, since it is associated with the thermalization process that neglects anomalous correlations. Furthermore, at variance with Ref. [48], the kinetic Eq. (8) accounting for anomalous correlators conserves the degree of polarization during evolution $\mathcal{P}(z) = \text{const}$, as it is fixed by the radius of the Poincaré sphere $S_0 = (\sum_{j=1}^3 S_j^2)^{1/2} = \mathcal{P}N$. It turns out that the nonlinear dynamics described by the AC-KE (8) is essentially periodic, featuring oscillations that involve a reversible exchange between the normal correlator $\Delta N(z)$ and the anomalous correlator $M(z)$, while conserving

$$\mathcal{P}^2 N^2 = \Delta N^2(z) + 4|M(z)|^2 = \text{const} \qquad (9)$$

This reversible dynamics is in agreement with the simulations of the NLSE (1); see Fig. 2(b). Note that the *fast*, reversible dynamics, governed by the quadratic nonlinearities in the AC-KE (4), (5), and (8) in Fig. 2, stands in contrast to the *slow*, irreversible thermalization driven by the cubic nonlinear KE (2) in Fig. 1.

Experiments—We have experimentally investigated the two different turbulent regimes that we have previously described. We use 100-ps long pulses made of temporally incoherent optical waves with a near Gaussian beam shape and the following spectrotemporal features: 558-nm central wavelength, full width at half maximum of about 1.93 THz [57]. The temporally incoherent pulse is then divided into two linear polarization states with relative tunable power and temporal delay (much larger than their coherence time). The incoherent waves are then injected into a 6.2-m-long weakly birefringent silica fiber ($\alpha \approx 0.565$ m^{-1} at 558 nm) whose propagation is governed by Eq. (1) [56]. The complex anomalous correlator M, as well as the optical powers $N_{x,y}$ and spectra, along x and y axes, are measured at the input and output of the fiber using an optical spectrum analyzer and a polarimeter.

Experiments on the thermalization regime: This regime is obtained when injecting the same power along both axes x and y, i.e., $\Delta N^0 = 0$. Figure 3(a) shows the measurements of the power difference $\Delta N/N$ and degree of polarization $\mathcal{P}$ at the fiber output, as a function of the input power. We observe that the degree of polarization is effectively determined by the power imbalance $\mathcal{P} \simeq \Delta N/N$ [see Eq. (3)], since the measured anomalous correlator is negligible ($|M| \simeq 0$), as expected for the thermalization regime under investigation. In Fig. 3(a) we can see that, as the input power increases, the degree of polarization $\mathcal{P}$ at the fiber output grows larger. This behavior is found to be in agreement with the numerical simulations of NLSE (1), performed by faithfully reproducing the details of the experimental conditions [57]. As predicted by the criterion 7)), a sufficiently large power difference may render the anomalous correlator unstable. It is worth emphasizing, however, that even for the largest power difference recorded at the fiber output in Fig. 3(a), the anomalous correlator remains stable [which is consistent with the criterion (7), as $\alpha L_{\rm nl} \approx 0.19 > (2/3)\Delta N/N \approx 0.12$].

Figure 3(b) shows the measured input and output spectra along the x axis of the fiber (similar results are obtained along the y axis), together with the spectrum obtained from NLSE simulations. The results show the formation of a spectral tail decaying with the power law $\sim\omega^{-2}$, which is a signature of light thermalization reflecting energy equipartition among the modes [1–5,18].

Experiments on the reversible turbulent regime: For small initial anomalous correlations $|M(z=0)| \simeq 0$, we have seen through the criterion (7) that $|M|$ can be unstable.

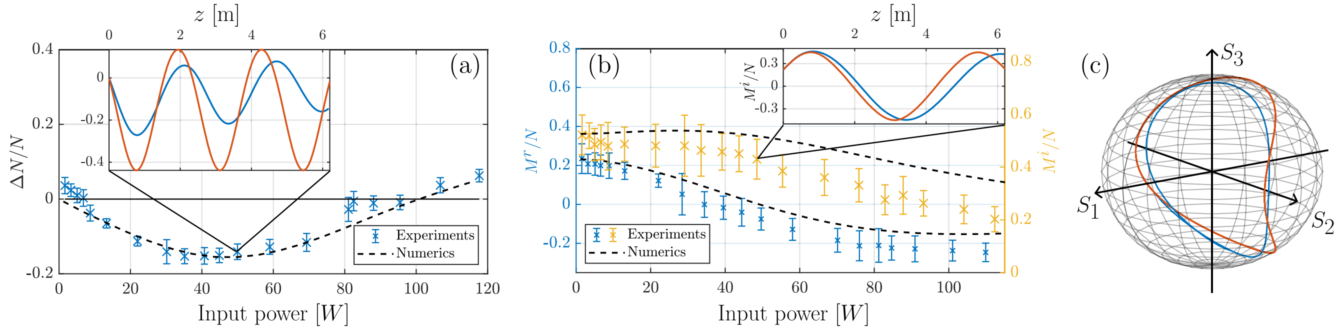

FIG. 4. Observation of the reversible turbulent regime. Measurements, at the fiber output and as a function of the input power, of (a) the power difference $\Delta N/N$ and (b) the real and imaginary parts of the normalized anomalous correlator M/N. The insets in (a),(b) show the dynamics along the fiber length $L = 6.2$ m for $\Delta N/N$ and M^i/N with an input power of 50 W ($L_{\rm nl} \approx 0.83$ m) in the strongly nonlinear regime $|E/U| \approx 1$: the blue and red lines report the simulations of NLSE (1) and AC-KE (8), respectively. (c) Corresponding field evolution on the Poincaré sphere for the NLSE (blue) and the AC-KE (8) (red). Note in the insets (a),(b) and in panel (c) that the AC-KE (8) (valid in the weakly nonlinear regime $|E/U| \gg 1$) still provides a qualitative description of the oscillatory dynamics even in the strongly nonlinear regime $|E/U| \approx 1$.

Experimental constraints, however, prevent simultaneously launching a strong unbalanced population $S_1 \simeq S_0$ with a small phase correlation $|M| \simeq 0$, precluding a direct experimental test of criterion (7). Nevertheless, in addition to the instability of the fixed point ($M = 0, S_1 = +S_0$), the AC-KE (8) also predicts a general nonlinear coupling between normal and anomalous correlators for arbitrary initial conditions on the sphere $\boldsymbol{S}(z = 0)$, even if the criterion (7) is not fulfilled. Here, we probe this nonlinear dynamics experimentally starting from $S_1(z = 0) \simeq 0$ with a large phase correlation $|M|$. We thus restore a large anomalous correlator by means of a polarizer, placed before injection of the laser beam into the fiber. Figures 4(a) and 4(b) report the experimental measurements of the power difference $\Delta N/N$ and complex anomalous correlator M/N at the fiber output, as functions of the input power. Their nonmonotonic evolutions agree well with the simulations of the NLSE (1). Such an agreement has been obtained by using a single adjustable parameter, namely the initial relative phase between u_x and u_y, which accounts for a minor, uncontrolled elliptical polarization of the injected incoherent optical beam. We have deliberately chosen to limit the number of adjustable parameters in order to highlight the physical mechanism at play. The quantitative discrepancy between simulations and experiments in Fig. 4(b) can also be attributed to the phase-sensitive nature of the anomalous correlator M/N, which makes it more vulnerable to experimental uncertainties and noise than $\Delta N/N$ shown in Fig. 4(a). The analysis of the evolution of $\Delta N/N$ and M/N along the fiber length $L = 6.2$ m uncovers the underlying oscillatory turbulent dynamics. This is illustrated in the insets of Figs. 4(a) and 4(b) for an input power of 50 W, which corresponds to a strongly nonlinear regime with $|E/U| \simeq 1$. The blue and red curves refer to simulations of the NLSE (1) and the AC-KE (8), respectively, whose corresponding evolution is also reported on the Stokes sphere; see Fig. 4(c). These results show that the AC-KE (8), which is rigorously valid in the weakly nonlinear regime $|E/U| \ll 1$ remains robust in describing qualitatively the oscillatory turbulent behavior even in the strongly nonlinear regime $|E/U| \simeq 1$.

Conclusion and perspectives—We have revealed, both theoretically and experimentally, that a phase-invariant Hamiltonian system of coupled waves may exhibit two distinct turbulent regimes: irreversible thermalization and reversible dynamics mediated by phase correlations. The experiments provide the first evidence of such contrasting turbulent behaviors in conservative (Hamiltonian) systems of dispersive nonlinear waves.

Our Letter establishes a basis for future advances in understanding the interplay of these fundamentally different regimes. Until now, turbulent regimes dominated by slow, irreversible thermalization and those governed by fast, correlation-driven reversible dynamics have been separately explored, leaving open the fundamental question about the role of anomalous correlators on thermalization, and the reciprocal impact of irreversible processes on the reversible dynamics. A key challenge for future research is therefore the development of a generalized theory that unifies these two aspects—reversible dynamics associated to anomalous correlators and irreversible thermalization—fully accounting for phase correlations.

Acknowledgments—The authors acknowledge Gang Xu and Adrien Fusaro for valuable discussions during the initial phase of this work. Funding was provided by Agence Nationale de la Recherche (Grants No. ANR-23-CE30-0021, No. ANR-21-ESRE-0040, and No. ANR-21-CE42-0026), by the Marie Sklodowska-Curie Actions project BESCLING (Grant No. 101209943), and by the Italian Ministry of University and Research (MUR) through the Italian Science Fund (FIS2), project SOFT

(No. H53C24001530001). Calculations were performed using HPC resources from Université Côte d'Azur's Center Azzura, and DNUM-CCUB (Université de Bourgogne Europe). S. W. acknowledges support from the European Research Council, project MULTIPROBE (No. 101224071).

Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

[1] V. E. Zakharov, V. S. L'vov, and G. Falkovich, *Kolmogorov Spectra of Turbulence I* (Springer, Berlin, 1992).

[2] V. Zakharov, F. Dias, and A. Pushkarev, One-dimensional wave turbulence, Phys. Rep. **398**, 1 (2004).

[3] S. Nazarenko, *Wave Turbulence*, Lectures Notes in Physics (Springer, New York, 2011).

[4] A. C. Newell and B. Rumpf, Wave turbulence, Annu. Rev. Fluid Mech. **43**, 59 (2011).

[5] S. Galtier, *Physics of Wave Trubulence* (Cambridge University Press, Cambridge, England, 2022).

[6] M. Onorato, Y. V. Lvov, G. Dematteis, and S. Chibbaro, Wave turbulence and thermalization in one-dimensional chains, Phys. Rep. **1040**, 1 (2023).

[7] V. S. L'vov, Y. V. Lvov, A. C. Newell, and V. E. Zakharov, Statistical description of acoustic turbulence, Phys. Rev. E **56**, 390 (1997).

[8] B. Rumpf, A. C. Newell, and V. E. Zakharov, Turbulent transfer of energy by radiating pulses, Phys. Rev. Lett. **103**, 074502 (2009).

[9] V. Labarre, G. Krstulovic, and S. Nazarenko, Wave-kinetic dynamics of forced-dissipated turbulent internal gravity waves, Phys. Rev. Lett. **135**, 014101 (2025).

[10] S. Galtier and S. V. Nazarenko, Turbulence of weak gravitational waves in the early universe, Phys. Rev. Lett. **119**, 221101 (2017).

[11] G. Dematteis, L. Rondoni, D. Proment, F. De Vita, and M. Onorato, Coexistence of ballistic and Fourier regimes in the β-Fermi-Pasta-Ulam-Tsingou lattice, Phys. Rev. Lett. **125**, 024101 (2020).

[12] M. Onorato, L. Vozella, D. Proment, and Y. V. Lvov, Route to thermalization in the α-Fermi–Pasta–Ulam system, Proc. Natl. Acad. Sci. U.S.A. **112**, 4208 (2015).

[13] J. Laurie, U. Bortolozzo, S. Nazarenko, and S. Residori, One-dimensional optical wave turbulence: Experiment and theory, Phys. Rep. **514**, 121 (2012).

[14] S. A. Babin, D. V. Churkin, A. E. Ismagulov, S. I. Kablukov, and E. V. Podivilov, Four-wave-mixing-induced turbulent spectral broadening in a long Raman fiber laser, J. Opt. Soc. Am. B **24**, 1729 (2007).

[15] S. K. Turitsyn, S. A. Babin, E. G. Turitsyna, G. E. Falkovich, E. Podivilov, and D. Churkin, Optical wave turbulence, in *Advances in Wave Turbulence* (World Scientific, Singapore, 2013).

[16] D. Churkin, I. Kolokolov, E. Podivilov, I. Vatnik, S. Vergeles, I. Terekhov, V. Lebedev, G. Falkovich, M. Nikulin, S. Babin, and S. Turitsyn, Wave kinetics of a random fibre laser, Nat. Commun. **2**, 6214 (2015).

[17] P. Suret, S. Randoux, H. R. Jauslin, and A. Picozzi, Anomalous thermalization of nonlinear wave systems, Phys. Rev. Lett. **104**, 054101 (2010).

[18] A. Picozzi, J. Garnier, T. Hansson, P. Suret, S. Randoux, G. Millot, and D. N. Christodoulides, Optical wave turbulence: Toward a unified nonequilibrium thermodynamic formulation of statistical nonlinear optics, Phys. Rep. **542**, 1 (2014).

[19] F. O. Wu, A. U. Hassan, and D. N. Christodoulides, Thermodynamic theory of highly multimoded nonlinear optical systems, Nat. Photonics **13**, 776 (2019).

[20] A. Fusaro, J. Garnier, K. Krupa, G. Millot, and A. Picozzi, Dramatic acceleration of wave condensation mediated by disorder in multimode fibers, Phys. Rev. Lett. **122**, 123902 (2019).

[21] E. V. Podivilov, F. Mangini, O. S. Sidelnikov, M. Ferraro, M. Gervaziev, D. S. Kharenko, M. Zitelli, M. P. Fedoruk, S. A. Babin, and S. Wabnitz, Thermalization of orbital angular momentum beams in multimode optical fibers, Phys. Rev. Lett. **128**, 243901 (2022).

[22] K. Baudin, A. Fusaro, K. Krupa, J. Garnier, S. Rica, G. Millot, and A. Picozzi, Classical Rayleigh-Jeans condensation of light waves: Observation and thermodynamic characterization, Phys. Rev. Lett. **125**, 244101 (2020).

[23] H. Pourbeyram, P. Sidorenko, F. Wu, L. Wright, D. Christodoulides, and F. Wise, Direct observations of thermalization to a Rayleigh–Jeans distribution in multimode optical fibres, Nat. Phys. **18**, 685 (2022).

[24] F. Mangini, M. Gervaziev, M. Ferraro, D. S. Kharenko, M. Zitelli, Y. Sun, V. Couderc, E. V. Podivilov, S. A. Babin, and S. Wabnitz, Statistical mechanics of beam self-cleaning in GRIN multimode optical fibers, Opt. Express **30**, 10850 (2022).

[25] M. Ferraro, F. Mangini, F. O. Wu, M. Zitelli, D. N. Christodoulides, and S. Wabnitz, Calorimetry of photon gases in nonlinear multimode optical fibers, Phys. Rev. X **14**, 021020 (2024).

[26] A. L. Muniz, F. Wu, P. Jung, M. Khajavikhan, D. Christodoulides, and U. Peschel, Observation of photon-photon thermodynamic processes under negative optical temperature conditions, Science **379**, 1019 (2023).

[27] M. S. Kirsch, G. G. Pyrialakos, R. Altenkirch, M. A. Selim, J. Beck, T. A. W. Wolterink, H. Ren, P. S. Jung, M. Khajavikhan, A. Szameit, M. Heinrich, and D. N. Christodoulides, Observation of Joule–Thomson photon-gas expansion, Nat. Phys. **21**, 214 (2025).

[28] N. Berti, K. Baudin, A. Fusaro, G. Millot, A. Picozzi, and J. Garnier, Interplay of thermalization and strong disorder: Wave turbulence theory, numerical simulations, and experiments in multimode optical fibers, Phys. Rev. Lett. **129**, 063901 (2022).

[29] K. Baudin, J. Garnier, A. Fusaro, N. Berti, C. Michel, K. Krupa, G. Millot, and A. Picozzi, Observation of light thermalization to negative temperature Rayleigh-Jeans equilibrium states in multimode optical fibers, Phys. Rev. Lett. **130**, 063801 (2023).

[30] M. Lian, Y. Geng, Y.-J. Chen, Y. Chen, and J.-T. Lü, Coupled thermal and power transport of optical waveguide arrays: Photonic Wiedemann-Franz law and rectification effect, Phys. Rev. Lett. **133**, 116303 (2024).

[31] A. Ramos, L. Fernández-Alcázar, T. Kottos, and B. Shapiro, Optical phase transitions in photonic networks: A spin-system formulation, Phys. Rev. X **10**, 031024 (2020).

[32] A. Y. Ramos, C. Shi, L. J. Fernandez-Alcazar, D. N. Christodoulides, and T. Kottos, Theory of localization-hindered thermalization in nonlinear multimode photonics, Commun. Phys. **6**, 189 (2023).

[33] A. Kurnosov, L. J. Fernández-Alcázar, A. Ramos, B. Shapiro, and T. Kottos, Optical kinetic theory of nonlinear multimode photonic networks, Phys. Rev. Lett. **132**, 193802 (2024).

[34] G. Yang, D. Bongiovanni, D. Song, R. Morandotti, Z. Chen, and N. K. Efremidis, Thermalization dynamics in photonic lattices of different geometries, APL Photonics **9**, 066115 (2024).

[35] M. Ferraro, K. Baudin, M. Gervaziev, A. Fusaro, A. Picozzi, J. Garnier, G. Millot, D. Kharenko, E. Podivilov, S. Babin, F. Mangini, and S. Wabnitz, Wave turbulence, thermalization and multimode locking in optical fibers, Physica (Amsterdam) **481D**, 134758 (2025).

[36] Y. Deng and Z. Hani, Full derivation of the wave kinetic equation, Inventiones Mathematicae **233**, 543 (2023).

[37] Y. Deng and Z. Hani, Long time justification of wave turbulence theory, arXiv:2311.10082.

[38] G. Staffilani and M.-B. Tran, On the wave turbulence theory for stochastic and random multidimensional KdV type equations, arXiv:2106.09819.

[39] E. Turitsyna, G. Falkovich, A. El-Taher, X. Shu, P. Harper, and S. Turitsyn, Optical turbulence and spectral condensate in long fibre lasers, Proc. R. Soc. A **468**, 2145 (2012).

[40] E. Turitsyna, S. Smirnov, S. Sugavanam, N. Tarasov, X. Shu, S. Babin, E. Podivilov, D. Churkin, G. Falkovich, and S. Turitsyn, The Laminar-turbulent transition in a fibre laser, Nat. Photonics **7**, 783 (2013).

[41] C. Rodda, C. Savaro, V. Bouillaut, P. Augier, J. Sommeria, T. Valran, S. Viboud, and N. Mordant, From internal waves to turbulence in a stably stratified fluid, Phys. Rev. Lett. **131**, 264101 (2023).

[42] D. Pierangeli, F. Di Mei, G. Di Domenico, A. J. Agranat, C. Conti, and E. DelRe, Turbulent transitions in optical wave propagation, Phys. Rev. Lett. **117**, 183902 (2016).

[43] L. Dieli, D. Pierangeli, E. DelRe, and C. Conti, Observation of two-dimensional dam break flow and a gaseous phase of solitons in a photon fluid, Phys. Rev. Lett. **133**, 183801 (2024).

[44] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Theory of superconductivity, Phys. Rev. **108**, 1175 (1957).

[45] V. E. Zakharov, V. S. L'Vov, and S. S. Starobinets, Stationary nonlinear theory of parametric excitation of waves, Zh. Eksp. Teor. Fiz. **59**, 1200 (1970) [Sov. Phys. JETP **32**, 656 (1971)].

[46] V. E. Zakharov, V. S. Lvov, and S. S. Starobinets, Spin-wave turbulence beyond the parametric excitation threshold, Sov. Phys. Usp. **17**, 896 (1975).

[47] V. S. L'vov, *Wave Turbulence Under Parametric Excitation —Application to Magnets* (Springer, Berlin, 1994).

[48] E. A. Kuznetsov and N. N. Noskov, Polarization effects in stimulated scattering of electromagnetic waves, Zh. Eksp. Teor. Fiz. **75**, 116 (1978) [Sov. Phys. JETP **48**, 57 (1978)].

[49] J. Zaleski, M. Onorato, and Y. V. Lvov, Anomalous Correlators in nonlinear dispersive wave systems, Phys. Rev. X **10**, 021043 (2020).

[50] M. Guasoni, J. Garnier, B. Rumpf, D. Sugny, J. Fatome, F. Amrani, G. Millot, and A. Picozzi, Incoherent Fermi-Pasta-Ulam recurrences and unconstrained thermalization mediated by strong phase correlations, Phys. Rev. X **7**, 011025 (2017).

[51] A. Villois, G. Dematteis, Y. V. Lvov, M. Onorato, and J. Shatah, Anomalous correlators, negative frequencies and non-phase-invariant Hamiltonians in random waves, Phys. Rev. Lett. **135**, 167201 (2025). Considering a scalar wave equation, it was proven in this work that phase invariant systems cannot develop anomalous correlations. The theorem can be extended to vector equations such as NLSE (1), for which phase invariance implies that, for given initial data $\boldsymbol{u}_0$, the solution $\boldsymbol{u}(z) = \boldsymbol{F}(z, \boldsymbol{u}_0)$, verifies $e^{i\theta}\boldsymbol{F}(z, \boldsymbol{u}_0) = \boldsymbol{F}(z, e^{i\theta}\boldsymbol{u}_0)$, $\theta \in \mathbb{R}$. Consequently, neither the u_x component nor the u_y component can individually develop anomalous correlations in the NLSE (1).

[52] L. Pitaevskii and S. Stringari, *Bose-Einstein Condensation and Superfluidity* (Oxford University Press, New York, 2016).

[53] R. Cominotti, A. Berti, C. Dulin, C. Rogora, G. Lamporesi, I. Carusotto, A. Recati, A. Zenesini, and G. Ferrari, Ferromagnetism in an extended coherently coupled atomic superfluid, Phys. Rev. X **13**, 021037 (2023).

[54] A. Zenesini, A. Berti, R. Cominotti, C. Rogora, I. G. Moss, T. P. Billam, I. Carusotto, G. Lamporesi, A. Recati, and G. Ferrari, False vacuum decay via bubble formation in ferromagnetic superfluids, Nat. Phys. **20**, 558 (2024).

[55] G. Millot, E. Seve, and S. Wabnitz, Polarization symmetry breaking and pulse train generation from the modulation of light waves, Phys. Rev. Lett. **79**, 661 (1997).

[56] G. Agrawal, *Nonlinear Fiber Optics*, 6th ed., (Academic Press, New York, 2019).

[57] See Supplemental Material at http://link.aps.org/supplemental/10.1103/tgtw-qnyq for presents the derivation of a reduced form of the WT KE (2), as well as the derivation of the AC-KE [(4), (5); (8)] and the corresponding stability analysis, Eq. (6), which includes Refs. [58–64]. It also provides technical details regarding the experimental procedures and measurements, as well as additional numerical simulations.

[58] A. Picozzi, Spontaneous polarization induced by natural thermalization of incoherent light, Opt. Express **16**, 17171 (2008).

[59] G. Millot, E. Seve, S. Wabnitz, and M. Haelterman, Observation of induced modulational polarization instabilities and pulse-train generation in the normal-dispersion regime of a birefringent optical fiber, J. Opt. Soc. Am. B **15**, 1266 (1998).

[60] G. Millot, E. Seve, S. Wabnitz, and M. Haelterman, Dark-soliton-like pulse-train generation from induced modulational polarization instability in a birefringent fiber, Opt. Lett. **23**, 511 (1998).

[61] L. Zanaglia, J. Garnier, I. Carusotto, V. Doya, C. Michel, and A. Picozzi, Spatio-temporal thermalization and adiabatic cooling of guided light waves, Phys. Rev. Lett. **136**, 053802 (2026).

[62] K. Krupa, A. Tonello, A. Barthélémy, V. Couderc, B. M. Shalaby, A. Bendahmane, G. Millot, and S. Wabnitz, Observation of geometric parametric instability induced by the periodic spatial self-imaging of multimode waves, Phys. Rev. Lett. **116**, 183901 (2016).

[63] N. B. Terry, T. G. Alley, and T. H. Russell, An explanation of SRS beam cleanup in graded-index fibers and the absence of SRS beam cleanup in step-index fibers, Opt. Express **15**, 17509 (2007).

[64] H. Pourbeyram, G. P. Agrawal, and A. Mafi, Stimulated Raman scattering cascade spanning the wavelength range of 523 to 1750 nm using a graded-index multimode optical fiber, Appl. Phys. Lett. **102**, 201107 (2013).

[65] The definition of the degree of polarization in Eq. (3) is equivalent to the usual one: $\mathcal{P}(z) = (1 - 4\,\det(\mathbf{J})/\mathrm{tr}(\mathbf{J})^2)^{1/2}$, where $\mathbf{J}_{ij} = \langle u_i u_j^* \rangle$ is the 2×2 coherence matrix ($j = x, y$), with 'det' and 'tr' the determinant and trace operations, see, e.g., L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).

[66] B. Daino, G. Gregori, and S. Wabnitz, Stability analysis of nonlinear coherent coupling, J. Appl. Phys. **58**, 4512 (1985).

[67] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th ed., (Pergamon Press, Oxford, 1975), see Sec. 78.