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A Mean Field Game Model for Renewable Investment Under Long-Term Uncertainty and Risk Aversion

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Abstract
We consider a stylized model for investment into renewable power plants under long-term uncertainty. We model risk-averse agents facing heterogeneous weather conditions and a common noise including uncertainty on demand trends, future fuel prices and the average national weather conditions. The objective of each agent is to maximize multistage profit by controlling investment in discrete time steps. We analyze this model in a noncooperative game setting with N players, where the interaction among agents occurs through the spot price mechanism. Our model extends to a mean field game with common noise when the number of agents is infinite. We prove that the N-player game admits a Nash equilibrium. Moreover, we prove that under appropriate assumptions, any sequence of Nash equilibria to the N-player game converges to the unique solution of the MFG game. Our numerical experiments highlight the impact of the risk aversion parameter and the importance of correctly specifying the distribution of the heterogeneity among agents. Moreover, we demonstrate that the results obtained by our model cannot be replicated by a model based on a representative agent with a unique parameter that would represent homogenized weather conditions. This emphasizes the importance of including explicit modeling of heterogeneity in prospective models when a heterogeneous parameter is expected to have a significant influence on the outcomes.

Keywords Stochastic control · Mean field games · Nash equilibrium · Renewable energy · Electricity markets

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1 Introduction

1.1 Context and Objectives

The achievement of carbon neutrality by 2050 requires significant and prompt reductions of emissions in the energy sector [47]. Energy-related emissions contribute to nearly 75% of global greenhouse gas emissions [32]. Decarbonization of the electricity sector through renewable energy investment plays a key role in all decarbonization strategies, given the considerable cost reductions of variable renewable energy (VRE) sources over the last decade IEA [33], Pörtner et al. [47]. Furthermore, fossil fuels are expected to be replaced by low-carbon electricity through electrification of sectors such as heating and transportation, thus increasing the role of electricity decarbonization to reduce greenhouse gas emissions. Consequently, most energy transition scenarios rely on heavy investments in low-carbon assets, including renewable power plants, such as onshore and offshore wind turbines, solar panels, and biomass, to attain the necessary emissions reductions [33]. Understanding whether market rules and regulatory arrangements in the energy sector allow to follow such scaling up of renewable investments is therefore a key issue for policy makers today [28].

Prospective models investigating investment in the energy sector—and more specifically in renewable power plants—should strive to include some essential features. Since the liberalization of the electricity sector, investment decisions are made by private risk-averse players, and are subject to a significant amount of uncertainty [43]. Future evolution of electricity markets is uncertain, regarding in particular future levels of electricity demand, evolution of power mix and market design, technology costs, fuel costs, or public policy. With increasing renewable penetration and subsequent decline in VRE unit revenues due to the cannibalization effect [39], the impact of uncertainties related to weather and demand variability is also becoming increasingly important to understand and to account for [50]. Such variability includes different time scales, going from very short-term volatility (hourly scale) to inter-annual variability [39]. All uncertainties translate into electricity price risk, and variable and uncertain profits, thus impacting the decisions of risk-averse investors.

Moreover, such investors are numerous and may have different characteristics, such as their geographical location, their level of risk aversion or their technology preferences. It is therefore important to allow for the representation of many competing heterogeneous agents who may share common features (such as national weather conditions) [8]. In particular, the question of whether renewable support systems such as feed-in tariffs and feed-in premiums should be geographically differentiated in order to avoid a higher concentration of renewable energy technologies in most productive areas (such as sunny or windy areas) has been raised [15]. Indeed, such geographical concentration could lead to congestion, increased costs in the longer term and decreased energy supply security.

Long investment horizons (several decades) require consideration of multiple investment decision periods to account for transformation pathways [31]. It is also essential to develop tractable models with effective numerical tools to analyze and use the results.

Finally, one could expect to have theoretical guarantees on the properties of the model output, such as whether the trajectory conforms to a Nash equilibrium, although it may be not required by some authors interested in studying systems out of equilibrium. This last feature distinguishes simulation models from optimization models. Our goal in this work is to incorporate many of these desirable features into a new model investigating investment in renewable power plants.

Three main approaches have been used to develop prospective models for the evolution of the energy sector.

First, equilibrium models are optimization models where the individual profit maximization problems of the different market players are solved simultaneously to obtain a Nash equilibrium [26]. Equilibrium models generally allow to represent different categories of uncertainties through a scenario-based approach. While such models can theoretically include various agents with heterogeneous characteristics and multistage investment decisions, their numerical complexity strongly limits the actual number of agents which can be considered [40]. Equilibrium models are also mostly two-stage, thus not representing transformation pathways.

Second, agent-based modeling (ABMs) represent a new and widely used approach in the energy sector. ABMs provide a bottom-up description of a complex, evolving system in which agents interact through a prescribed set of rules [30]. They allow the representation of heterogeneous agents and a very high degree of realism in describing market structure and real-world aspects such as asymmetric information, collective learning, or market power [51, 53]. However, mimicking real-world behavior and realistic markets with a high level of detail quickly leads to numerically intensive simulations and less transparent modeling. Moreover, ABMs are simulation models, not optimization models. Therefore, they do not provide theoretical guarantees for characterizing the model output, for example, proving that it is a Nash equilibrium (that is to say a situation where any agent cannot profitably deviate)

Lastly, a more recent approach coming from the applied mathematics community is mean field games (MFGs), which were introduced in Lasry and Lions [38]. They are stochastic games where an infinite number of agents interact symmetrically through the average density of the players. MFG models therefore include by design a continuum of agents, and the mathematical framework also allows to derive strong theoretical guarantees, such as the existence of a Nash equilibrium. However, such complex mathematical framework may require sacrificing model realism or interpretability, and the MFG literature lacks effective numerical tools to analyze the model outputs in more complex settings than the linear quadratic setting.

Our goal is to provide a middle way approach trying to include as many of previously listed features as possible to model investment into renewable power plants by heterogeneous agents under long-term uncertainty and risk aversion. Our approach relies on the MFG framework, thus allowing by essence the representation of numerous heterogeneous agents and deriving theoretical guarantees regarding existence of a Nash equilibrium and convergence of the N-player game toward the limiting MFG. On the other hand, we strive to provide a realistic and interpretable model, where many sources of uncertainties are included, where the specificity of renewables variability is captured, and which is numerically tractable. Before explaining in detail our original contributions to the topic, we shall first give the state of the art with the previously introduced three approaches, both on the modeling viewpoint and the mathematical analysis viewpoint.

1.2 State of the Art in Energy Economics and MFGs

Equilibrium models were firstly developed in response to the need to adapt traditional cost minimization problems taking the perspective of a social planner to the new competitive investment environment, with rising uncertainties and overwhelming risk [26]. While the theoretical framework for such models is introduced with an arbitrary number of agents, numerical experiments are often made with very few players, for example, two players in Mays et al. [40] and Abada et al. [1] and three players in Ehrenmann and Smeers [26].

All such papers also consider a two-stage setting, and while Ehrenmann and Smeers [26] highlight that extending this to a multistage setting is not conceptually difficult, this would result in a exponential increase (in the number of stages) of computational effort. Ambrosius et al. [7] explore the interaction between different electricity market designs and risk aversion in a stochastic multi-level equilibrium model. To circumvent the problem mentioned above of computational complexity of representing multiple agents, the authors make the quite demanding assumption that financial markets are complete, thus simplifying the equilibrium decisions into welfare-maximizing decisions. The same hypothesis is made in Munoz et al. [42] and Möbius et al. [41]. However, this assumption does not hold in real electricity markets, where there are, for example, no long-term contracts to cope with risk management over several years [1, 24].

As mentioned above, ABMs allow for greater level of detail in the modeling of wholesale electricity markets than equilibrium models, and can include heterogeneous agents [51]. Such models make it possible to investigate the interdependencies between risk aversion and market design. They have been used to study more specifically the effects of uncoordinated changes in market design in a multi-country model [31], the impact of imperfect information and firms' heterogeneous attributes [8], and the effect of investors' risk aversion on the performance of support schemes [29]. Many sources of uncertainty can be included in such models, such as load growth [8, 14, 34], inter-annual weather variability [31, 45], fuel prices [22, 29, 34] and carbon prices [22, 34]. Uncertainties are usually represented through scenario trees or Monte Carlo simulations. However, as stated previously, ABMs are not optimization models and therefore do not provide any theoretical insights on the simulated trajectory.

MFG models have been developed and applied to the energy sector. Alasseur et al. [5] explore how to optimally control a storage device in a noncooperative game setting where nodes compete through the electricity price. The trade-offs between higher and more stable revenues from fossil fuel thermal power plants and the negative externalities of a carbon tax for electricity producers are studied in Carmona et al. [21]. Three papers should be emphasized as they consider settings close to the one developed in this paper. Bonnans et al. [13] analyze a model with risk-averse agents optimizing a linear discrete-time dynamical system and interacting through a price which depends on the aggregate demand and through a congestion function. This paper mostly focuses on proving existence of a solution in a general theoretical setting. Aïd et al. [4] introduce a game where renewable and conventional producers compete through optimal entry and exit times. Their optimal stopping MFG setting allows to understand the dynamics of investment and divestment in an uncertain context, where conventional producers face uncertain costs represented through a Cox–Ingersoll–Ross process and renewable producers face intermittent output characterized by a slow climatic variation through a Jacobi process. Dumitrescu et al. [25] improve on the previous paper by adding common noise regarding transition scenario uncertainty in a discrete-time setting.

MFG also provides a theoretical framework for deriving existence and convergence results. Many results regarding existence of solutions to mean field games have been developed over the last decade. Carmona and Delarue [18] develop a probabilistic approach to the MFG problem. They derive a specific form of the stochastic maximum principle, and use fixed point arguments in the space of flows of probability to conclude to the existence of the value function. Their result is improved in Carmona et al. [20] to further include common noise.

The other main approach to constructing equilibria in MFG recasts the fixed point problem in terms of a forward-backward stochastic partial differential McKean–Vlasov equations and relies on a monotonicity condition to prove existence [17].

The convergence problem is one of the current research questions in MFG. It asks whether the N-player equilibria converge to a solution of the MFG. While it has been shown in various settings that MFG solutions provide approximate Nash equilibria to the N-player game [18], the convergence problem is significantly harder. Convergence has been known in some very specific settings, for example, in the linear-quadratic setting [9], or for ergodic MFG [10]. Convergence results depend on the type of controls considered for the N-player game, whether we consider open-loop or closed-loop controls. With open-loop controls, players choose their control as an adapted process on some given filtered probability space. The convergence for open-loop MFG has been extensively studied in Lacker [35]. In closed-loop controls, players choose control as a feedback function of the state of the system. The convergence problem was solved in this setting relying on the master equation framework. However, this requires a sufficiently smooth solution to the value function [17]. These shortcomings are addressed in Lacker and Flem [37], which relies on the probabilistic approach developed in Lacker [35] and on the notion of weak MFG solutions to prove that all closed-loop approximative equilibria converge to a weak mean field equilibrium.

1.3 Contributions of This Work

We develop a stylized MFG model to study and understand investment in renewable energy under long-term uncertainty regarding evolution of wholesale electricity markets. We consider heterogeneous risk-averse agents investing in renewable capacity in discrete time steps (corresponding to 5-year time steps), while being exposed to a common long-term uncertainty and to the variability of renewable production at the hourly scale. The rest of the power mix is assumed to evolve exogenously, as we expect that more stringent climate regulations will dictate evolution in fossil-fuel capacities (such as gas), rather than market-based interactions. A representative agent indexed by i controls her investment strategy $\left(q_t^i\right)_{1\leq t\leq T-1}$ and minimizes the following objective, knowing other agents' strategy $\boldsymbol{q}^{-i}$:

$$\mathcal{J}^i(q^i, \boldsymbol{q}^{-i}) = \mathbb{E}\left[\sum_{t=1}^{T-1} L^\beta(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, D_{t-1}, \boldsymbol{Y}_t) + Q_T^i e^{-rT} g(m_T^{N,i}, D_{T-1}, (\boldsymbol{Y}_t)_{T\leq t\leq T'})\right],$$

where the running cost L^β has the following general form

$$L^\beta(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, D_{t-1}, \boldsymbol{Y}_t) = e^{-rt}\left(Q_t^i f^\beta(t, m_t^{N,i}, D_t, D_{t-1}, \boldsymbol{Y}_t) + c_t q_t^i + \tilde{c}(q_t^i)^2\right).$$

Running cost L^β captures the (negative) agent's profit $Q_t^i f^\beta(t, m_t^{N,i}, D_t, D_{t-1}, \boldsymbol{Y}_t)$, and the convex investment costs $c_t q_t^i + \tilde{c}(q_t^i)^2$, see Sect. 2.3 for details. Q_t^i represents agent i's total invested capacity weighted by the agent's average capacity factor Γ^i (see Eq. (3)). Γ^i represents whether agent i faces on average better or worse weather conditions than the national average, and allows to capture heterogeneity among agents. Profit at time t depends on agent i's total invested capacity Q_t^i, and on the electricity spot price, which itself depends on aggregate renewable production $m_t^{N,i}$ (see Eq. (5)). This is where interaction with the other agents takes place. Profit is specified as the sum of hourly profits and relies on historical weather and demand data, thus capturing the daily and seasonal patterns characterizing weather and demand variability and impacting electricity prices. The model allows to capture many types of uncertainties through the presence of common noise. The first kind of uncertainty D_t corresponds to evolution of market trends. It includes uncertainty on future levels of demand and the correlated uncertain evolution of power mix (e.g., evolution in gas capacity), as well as uncertainty on fuel prices. Our model also includes uncertainty on annual weather scenario $\boldsymbol{Y}_t$, as we emphasized previously the essential role inter-annual variability plays in power systems with a large renewable share. Running cost L^β includes in mapping

f^{β} a convex combination between the random profit at time t and the conditional expected shortfall for the same profit, determined by uncertain market trends D_t and uncertain weather scenario $\boldsymbol{Y}_t$, thus capturing agents' risk aversion. Finally, the objective is multistage, allowing to explore the impact of transformation pathways.

The main contribution of this paper is to provide a modeling framework trying to reconcile the different advantages of the previously mentioned approaches (ABM, equilibrium models, MFG). Our model, while stylized, includes many essential features for a realistic description of electricity markets. Long-term uncertainties, inter-annual weather variability and agents' risk aversion are represented, while the multistage objective captures transformation pathways. The MFG framework allows to model explicitly a large number of agents while avoiding any numerical complexity. In particular, the MFG framework allows to capture explicitly heterogeneity among agents through their heterogeneous weather conditions. The specific though sufficiently general formulation of the representative agent's problem avoids usual shortfalls of MFG models by offering interpretability of the model output, and effective numerical implementation. On the other hand, the MFG framework allows the derivation of strong theoretical guarantees in our specific setting characterizing the output of the model. In particular, we prove that the N-player game admits at least one equilibrium solution, which we characterize through a closed-form condition. We then introduce the corresponding MFG game and prove that under proper assumptions, this limiting game admits a unique solution, also characterized by a closed-form condition. Finally, we prove that any sequence of Nash equilibria to the N-player game converges to the unique solution of the MFG game. The uniqueness guarantee for the limiting MFG game cannot be extended to the N-player game. A more detailed explanation of the distinctions between the N-player game and the limiting game can be found in Remark 5. A second contribution consists in using our model to derive practical insights. In particular, we show that our results derived in the MFG framework with heterogeneity regarding geographical localization cannot be reproduced with a representative agent's model, i.e., a model which would be solved by considering a unique parameter Γ representing homogenized weather conditions instead of the whole distribution. This highlights the importance of explicitly modeling heterogeneity in energy prospective models. We develop a toy model applied to the specific case of France electricity market. Our numerical experiments highlight the importance of the risk aversion parameter and the analysis of the spread of invested capacity across agents, allowed by the explicit modeling of heterogeneity.

We want to emphasize the specificity of our contributions in comparison to the three papers Aïd et al. [4], Dumitrescu et al. [25] and Bonnans et al. [13] mentioned in Sect. 1.2 as having settings close to the one considered in this paper. While Aïd et al. [4] and Dumitrescu et al. [25] also consider long-term evolution of electricity markets under uncertainty and large renewable penetration, they differ in many aspects from our setting. First, from a modeling perspective, they only consider renewable uncertain output through a slow climate variability, but they fail to consider the weather variability (especially inter-annual). Second, they do not include risk aversion. Third, they rely on an optimal stopping framework, therefore not representing the choice of the size of invested capacity. From a numerical perspective, the choice of a continuous setting with relaxed solutions of optimal stopping MFG in Aïd et al. [4] makes numerical implementation harder, and the model less interpretable for practitioners. While Bonnans et al. [13] consider a similar discrete-time setting with risk-averse agents interacting through a price function depending on their average density, their setting differs fundamentally from our work in different ways. First, they focus on generic risk measures, and include an additional congestion term, preventing them from finding a closed-form solution for the optimal control. They focus on idiosyncratic noise and do not include common noise.

Finally, their generic theoretical setting only allows them to derive an existence result, and no result on uniqueness or convergence.

1.4 Notations

We set the investment time grid $\mathcal{T} := \{1, \ldots, T-1\}$. In practice, T corresponds to the usual prospective time horizon of 2050. The interval $[t; t+1]$ corresponds to 5 years in the model. We set H the number of hours in a unit time interval $[t; t+1)$. We denote $\mathcal{H} = \{0, \ldots, H-1\}$ the set of hours included in time interval $[t; t+1)$. We write $T' > T$ for the time after which all invested capacities at time T are decommissioned, and we denote $\widetilde{\mathcal{T}} := \left\{1, \ldots, T'\right\}$. For any $t \in \mathcal{T}$ and any vector $(x_0, \ldots, x_t)$ we denote

$$x_{[t]} = (x_0, \ldots, x_t).$$

Let $\mathcal{N} := \{1, \ldots, N\}$ be the set of agents. We denote $\mathcal{N}^{-i} := \mathcal{N} \backslash \{i\}$, for $i \in \mathcal{N}$. For any vector $\boldsymbol{x} := (x^1, \ldots, x^N)$, we denote

$$\boldsymbol{x}^{-i} = (x^1, \ldots, x^{i-1}, x^{i+1}, \ldots, x^N).$$

We define the positive part function as $(x)_+ = x$ for non-negative x and zero otherwise.

Control We consider a given filtration $\mathbb{F}$. We denote by $\boldsymbol{A}$ a set of $\mathbb{F}$-progressively adapted controls (q_t). We define the ℓ_2 norm and ℓ_∞ norm as $\|q\|_2 := \left(\mathbb{E}\left[\sum_{t=1}^{T-1} q_t^2\right]\right)^{1/2}$, $\|q\|_\infty :=$ ess $\sup_{t,\omega} q_t(\omega)$. We denote by $\boldsymbol{A}_C$ the set of controls $q \in \boldsymbol{A}$ such that $\|q\|_2^2 < C$.

Probability Measures Let $\mathcal{X}$ be a subset of $\mathbb{R}^d$ and $\mathcal{M}_0(\mathcal{X})$ denote the set of probability measures on $\mathcal{X}$. Given $p \in [1, \infty)$, we define the set of finite p-th order moment measures $\mathcal{M}_p(\mathcal{X}) := \left\{m \in \mathcal{M}_0(\mathcal{X}), \int_{\mathcal{X}} |x|^p \mathrm{d}m(x) < +\infty\right\}$, that we endow with the Kantorovich–Rubinstein distance [52, p. 94], defined by

$$d_1(\mu, \nu) := \sup_{\psi \in 1-\mathrm{Lip}} \int_{\mathcal{X}} \psi(x) \mathrm{d}(\mu - \nu)(x), \tag{1}$$

for any μ and $\nu \in \mathcal{M}_1(\mathcal{X})$. As usually, a function φ is $C-$ Lip if φ is Lipschitz with a Lipschitz constant smaller than C.

Remark 1 Note that if X_1 and X_2 are random variables defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that the law of X_i is m_i, then $d_1(m_1, m_2) \leq \mathbb{E}[|X_1 - X_2|]$, because, for any 1-Lipschitz map $\psi : \mathcal{X} \to \mathbb{R}$, $\int_{\mathcal{X}} \psi(x) \mathrm{d}(m_1 - m_2)(x) = \mathbb{E}[\psi(X_1) - \psi(X_2)] \leq \mathbb{E}[|X_1 - X_2|]$.

Remark 2 Consider E a compact subspace of a metric space and denote $K = \sup_{x \in E} \|x\|$. For all $N \geq 1, \alpha \in E$ and $\mu \in \mathcal{M}_0(E)$, the metric d_1 satisfies

$$d_1\left(\mu, \frac{N-1}{N}\mu + \frac{1}{N}\delta_\alpha\right) \leq \frac{2K}{N}. \tag{2}$$

2 The Investment Model

We consider a stylized model for investment. In this section, we introduce exhaustively the different modeling assumptions related to agents' dynamics and to electricity markets. The cost minimization problem solved by each agent is introduced in Sect. 2.4.

2.1 Agent's Dynamics

In order to model the state dynamics, we consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ on which we define a common noise B^0:

$$B_t^0 = (D_t, \boldsymbol{Y}_t), \quad \boldsymbol{Y}_t = \left(\Gamma_{t,h}, \tilde{D}_{t,h}\right)_{h \in \mathcal{H}} \in [0;1]^H \times \mathbb{R}^H.$$

For the sake of simplicity, we will assume in this section that D_t corresponds to average demand during time interval $[t; t+1)$. More uncertainties are included in D_t in the numerical experiments in Sect. 4, allowing for greater realism without modifying the theoretical results obtained in Sect. 3. $\boldsymbol{Y}_t$ corresponds to the random variable representing the weather and centered demand scenario during time interval $[t; t+1)$. It includes information on average national hourly capacity factor $\Gamma_{t,h}$, and centered demand $\tilde{D}_{t,h}$ around the average value D_t, for all hours $h \in \mathcal{H}$. Capacity factor corresponds to the dimensionless ratio of actual electrical energy output over a given period of time to the theoretical maximum electrical energy output over that period. Variable $\boldsymbol{Y}_t$ therefore captures inter-annual weather variability.

We also consider N independent identically distributed (i.i.d) random variables Γ^i which are independent of B^0 and which follow law λ_Γ. Γ^i corresponds to the typical capacity factor for agent i, so that for a given hour h, agent i faces a final capacity factor of $\Gamma^i \times \Gamma_{t,h}$. $\Gamma^i > 1$ (resp. $\Gamma^i < 1$) implies that agent i faces better (resp. worse) weather conditions than the national average.

We denote $\mathbb{F}^{\mathcal{N}} = (\mathcal{F}_t^{\mathcal{N}})_{t \in \mathcal{T}}$ the filtration defined by $\mathcal{F}_t^{\mathcal{N}} := \sigma((\Gamma^i)_{i \in \mathcal{N}}, D_{[t-1]}, \boldsymbol{Y}_{[t-1]})$. Because D_{t-1} and $\boldsymbol{Y}_{t-1}$ depend on average and hourly data on the interval $[t-1, t)$, observe that $\mathcal{F}_t^{\mathcal{N}}$ represents indeed the information available at time t for decisions. We make the reasonable assumption that all agents are aware of the geographical conditions of other agents (the variables Γ^i), and that such conditions do not evolve over time.

We make some assumptions on the common noise.

Assumption 1 The stochastic process $(D_t)_{t \in \mathcal{T}}$ is a Markov process, with law λ_D.

Assumption 2 The random variables $(\boldsymbol{Y}_t)_{t \in \tilde{\mathcal{T}}}$ are independent and identically distributed with common distribution λ_Y, and $(\boldsymbol{Y}_t)_{t \in \tilde{\mathcal{T}}}$ is independent from $(D_t)_{t \in \mathcal{T}}$.

Assumption 3 We assume that all stochastic processes are defined on a finite support. We denote by $\mathcal{D}, \mathcal{Y}, \mathcal{G}$ finite sets such that, a.s., $D_t \in \mathcal{D}$ for all $t \in \mathcal{T}$, $Y_t \in \mathcal{Y}$ for all $t \in \widetilde{\mathcal{T}}$, and $\Gamma^i \in \mathcal{G}$ for all $i \in \mathcal{N}$, and assume that $\mathcal{G} \subset \left[\Gamma^{min}; \Gamma^{max}\right]$ with $\Gamma^{min} > 0$.

Assumption 3 is a technical assumption for the proofs; note that in applied settings, it is not really a restriction.

Each agent $i \in \mathcal{N}$ chooses strategy $q^i = (q_t^i)_{t \in \mathcal{T}}$, where q_t^i corresponds to invested capacity at time t. A $\mathcal{F}_0^N$-adapted random variable Γ^i defines agent i's capacity factor, as described above. Agent i's state $Q^i := (Q^i_t)_{t\in\widetilde{\mathcal{T}}}$ verifies the following equation:

$$\begin{aligned} &Q^i_0 = 0, \\ &Q^i_t = (1-\nu)Q^i_{t-1} + \Gamma^i q^i_t, \quad \forall t \in \mathcal{T}, \\ &Q^i_t = (1-\nu)Q^i_{t-1}, \quad \forall T \le t \le T', \end{aligned} \tag{3}$$

where $\nu > 0$ represents the depreciation of the installed capacities with time. Q^i_t corresponds to installed capacity, weighted by the agent's capacity factor. We note that after time $T-1$, state dynamics are uncontrolled: agents do not plan investments after time $T-1$, but due to installations' long lifetime, it is necessary to account for profits stemming from the installed capacities after time $T-1$ in order to avoid boundary effects.

A strategy q^i is deemed admissible if it belongs to the set $\boldsymbol{A}^{\mathcal{N}}$ which consists of the $\mathbb{F}^{\mathcal{N}}$-progressively adapted non-negative real-valued processes $(q_t)_{t\in\mathcal{T}}$ satisfying $\|q\|_2^2 < \infty$.

Lemma 4 *For any control $q^i \in \boldsymbol{A}^{\mathcal{N}}$, the state dynamics (3) are well-posed, and $Q^i \in \boldsymbol{A}^{\mathcal{N}}$. Moreover, if there exists $C > 0$ such that $q^i \in \boldsymbol{A}^{\mathcal{N}}_C$, then there exists $C' > 0$ such that $Q^i \in \boldsymbol{A}^{\mathcal{N}}_{C'}$.*

Proof The proof is easy and available in the supplementary material [27, Section 5.1]. □

An admissible strategy for all players $\boldsymbol{q} = \left(q^i\right)_{i\in\mathcal{N}}$ is such that $q^i \in \boldsymbol{A}^{\mathcal{N}}$ for all $i \in \mathcal{N}$. For an admissible strategy $\boldsymbol{q}$, the coupled state processes $\boldsymbol{Q} := (Q^i)_{i\in\mathcal{N}}$ are governed by the equations (3). We introduce the random empirical measure of the positions of all agents by

$$m^N_t = \frac{1}{N}\sum_{i'\in\mathcal{N}} \delta_{Q^{i'}_t}. \tag{4}$$

For all $i \in \mathcal{N}$, we denote similarly the random empirical measure of the positions of all agents excluding agent i by

$$m^{N,i}_t = \frac{1}{N-1}\sum_{i'\in\mathcal{N}^i} \delta_{Q^{i'}_t}, \quad \text{and} \quad m^{N,i} = \left(m^{N,i}_1, \ldots, m^{N,i}_{T-1}\right). \tag{5}$$

2.2 Market Mechanisms

In this section, we detail some modeling assumptions regarding the electricity market from which agents' profits are stemming.

Assumption 5 We make a price-taking assumption, according to which the i^{th} agent does not consider her impact on market price. This assumption is justified in a setting where the number of agents N is large. It is often made in a context of perfect competition, when each agent's influence on the price is negligible. Moreover, in our setting, the impact of this assumption is very limited. This will be justified in more detail in Sect. 3.1, in particular Proposition 9.

Residual Demand The hourly residual demand corresponds to the demand left to satisfy after accounting for renewable production, and it is defined by the following mapping $R : \mathcal{H} \times \mathcal{M}_2(\mathbb{R}^+) \times \mathcal{D} \times \mathcal{Y} \to \mathbb{R}$ by

$$R(h, m_t, D_t, \boldsymbol{Y}_t) = \left(D_t + \tilde{D}_{t,h} - \Gamma_{t,h}\int x\,\mathrm{d}m_t(x)\right)_+. \tag{6}$$

The hourly residual demand considered by agent i in her optimization problem reads as $R(h, m_t^{N,i}, D_t, \boldsymbol{Y}_t)$ by the price-taking assumption. Note that average installed capacity is considered in the definition of residual demand instead of total installed capacity, as there is an equivalence between the two modeling choices as discussed in Remark 3.

Electricity Price We define an aggregate supply function for conventional producers (coal, gas, nuclear, hydroelectricity) $F(t, P) : \mathbb{R}_+ \to \mathbb{R}_+$ corresponding to the offered capacity for a given price P at time t. It is obtained by summing all available capacities by increasing marginal cost. Dependence in t indicates that the supply function evolves over time, as the rest of the electricity mix is assumed to change exogenously.

Assumption 6 For all $t \in \tilde{\mathcal{T}}$, we define an aggregate supply function $P \in [0; \infty] \to F(t, P) \in [0; \infty]$ which is smooth and strictly increasing in P. We assume that there exists $L_F > 0$ such that for all $t \in \widetilde{\mathcal{T}}$, $F(t, .)$ is L_F-Lipschitz. This implies that $F(t, .)$ admits an inverse function which is smooth and strictly increasing. Moreover, we assume that $F^{-1}(t, .)$ is also L_F-Lipschitz for all $t \in \widetilde{\mathcal{T}}$.

In a simplified representation of electricity markets, the electricity price on the spot market is obtained by a cost minimization problem where the solution is provided by the merit-order dispatching rule [23, Chapter 8]. According to this rule, the dispatching of power is ordered from the least variable cost power plant to those with higher variable costs. Therefore, the electricity price corresponds to the intersection between the residual demand and the supply function.

Definition 1 The price mapping is defined following the merit-order rule, by the following mapping: $\phi : \widetilde{\mathcal{T}} \times \mathcal{H} \times \mathcal{M}_2(\mathbb{R}^+) \times \mathcal{D} \times \mathcal{Y} \to \mathbb{R}$ such that:

$$\phi(t, h, m_t, D_t, \boldsymbol{Y}_t) := F^{-1}(t, R(h, m_t, D_t, \boldsymbol{Y}_t)) \wedge \bar{P} \geq 0. \tag{7}$$

where $\bar{P}$ is the price cap.

Note that the price mapping is defined at the hourly scale and is bounded. A regulator impacts agents' profit through a deterministic additive process $(\alpha_t)_{t \in \widetilde{\mathcal{T}}} \in \mathbb{R}$ (see Eq. (9)). This corresponds, for example, to feed-in premium or feed-in tariffs [6]. We assume that α_t is bounded.

2.3 Costs and Objective Function

Investment Costs They are defined as a convex function of the invested capacity q:

$$C_t(q) = c_t q + \tilde{c} q^2. \tag{8}$$

The quadratic component $\tilde{c} q^2$ captures the fact that marginal investment costs for a given agent are increasing. We assume that the parameters c_t and $\tilde{c}$ are positive.

Producer Profit We define the negative unit profit over time interval $[t; t + 1)$ by

$$f(t, m_t, D_t, \boldsymbol{Y}_t) = -\sum_{h \in \mathcal{H}} (\phi(t, h, m_t, D_t, \boldsymbol{Y}_t) + \alpha_t) \Gamma_{t,h}. \tag{9}$$

Therefore, agent i earns $-Q_t^i f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t)$ total 5-year profits.

Producer's Cost Function Each producer has the same discount rate $r \geq 0$ capturing the value of time. The running cost of the producer $L : \mathcal{T} \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathcal{M}_2(\mathbb{R}^+) \times \mathcal{D} \times \mathcal{Y} \to \mathbb{R}$ is then defined by

$$L(t, Q_t, q_t, m_t, D_t, \boldsymbol{Y}_t) = e^{-rt} \left(Q_t f(t, m_t, D_t, \boldsymbol{Y}_t) + c_t q_t + \tilde{c} q_t^2 \right). \tag{10}$$

The first part of the running cost corresponds to the negative producer profit. The second part of the running cost corresponds to the investment costs.
The terminal cost of the producer is defined by the mapping $g : \mathcal{M}_2(\mathbb{R}^+) \times \mathcal{D} \times \mathcal{Y}^{T'-T+1} \to \mathbb{R}$

$$g(m_T, D_{T-1}, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) = \sum_{t=T}^{T'} e^{-r(t-T)} (1-\nu)^{t-T} f\left(t, m_T\left(\frac{\cdot}{(1-\nu)^{t-T}}\right), D_{T-1}, \boldsymbol{Y}_t\right). \tag{11}$$

This terminal cost corresponds to the sum of negative discounted unit profits over time interval $[T; T']$, and allows to avoid a discontinuity at the terminal date $T-1$. We make the assumption that average demand after time T stays equal to its value over interval $[T; T']$. This assumption can be justified as there are very few forecasts for the evolution of electricity demand after 2050.

Remark 3 In an initial game where total installed capacity is considered instead of average installed capacity in the definition of residual demand (6), the quadratic component in investment costs in (8) initially scales as N such as investment costs write as $C_t(q) = c_t q + N\tilde{c}q^2$. In this initial game, the running cost then writes as

$$L^{tot}(t, Q_t^i, q_t^i, Nm_t^{N,i}, D_t, \boldsymbol{Y}_t) = e^{-rt} \left(Q_t^i f(t, Nm_t^{N,i}, D_t, \boldsymbol{Y}_t) + c_t q_t^i + \tilde{c} N (q_t^i)^2 \right).$$

A change of variable $q_t^i \leftarrow \frac{q_t^i}{N}$ in the above yields

$$\begin{aligned} L^{tot}(t, Q_t^i, q_t^i, (N-1)m_t^{N,i}, D_t, \boldsymbol{Y}_t) &= e^{-rt} \left(\frac{Q_t^i}{N} f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t) + c_t \frac{q_t^i}{N} + N\tilde{c} \left(\frac{q_t^i}{N} \right)^2 \right) \\ &= \frac{1}{N} L(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, \boldsymbol{Y}_t). \end{aligned}$$

We obtain an equivalence up to a scaling factor between the running cost with total capacity, and the running cost with average capacity defined in (10), and investment costs as introduced in (8). The same holds for the terminal cost (11). This justifies the assumption of considering average installed capacity instead of total installed capacity. As it will be seen in Sect. 3.2, modeling with average capacity allows for MFG arguments.

2.4 Agent Minimization Problem

Following Rockafellar and Uryasev [48], we use the following definition of the expected shortfall $\mathbf{ES}^\alpha$ of a random variable X:

$$\mathbf{ES}^\alpha(X) = \inf_{x \in \mathbb{R}} \mathbb{E}\left[x + \frac{(X-x)^+}{\alpha} \right]. \tag{12}$$

Note that expected shortfall was initially named as the conditional value-at-risk in the literature [48]. However, this definition made it confusing to consider additional conditioning, leading to the less ambiguous denomination of expected shortfall.

In our case, X corresponds to negative profits (i.e., $X \leq 0$). Therefore, the $\mathbf{ES}^\alpha$ corresponds to the conditional expectation of negative profits below the amount $\mathbf{VaR}^\alpha$, where $\mathbf{VaR}^\alpha$ is the lowest amount such that, with probability $1-\alpha$, the negative profits will not exceed this amount. The $\mathbf{ES}^\alpha$ therefore identifies the worst (100α) percent of profit outcomes for an agent. Typical values for α are 0.05. The choice of $\mathbf{ES}^\alpha$ instead of $\mathbf{VaR}^\alpha$ allows to use a coherent risk measure.

For any admissible strategy $\boldsymbol{q}^{-i} \in \prod_{i \in \mathcal{N}^i} \boldsymbol{A}^{\mathcal{N}}$, and for a control $q^i \in \boldsymbol{A}^{\mathcal{N}}$ the multistage cost of agent i is given by

$$\begin{aligned}\mathcal{J}^i(q^i, \boldsymbol{q}^{-i}) := \mathbb{E}&\left[\beta \sum_{t=1}^{T-1} L(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, \boldsymbol{Y}_t)\right]\\ &+ \mathbb{E}\left[(1-\beta)\sum_{t=1}^{T-1} \mathbf{ES}^\alpha\left(L(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, \boldsymbol{Y}_t) \mid \mathcal{F}_t^{\mathcal{N}}, q_t^i, Q_t^i\right)\right]\\ &+ \mathbb{E}\left[Q_T^i e^{-rT} g(m_T^{N,i}, D_{T-1}, (\boldsymbol{Y}_t)_{T \leq t \leq T'})\right] \end{aligned} \tag{13}$$

for $\beta \in (0,1)$. The first part of the objective corresponds to a risk-neutral assessment of expected cost. The second part of the objective corresponds to a sum of conditional $\mathbf{ES}^\alpha$. This part of the objective corresponds to a time-consistent risk measure. Using a convex combination of expectation and of $\mathbf{ES}^\alpha$ is a common modeling assumption when evaluating the feasibility of an investment (see Munoz et al. [42]; Mays et al. [40]; Möbius et al. [41]; Fraunholz et al. [31])

Note that the objective function in Equation (13) is multistage, implying that the optimal decision at time t includes anticipation of future time steps and future optimal decisions for $t' \geq t+1$.

Deriving the Expected Shortfall We derive in the following preliminary discussion a new form for the objective (13). We have

$$\begin{aligned}\mathbf{ES}^\alpha\left(L(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, \boldsymbol{Y}_t) \mid \mathcal{F}_t^{\mathcal{N}}, q_t^i, Q_t^i\right) &= e^{-rt} Q_t^i \mathbf{ES}^\alpha_{\mathcal{F}_t^{\mathcal{N}}}\left(f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t)\right)\\ &\quad + e^{-rt}\left(c_t q_t^i + \tilde{c}(q_t^i)^2\right), \end{aligned} \tag{14}$$

where we denote $\mathbf{ES}^\alpha_{\mathcal{F}_t^{\mathcal{N}}}\left(f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t)\right) = \inf_{x \in \mathbb{R}} \mathbb{E}\left[x + \frac{(f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t) - x)^+}{\alpha} \mid \mathcal{F}_t^{\mathcal{N}}\right]$. Note that since D is Markovian, and since Y_t is independent from $Y_{[t-1]}$ and from D_{t-1}, we have

$$\mathbf{ES}^\alpha_{\mathcal{F}_t^{\mathcal{N}}}\left(f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t)\right) = \inf_{x \in \mathbb{R}} \mathbb{E}\left[x + \frac{(f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t) - x)^+}{\alpha} \mid D_{t-1}\right].$$

One should note that the expectation in the expected shortfall does not include the random variable $m_t^{N,i}$, which is fixed given $\boldsymbol{Q}_t^{-i}$. Consequently, we will write

$$\mathbf{ES}^\alpha_{\mathcal{F}_t^{\mathcal{N}}}\left(f(t, m_t^{N,i}, D_t, \boldsymbol{Y}_t)\right) = \mathbf{ES}^\alpha_{D_{t-1}}\left(f(t, m_t^{N,i}, ., .)\right).$$

where we used the notation "." to avoid confusions.

We define the following mapping and corresponding running cost

$$f^\beta(t, m, D, D', \boldsymbol{Y}) = \beta f(t, m, D, \boldsymbol{Y}) + (1-\beta)\mathbf{ES}^\alpha_{D'}(f(t, m, ., .)), \tag{15}$$

$$L^{\beta}(t, Q, q, m, D, D', \boldsymbol{Y}) = e^{-rt}\left(Qf^{\beta}(t, m, D, D', \boldsymbol{Y}) + c_t q + \tilde{c}q^2\right). \tag{16}$$

With (13) and (14), we finally obtain the following simplified multistage cost:

$$\mathcal{J}^i(q^i, \boldsymbol{q}^{-i}) = \mathbb{E}\left[\sum_{t=1}^{T-1} L^{\beta}(t, Q_t^i, q_t^i, m_t^{N,i}, D_t, D_{t-1}, \boldsymbol{Y}_t) + Q_T^i e^{-rT} g(m_T^{N,i}, D_{T-1}, (\boldsymbol{Y}_t)_{T\le t\le T'})\right]. \tag{17}$$

3 Theoretical Results

3.1 *N*-Player Game

This section focuses on the N-player game. We consider that players have access to open-loop controls. This implies that their controls are only specified as $\mathcal{F}_t$-adapted processes and do not depend on the state of the system. A detailed discussion on the difference between closed-loop and open-loop controls can be found in Carmona and Delarue [19].

In the following, we will consider Nash equilibriums, with the following definition.

Definition 2 We say that $\boldsymbol{q^*} = \left(q^{1,*}, \ldots, q^{N,*}\right)$ is a Nash equilibrium for the N-player game if for any $i \in \mathcal{N}$, for any $q \in \boldsymbol{A}^{\mathcal{N}}$: $\mathcal{J}^i(q, \boldsymbol{q}^{-i,*}) \ge \mathcal{J}^i(q^{i,*}, \boldsymbol{q}^{-i,*})$.

Existence of optimum for the producer problem. We consider a given strategy $\boldsymbol{q}^{-i} \in \prod_{i' \in \mathcal{N}^i} \boldsymbol{A}^{\mathcal{N}}$ with associated state dynamics $\boldsymbol{Q}^{-i} = (Q_{i'})_{' \in \mathcal{N}^i}$ given by the equations (3) for $i' \in \mathcal{N}^{-i}$. The control problem of agent i is obtained by solving

$$\inf_{q^i \in \boldsymbol{A}^{\mathcal{N}}} \mathcal{J}^i(q^i, \boldsymbol{q}^{-i}), \tag{$\mathcal{P}^N$}$$

where $\mathcal{J}^i$ was defined in (17). We will prove that Problem ($\mathcal{P}^N$) admits a unique minimizer, and that this unique minimizer can be characterized through a closed-form expression.

Proposition 7 *Let $\boldsymbol{q}^{-i} \in \prod_{i' \in \mathcal{N}^i} \boldsymbol{A}^{\mathcal{N}}$ and $m^{N,i}$ be the associated empirical measure defined in (5). The function $\mathcal{J}^i(., \boldsymbol{q}^{-i})$ admits a unique minimizer denoted by $q^{i,*} \in \boldsymbol{A}^{\mathcal{N}}$ given by:*

$$q_t^{i,*} = \frac{1}{2\tilde{c}}(H_t)_+, \qquad \forall t \in \mathcal{T}, \tag{18}$$

with

$$H_t = -\mathbb{E}\left[\sum_{t'=t}^{T-1} x^{t'-t}\Gamma^i f^{\beta}(t', m_{t'}^{N,i}, D_{t'}, D_{t'-1}, \boldsymbol{Y}_{t'}) + x^{T-t}\Gamma^i g(m_T^{N,i}, D_{T-1}, (\boldsymbol{Y}_t)_{T\le t\le T'}) \mid \mathcal{F}_t^N\right] - c_t,$$

and $x = e^{-r}(1-\nu)$.

Proof See Sect. 5.1. □

Corollary 8 *Let $q^* \in \boldsymbol{A}^{\mathcal{N}}$ be the unique minimizer of Problem ($\mathcal{P}^N$). There exist constants $C, C' > 0$ independent of N such that $q^* \in \boldsymbol{A}_C^{\mathcal{N}}$, and $Q^* \in \boldsymbol{A}_{C'}^{\mathcal{N}}$.*

Proof The proof directly stems from the boundedness of mapping f^β in Lemma 24, and from the closed-form expression of q^* in (18). It is clear that the constant C does not depend on N. Lemma 4 proves that there exists $C' > 0$ such that $Q^* \in \boldsymbol{A}^{\mathcal{N}}_{C'}$. □

We will now justify in more detail the price-taking Assumption 5. Consider the alternative optimization problem

$$\inf_{q^i \in \boldsymbol{A}^{\mathcal{N}}} \bar{\mathcal{J}}^i(q^i, \boldsymbol{q}), \qquad (\mathcal{Q}^N)$$

where

$$\bar{\mathcal{J}}^i(q^i, \boldsymbol{q}) = \mathbb{E}\left[\sum_{t=0}^{T-1} L^\beta(t, Q^i_t, q^i_t, m^N_t, D_t, D_{t-1}, \boldsymbol{Y}_t) + Q^i_T e^{-rT} g(m^N_T, D_{T-1}, (\boldsymbol{Y}_t)_{T \le t \le T'})\right].$$

This optimization problem depends on the empirical measure (4) of the positions of all agents, without excluding agent i. Therefore, the price-taking assumption for the N-player game is removed. It cannot be solved through a closed-form solution as was done in Proposition 7. However, Proposition 9 shows that any ϵ-solution to Problem ($\mathcal{Q}^N$) (in the sense of Eq. (32)) can be approximated by the optimal solution of Problem ($\mathcal{P}^N$) with a rate $\frac{1}{N}$. This further justifies Assumption 5 for large number N of players.

Proposition 9 *Let $\boldsymbol{q}^{-i} \in \prod_{i' \in \mathcal{N}^i} \boldsymbol{A}^{\mathcal{N}}_C$. Let $q^{i,*}$ be the unique optimal solution of ($\mathcal{P}^N$). Let $\epsilon > 0$, and $\bar{q}^i$ be an ϵ-solution of ($\mathcal{Q}^N$). Then, the following holds:*

- *There exists a constant C independent from N such that $\bar{q}^i \in \boldsymbol{A}^{\mathcal{N}}_C$ and $q^{i,*} \in \boldsymbol{A}^{\mathcal{N}}_C$.*
- *There exists a constant $\kappa > 0$ (independent on N and ϵ) such that*

$$\left\|\bar{q}^i - q^{i,*}\right\|_2 \le \frac{1}{N}\left(e^{-rT}\tilde{c}\right)^{-1}\kappa + \left(e^{-rT}\tilde{c}\right)^{-\frac{1}{2}}\epsilon^{\frac{1}{2}}. \qquad (19)$$

Proof See Sect. 5.2. □

Existence of Nash equilibrium for the N-player game. We state a result for which the proof is similar to the one in Theorem 13 introduced later in Sect. 3.2.

Proposition 10 *There exists at least one Nash equilibrium for the N-player game.*

Proof The proof is completely similar to the proof of Theorem 13. Details are left to the reader. □

3.2 Limiting Game

In this section, we introduce a limiting mean field game. We introduce another random variable Γ, independent of $(B^0_t)_{t\in\mathcal{T}}$. Similarly as for the N-player game, this variable represents the typical capacity factor, and its distribution represents the distribution of the capacity factor over the distribution of agents.

We denote $\mathbb{F} = (\mathcal{F}_t)_{t\in\mathcal{T}}$ the filtration defined by $\mathcal{F}_t := \sigma(\Gamma, D_{[t-1]}, \boldsymbol{Y}_{[t-1]})$ and $\mathcal{F}^B_t := \sigma(D_{[t-1]}, \boldsymbol{Y}_{[t-1]})$ the filtration associated with the common noise.

As in Lacker and Zariphopoulou [36], there are two separate sources of randomness in this limiting model. The first is due to the random processes D_t and $\boldsymbol{Y}_t$, while the second source is static and comes from the random variable Γ, which represents the distribution of the capacity factor across the continuum of agents. The agents can be thought of as a continuum, each assigned an independently and identically distributed capacity factor vector at the outset, and they interact after these assignments are made. We adopt the MFG approach of depicting a single representative agent as a random sample from the population, instead of explicitly modeling the continuum of agents. Note that this is the extension of the N-player game, where parameter Γ^i was fixed for each player and known by all agents, and the equilibrium strategy of player i depended on the distribution $m^{N,i}$ of the finite set of other players.

For any control q, we denote $Q^q := (Q_t^q)_{t\in\mathcal{T}}$ the state process corresponding to q, driven by the following dynamics:

$$\begin{aligned} Q_0^q &= 0, \\ Q_t^q &= (1-\nu)Q_{t-1}^q + \Gamma q_t, \quad \forall t \in \mathcal{T}, \\ Q_t^q &= (1-\nu)Q_{t-1}^q, \quad \forall T \le t \le T', && (20) \\ m_t^q &= \mathcal{L}(Q_t^q \mid \mathcal{F}_t^B), \quad \forall t \in \mathcal{T}, && (21) \end{aligned}$$

where $\mathcal{L}(. \mid \mathcal{F}_t^B)$ denotes the conditional law given $\mathcal{F}_t^B$.

A strategy q is deemed admissible if it belongs to the set $\boldsymbol{A}$ which consists of the $\mathbb{F}$-progressively adapted non-negative real-valued processes $(q_t)_{t\in\mathcal{T}}$ satisfying $\|q\|_2 < \infty$. It is easy to see that when $q \in \boldsymbol{A}$, m_t^q has finite second moment a.s., i.e., $m_t^q \in \mathcal{M}_2(\mathbb{R}^+)$ a.s.. Therefore, $m_t^q \in \boldsymbol{\mathcal{L}}(\Omega, \mathcal{F}_t, \mathbb{P}, \mathcal{M}_2(\mathbb{R}^+))$, i.e., m_t^q is an $\mathcal{F}_t$-adapted random variable taking values in $\mathcal{M}_2(\mathbb{R}^+)$.

Problem Definition The limiting game with common noise (MFG) is defined as follows. Find a control $\hat{q} \in \boldsymbol{A}$ such that, given $m^{\hat{q}} := \left(m_t^{\hat{q}}\right)_{t\in\mathcal{T}}$, $\hat{q}$ is an optimal control for the stochastic control problem with state process (20) and cost

$$\mathcal{J}^{mfg}(q, m^{\hat{q}}) = \mathbb{E}\left[\sum_{t=1}^{T-1} L^\beta(t, Q_t^q, q_t, m_t^{\hat{q}}, D_t, D_{t-1}, \boldsymbol{Y}_t) + Q_T^q e^{-rT} g(m_T^{\hat{q}}, D_{T-1}, (\boldsymbol{Y}_t)_{T\le t\le T'})\right]. \tag{22}$$

We write

$$\inf_{q\in\boldsymbol{A}} \mathcal{J}^{mfg}(q, m^{\hat{q}}). \tag{$\mathcal{P}$}$$

Alternatively, we can view Problem (MFG) as a fixed point problem as follows: given a strategy $\hat{q} \in \boldsymbol{A}$, find an optimal control q for the stochastic control problem with state process (20) and cost $(\mathcal{P})$ where $m^{\hat{q}}$ is defined as in Equation (21). Then, $\hat{q}$ is a solution of Problem (MFG) if and only if it is a fixed point of the following map:

$$\hat{q} := (\hat{q}_t)_{t\in\mathcal{T}} \to Q^{\hat{q}} = Q^q(\hat{q}) \to m^{\hat{q}} := (m_t^{\hat{q}})_{t\in\mathcal{T}} \to q := \operatorname{argmin}_{q\in\boldsymbol{A}} \mathcal{J}^{mfg}(q, m^{\hat{q}}) \tag{23}$$

We will define the map (23) formally later in (25). Our main result in this section is to show existence and uniqueness of a solution to Problem (MFG).

We first suppose we are given a vector of random probability measures. We prove in the following proposition that the standard control problem admits a unique minimizer, and we give a closed-form expression for this minimizer.

Proposition 11 *Let $m^q := (m_t^q)_{t\in\mathcal{T}}$ with $m_t^q \in \boldsymbol{\mathcal{L}}(\Omega, \mathcal{F}_t, \mathbb{P}, \mathcal{M}_2(\mathbb{R}^+))$ be a vector of random probability measures taking values in $\mathcal{M}_2(\mathbb{R}^+)$. The function $\mathcal{J}^{mfg}(., m^q)$ admits a unique* *minimizer denoted by* $q^* \in \boldsymbol{A}$ *given by:*

$$q^*_t = \frac{1}{2\tilde{c}} (H_t)_+ , \qquad \forall t \in \mathcal{T} \tag{24}$$

with

$$H_t = -\mathbb{E}\left[\sum_{t'=t}^{T-1} \Gamma x^{t'-t} f^\beta(t', m^q_{t'}, D_{t'}, D_{t'-1}, \boldsymbol{Y}_{t'}) + x^{T-t}\Gamma g(m^q_T, D_{T-1}, (\boldsymbol{Y}_t)_{T\le t\le T'}) \mid \mathcal{F}_t \right] - c_t,$$

and $x = e^{-r}(1-\nu)$.

Proof The proof is exactly the same as the one for the N-player Nash game, see Sect. 5.1. □

Corollary 12 *Let* $q^* \in \boldsymbol{A}$ *be the unique minimizer of Problem* $(\mathcal{P})$. *There exists* $C > 0$ *such that* $q^* \in \boldsymbol{A}_C$, *and* $C' > 0$ *such that* $Q^* \in \boldsymbol{A}_{C'}$.

Proof The proof is the same as the proof for Corollary 8. □

3.2.1 Existence of a MFG Solution

We begin by proving existence of a solution. We adopt an approach relying on Schauder's fixed point theorem. It should be emphasized that our proof is allowed by the fact that we consider a discrete-time problem, and Proposition 11 gives a closed-form solution of the optimization problem. We also strongly rely on the fact that the price process is bounded and the support of all random variables is finite, therefore working on compact convex sets. This allows simpler proofs than in classical continuous-time setting with common noise like in Carmona et al. [20].

We define the map $\Psi : \boldsymbol{A} \to \boldsymbol{A}$ as follows; given $\hat{q} \in \boldsymbol{A}$, we define $Q^{\hat{q}}$ to be the state process corresponding to $\hat{q}$ as defined by (20), and $m^{\hat{q}}$ the (random) conditional probability measure given by (21). We then solve Problem $(\mathcal{P})$, and we set

$$\Psi(\hat{q}) = q. \tag{25}$$

By Proposition 11, Problem $(\mathcal{P})$ admits a unique minimizer, so the map Ψ is well defined. Furthermore, a fixed point of Ψ clearly gives a solution of Problem (MFG).

Theorem 13 *There exists a solution to Problem* (MFG).

Proof See Sect. 5.3. □

We now state a lemma necessary for Theorem 17.

Lemma 14 *The following constants* C_e *and* C'_e *are finite:*

$$C_e := \max\left[\sup_{q \text{ s.t. } \Psi(q)=q} \|q\|_\infty , \sup_N \sup_{q \text{ s.t. } \Psi^N(q)=q} \|q\|_\infty \right],$$

$$C'_e := \max\left[\sup_{q \text{ s.t. } \Psi(q)=q} \|Q(q)\|_\infty , \sup_N \sup_{q \text{ s.t. } \Psi^N(q)=q} \|Q(q)\|_\infty \right].$$

Proof The proof follows from the proof of Corollary 12. (The bound does not only hold in ℓ_2 norm but also in ℓ_∞ norm.) The bound is independent of N. □

Remark 4 (Important) Our MFG problem is not equivalent to a game with a representative player, i.e., with a unique parameter $\bar{\Gamma}$ that would represent homogenized weather conditions, and with corresponding equilibrium state $Q^{\bar{\Gamma}}$. Indeed, one can note that the optimal control defined in (24) depends linearly in parameter Γ, therefore writing in a simplified form $q_t = \Gamma q_t^1 + q_t^0$. Similarly, by the state equation, we can write $Q_t = \Gamma^2 Q_t^1 + \Gamma Q_t^0$. If problem (MFG) was equivalent to a representative agent game, the equivalence of the optimal controls would require that $\mathbb{E}[Q] = Q^{\bar{\Gamma}}$. This would yield that $\mathbb{E}\left[\Gamma^2\right] Q_1 + \mathbb{E}[\Gamma] Q_0 = \bar{\Gamma}^2 Q_1 + \bar{\Gamma} Q_0$. In full generality on Q_0 and Q_1, such an equality can hold only when $\mathbb{E}\left[\Gamma^2\right] = \bar{\Gamma}^2 = \mathbb{E}[\Gamma]^2$, i.e., Γ is constant, discarding interesting settings with heterogeneous players.

This will be further discussed in the numerical experiments in Sect. 4.

3.2.2 Uniqueness

To prove uniqueness, previous papers often use a monotonicity condition. See Cardaliaguet et al. [17]; Ahuja [3]. We introduce the following assumptions needed in this section. The first assumption denotes the fact that the function $F^{-1}(t, .)$ is strictly increasing and bounded from below by a linear function. This is a very reasonable assumption based on the fact that $F^{-1}(t, .)$ is increasing smoothly.

Assumption 15 Let $t \in \widetilde{\mathcal{T}}$, and $C'_e > 0$ as defined in Lemma 14. Let $0 < \bar{Q}_1 < \bar{Q}_2 \leq C'_e$. There exists $C_{(26)} > 0$ such that

$$F^{-1}(t, \bar{Q}_2) - F^{-1}(t, \bar{Q}_1) \geq C_{(26)}(\bar{Q}_2 - \bar{Q}_1). \tag{26}$$

The next assumption is needed to obtain a strict lower bound in the proof of Theorem 17.

Assumption 16 Let $C'_e > 0$ as defined in Lemma 14. There exists $\epsilon > 0$ such that, for all $t \in \widetilde{\mathcal{T}}$, for all $\boldsymbol{Y}_t \in \mathcal{Y}$,

$$\mathcal{H}_t^\epsilon = \left\{ h \in \mathcal{H} \mid \forall D \in \mathcal{D}, \Gamma_{t,h} C'_e \leq D + \tilde{D}_{t,h} \leq F(t, \bar{P}) - \epsilon, \quad \Gamma_{t,h} > \epsilon \right\}$$

satisfies $\mathcal{H}_t^\epsilon \neq \emptyset$.

Assumption 16 justifies that for all possible weather annual scenarios, there exists a subset of hours where average national renewable production is nonzero, renewables are not sufficient to cover all demand, and demand does not reach market price cap. This assumption is very realistic based on historical data and projections.

We can now state the following.

Theorem 17 *Under Assumptions 15 and 16, the solution to Problem (MFG) is unique.*

Proof See Sect. 5.4. □

Remark 5 The uniqueness property is specific to this limiting MFG game. Indeed, for the N-player game, the fixed point equation solving Problem $(\mathcal{P}^N)$ defines the optimal control for each player i conditionally on the measure restricted to other players (i.e., with the price-taking assumption). This yields existence of multiple equilibria. Moreover, solving this game would require iterating on N fixed point conditions, resulting in substantial numerical computation challenges as N increases. Therefore, the point of the limiting MFG game is to get strong theoretical guarantees such as the uniqueness result, and to provide simple computation. Note that the N-player game without the price-taking assumption—with corresponding optimal control problem $\mathcal{Q}^N$—does not yield any closed-form expression.

3.3 Algorithm to Find the MFG Equilibrium

Common algorithms to derive an MFG equilibrium include Fictitious play [16]. Algorithm 1 describes the process. Following Bonnans et al. [11], we use the Frank-Wolfe learning rate of $\frac{2}{k+2}$ which demonstrates sharper convergence results. Note that in Algorithm 1, notations m_t^k no longer refer to the solution of the Nash equilibrium, but to the consecutive iterations of the algorithm.

Algorithm 1 Fictitious play

Input: number of time steps $T-1$, initial policy m^0
for $k = 0, \ldots, K-1$ **do**
Compute $q^{k+1} \in \arg\min_q \mathcal{J}^{mfg}(q, m^k)$
Compute $m_t^{q^{k+1}} = \mathcal{L}(Q_t^{q^{k+1}} \mid \mathcal{F}_t), \quad \forall t = 1, \ldots, T-1$
Update $m_t^{k+1} = \frac{1}{k+2} m_t^{q^{k+1}} + \frac{k+1}{k+2} m_t^k$
end for
Return: m^K, q^K

Remark 6 Proposition 11 gives a recursive form for the solution of the control problem, which facilitates the calculus in the algorithm.

3.4 Convergence of the *N*-Player Game

This section focuses on proving that when N tends to infinity, any sequence of Nash equilibria to the N-player game converges to the unique solution of the MFG problem. The following section is greatly inspired from Lacker [35], where the author studies convergence of open-loop N-player game to the corresponding MFG.

Definition 3 We will now write an MFG solution as a tuple $(\Omega, \mathbb{F}, \mathbb{P}, D, \boldsymbol{Y}, \Gamma, m, q, Q)$, where $(\Omega, (\mathcal{F}_t)_{t\in\mathcal{T}}, \mathbb{P})$ is a complete filtered probability space supporting $(D, \boldsymbol{Y}, \Gamma, m, q, Q)$. This MFG solution satisfies:

- $D := (D_t)_{t\in\mathcal{T}}$ and $\boldsymbol{Y} := (\boldsymbol{Y}_t)_{t\in\tilde{\mathcal{T}}}$ are $\mathcal{F}_t$-adapted processes following, respectively, the laws λ_D and $\lambda_Y^{\otimes T'}$.
- Γ is a random variable with law λ_Γ.
- Γ and $D, \boldsymbol{Y}$ are independent.
- q_t is an $\mathcal{F}_t$-adapted process such that $q \in \boldsymbol{A}$.
- (q, Γ, Q) satisfy the state equation (20).
- m is the conditional law of Q given $\mathbb{F}^B$: $m_t = \mathcal{L}(Q_t \mid \mathcal{F}_t^B)$.
- For all $q' \in \boldsymbol{A}$, we have $\mathbb{E}\left[\mathcal{J}(q', m)\right] \geq \mathbb{E}\left[\mathcal{J}(q, m)\right]$.

From Lemma 14, we know that if q is an MFG solution, $\|q\|_\infty \leq C_e$ and $\|Q\|_\infty \leq C_e'$. From now on, we will denote $E = \left[0, \max\left[C_e, C_e'\right]\right]$ the compact convex subspace of $\mathbb{R}$.

Given an MFG solution $(\Omega, \boldsymbol{\mathcal{F}}, \mathbb{P}, D, \boldsymbol{Y}, m, \Gamma, q, Q)$, we may view $D, \boldsymbol{Y}, \Gamma, m, q, Q$ as a random element of the canonical space

$$\Omega := (\mathcal{D})^{T-1} \times (\mathcal{Y})^{T'} \times \mathcal{G} \times (\mathcal{M}_2(E))^{T-1} \times E^{T-1} \times E^{T-1}. \tag{27}$$

Note that Ω is a metric space, as product of metric spaces.

An MFG solution thus induces a probability measure on Ω, which itself we would like to call a MFG solution. Following Lacker [35], we give the following equivalent definition for an MFG solution.

Definition 4 If $P \in \mathcal{M}_0(\Omega)$ satisfies $P = \mathbb{P} \circ (D, \boldsymbol{Y}, \Gamma, m, q, Q)^{-1}$ for some MFG solution $(\Omega, \boldsymbol{\mathcal{F}}, \mathbb{P}, D, \boldsymbol{Y}, \Gamma, m, q, Q)$, then we refer to P as the MFG solution.

From now on, we let $D, \boldsymbol{Y}, \Gamma, m, q, Q$ denote the identity maps on $(\mathcal{D})^{T-1}$, $(\mathcal{Y})^{T'}$, $\mathcal{G}$, $(\mathcal{M}_2(E))^{T-1}$, E^{T-1}, and E^{T-1}, respectively. We define the objective functional

$$\Lambda(D, \boldsymbol{Y}, \Gamma, m, q, Q) := \sum_{t=1}^{T-1} L^{\beta}(t, Q_t^q, q_t, m_t, D_t, D_{t-1}, \boldsymbol{Y}_t) + Q_T^q e^{-rT} g(m_T, D_{T-1}, (\boldsymbol{Y}_t)_{T \le t \le T'}), \tag{28}$$

and we define the reward associated with an element $P \in \mathcal{M}_0(\Omega)$ by

$$J(P) := \mathbb{E}^P \left[\Lambda(D, \boldsymbol{Y}, \Gamma, m, q, Q)\right]. \tag{29}$$

We will now define some subsets of $\mathcal{M}_0(\Omega)$. Let $\mathcal{M}_0(\Omega')$ denote the set of $\rho \in \Omega' := (\mathcal{D})^{T-1} \times (\mathcal{Y})^{T'} \times \mathcal{G} \times (\mathcal{M}_2(E))^{T-1}$ satisfying $\rho \circ (D, \boldsymbol{Y}, \Gamma)^{-1} = \lambda_D \otimes \lambda_Y^{\otimes T'} \otimes \lambda_\Gamma$.

For any $\rho \in \mathcal{M}_0(\Omega')$, the class $\mathcal{RA}(\rho)$ is the set of admissible joint laws for the optimal control problem associated with ρ. Specifically, it is the set of probability measures P on Ω satisfying:

1. $P \circ (D, \boldsymbol{Y}, \Gamma, m)^{-1} = \rho$,
2. $\mathbb{E}^P \left[\sum_{t=1}^{T-1} q_t^2 \right] < \infty$,
3. $P \circ (D, \boldsymbol{Y}, \Gamma, m, q, Q)^{-1} \in \mathcal{M}_0(\Omega)$ denotes the joint law of the solution and the inputs to (20) (i.e., the canonical processes Γ, q, Q satisfy the state equation (20)).

Define the set of optimal controls corresponding to ρ by

$$\mathcal{RA}^*(\rho) := \arg \min_{P \in \mathcal{RA}(\rho)} J(P).$$

By Proposition 11, we know that $\mathcal{RA}^*(\rho)$ is reduced to a singleton. (We are simply changing the probability space that we consider.)

The first lemma gives a characterization of an MFG solution.

Lemma 18 *Let $P \in \mathcal{M}_0(\Omega)$, and define $\rho := P \circ (D, \boldsymbol{Y}, \Gamma, m)^{-1}$. If P satisfies the following conditions:*

1. *ρ is in $\mathcal{M}_0(\Omega')$,*
2. *$P \in \mathcal{RA}(\rho)$,*
3. *$m = P(Q \in . \mid D, \boldsymbol{Y})$. That is, m is a version of the conditional law of Q given $D, \boldsymbol{Y}$,*
4. *$P \in \mathcal{RA}^*(\rho)$,*

then P is an MFG solution.

Proof It directly follows from Definitions 3 and 4. □

We now state some useful results following Lacker [35].

Lemma 19 *The map $J : \mathcal{M}(\Omega) \to \mathbb{R}$ is continuous.*

Proof The map Λ is bounded and continuous on Ω. This directly yields the continuity of J on $\mathcal{M}(\Omega)$ (for the topology defined by the weak convergence). □

We define the N-player environment:

$$\mathcal{E}_N := \left(\Omega, (\mathcal{F}_t^{\mathcal{N}})_t, \mathbb{P}, D, \boldsymbol{Y}, (\Gamma^i)_{i\in 1,\dots,N}\right).$$

We now consider a sequence of Nash equilibria. By Proposition 10, we know that there exists at least one Nash equilibrium for the N-player game. For each N, let $q^{1,N}, \dots, q^{N,N} \in \boldsymbol{A}_C^{\mathcal{N}}$ denote such a Nash equilibrium. We let

$$P_N := \frac{1}{N}\sum_{i=1}^{N} \mathbb{P} \circ \left(D, \boldsymbol{Y}, \Gamma^i, m^{N,i}, q^{i,N}, Q^{i,N}\left[q^{i,N}\right]\right)^{-1}. \tag{30}$$

We can now prove the following lemma.

Lemma 20 *The sequence $(P_N)_{N\in\mathbb{N}} \in \mathcal{M}_0(\Omega)$ is relatively compact.*

Proof Ω is a compact metric space, since the random variables D, Y and Γ have respective finite support $\mathcal{D}$, $\mathcal{Y}$ and $\mathcal{G}$. It follows from [44, Theorem 6.4] that $\mathcal{M}_0(\Omega)$ is also a compact metric space. Therefore, the sequence $(P_N)_{N\in\mathbb{N}} \in \mathcal{M}_0(\Omega)$ is relatively compact. □

By Lemma 20, we have that every subsequence P_{N_k} contains a further subsequence such that this subsequence converges weakly to a limit point $P \in \mathcal{M}_0(\Omega)$.

Lemma 21 *Each limit point P of any converging subsequence of $(P_N)_{N\in\mathbb{N}}$ is an MFG solution.*

Proof See Sect. 5.5. □

Proposition 22 *The sequence $(P_N)_{N\in\mathbb{N}}$ converges to a unique limit solution which is the solution to the (MFG) problem.*

Proof Lemma 21 and Lemma 18 prove that the limit point P is an MFG solution. Therefore, we have proven that each subsequence of P_N contains a further subsequence converging weakly to P where P is an MFG solution. By Proposition 11, we know that Problem (MFG) admits a unique solution. Therefore, we conclude that $(P_N)_{N\in\mathbb{N}}$ converges weakly to P where P is the unique MFG solution. □

3.5 Extension to the Case of Multiple Clusters

While the results derived in the previous subsections hold for the case where a single type of producers is considered, an interesting application of the model includes the representation of multiple clusters (e.g., producers of wind and solar). Our results of the existence of a Nash equilibrium for the N-player game and for the MFG game can easily be extended to the case of multiple clusters. The extension of the uniqueness result is left for future research. However, we observed empirically in our simulations that uniqueness seems to hold in the case of multiple clusters.

From a numerical perspective, including more than one type of renewable producers requires only marginal adjustments. The random variable Γ now contains all variables that determine the weather conditions important for all producers. For example, considering wind producers and solar producers, we can define the optimal control problem for wind producers as

$$\min_{q \in \boldsymbol{A}} \mathcal{J}^{mfg}_{wind}(q, m^k_{wind} + m^k_{solar})$$

The objective function depends on the sum of measures for both types of producers, as it is the total renewable invested capacity which drives the profit on electricity markets. The same holds for solar producers. The Fictitious Play algorithm can then be adapted as follows:

Algorithm 2 Fictitious play for multiple producers

Input: number of time steps $T-1$, initial policy m^0
for $k = 0, \ldots, K-1$ **do**
Compute $q^{k+1}_{wind} \in \arg\min_q \mathcal{J}^{mfg}_{wind}(q, m^k_{wind} + m^k_{solar})$
Compute $q^{k+1}_{solar} \in \arg\min_q \mathcal{J}^{mfg}_{solar}(q, m^k_{wind} + m^k_{solar})$
Compute $m^{q^{k+1}}_{t,wind} = \mathcal{L}(Q^{q^{k+1}}_{t,wind} \mid \mathcal{F}_t), \quad \forall t = 1, \ldots, T-1$
Compute $m^{q^{k+1}}_{t,solar} = \mathcal{L}(Q^{q^{k+1}}_{t,solar} \mid \mathcal{F}_t), \quad \forall t = 1, \ldots, T-1$
Update $m^{k+1}_{t,wind} = \frac{1}{k+2} m^{q^{k+1}}_{t,wind} + \frac{k+1}{k+2} m^k_{t,wind}$
Update $m^{k+1}_{t,solar} = \frac{1}{k+2} m^{q^{k+1}}_{t,solar} + \frac{k+1}{k+2} m^k_{t,solar}$
end for
Return: $m^K_{wind}, q^K_{wind}, m^K_{solar}, q^K_{solar}$

4 Numerical Simulations

The objective of this section is to provide a toy example inspired by the French electricity sector to illustrate our model, rather than utilizing it for generating practical predictions. These predictions will instead be the focus of forthcoming research.

The main energy sources that we consider are gas, coal, nuclear and intermittent renewables, consisting of solar, onshore and offshore wind, and run-of-river. Initial capacities of renewables are taken to be equal to 10GW, 18GW, 0 GW and 10 GW, respectively. We therefore consider an economy consisting of only these sources of electricity, without considering other sources of flexibility such batteries, electrolyzers or imports. This allows us to derive a practical toy model.

4.1 Setting and Data Gathering

Electricity and Mix Projections We use RTE's (French transmission network operator) latest study "Futurs Energétiques" [49] to obtain forecasts of evolution of electricity demand. Three main scenarios are considered: a "Sobriety" scenario, a "Reference" scenario, and a "Reindustrialization" scenario. Final electricity demand in 2050 amounts to a range between 555 TWh and 750 TWh according to the scenarios. This takes into account foreseeable change in the demand profile up to 2050, including an increased demand for electric vehicles and for heating.

All capacities but onshore wind and solar capacities are assumed to evolve exogenously, following scenario N1 of the “Futurs Energétiques’s” study. This study provides a variation around the central N1 scenario to consider adjustment of the mix to the demand scenario (e.g., offshore capacities are considered lower in the “Sobriety” scenario than in the “Reference” scenario, as demand is lower).

We rely on such assumptions to create the aggregate supply function $F(t, .)$. Note that as exogenous power mix is taken to evolve over time following RTE’s given scenario, the corresponding supply function also evolves in a deterministic manner over time. We build this supply function as a piecewise affine function, according to the merit-order rule. Cutoff points follow available capacity, while the slope of the function is determined by each technology’s variable cost. We consider a variable cost for gas of 23.2 EUR/MWh, following IEA’s projections for 2040 [33]. We model different gas power plants with varying efficiency between 0.4 (for open-cycle gas turbines) and 0.58 (for combined-cycle gas turbines), to capture the heterogeneity of gas supply across France. Because of maintenance constraints, nuclear capacity is not available for all hours. We consider a corresponding capacity factor of 0.9. Finally, we assume a market price cap of 10,000 EUR/MWh.

Demand and Fuel Prices Uncertainty We use a Markovian model for demand uncertainty, with a transition matrix between the different demand scenarios introduced beforehand. Specifically, we consider that when on the “Reference” scenario, there is a $\frac{1}{2}$ probability of staying on that trajectory, and a $\frac{1}{4}$ trajectory of shifting trajectory to either the lower or higher demand scenario. When on an extreme scenario (either “Sobriety” or “Reindustrialization”), there is a $\frac{3}{4}$ of staying on that trajectory, and a $\frac{1}{4}$ probability of shifting to the “Reference” scenario trajectory. We do not consider transition from the “Sobriety” to the “Reindustrialization” scenario, as such transitions would be very unrealistic. While a very simplistic modeling of uncertainty, this model allows to explore the impact of demand uncertainty on investment decisions. Moreover, the jump from one trajectory to another may represent either political decisions (e.g., rapid reindustrialization), or society shifts (e.g., decrease of consumption). Moreover, parametrizing such an uncertainty is a difficult task.

We model fuel prices uncertainties by considering that variable gas price is a random uniform variable with mean of 23.2 EUR/MWh (as stated before) and spread of 10EUR/MWh.

Inter-annual Variability of Hourly Profiles As emphasized, our model relies on an hourly time scale to calculate profits for producers, in order to capture the variability of demand and intermittent renewables. This is captured in the random variable $\boldsymbol{Y}_t$. Hourly capacity for offshore wind, onshore wind and solar PV was prepared using the renewables.ninja website, [1] which provides the hourly capacity factor profiles of solar and wind power from 1990 to 2019 at the geographical scale of French counties, following the methods elaborated by Pfenninger and Staffell [46]. The hourly electricity demand profile was provided by RTE.

Estimating demand heat sensitivity is a very challenging task. Therefore, we choose not to capture this in the model, and we focus on the inter-annual variability of renewable capacity factors. The average correlation between demand and weather is still captured through the average electricity profile.

Variability of Capacity Factor We test different distributions for the capacity factor, with a range of [0.5; 1.5]. In particular, we consider a distribution made of two Diracs at 0.7 and

[1] https://www.renewables.ninja/.

1.3, with equal ($p = \frac{1}{2}$) or asymmetric probability ($p = \frac{1}{4}$ and $p = \frac{3}{4}$, and $p = \frac{3}{4}$ and $p = \frac{1}{4}$). We also consider Beta Binomial distributions with law

$$f(x + (y - x)\frac{k}{n} \mid a, b) := \binom{n}{k} \frac{B(k + a, n - k + b)}{B(a, b)}, \quad \forall 0 \leq k \leq n,$$

with $x = 0.5$, $y = 1.5$ and $n = 12$. We explore different combinations of parameters a and b (namely $a = 0.5$ and $b = 0.5$, $a = 0.15$ and $b = 0.3$ and $a = 0.3$ and $b = 0.15$). In addition, we also model representative agent model by taking singletons as distributions. We consider different singletons distributions, spanning the whole range of the other distributions, with a step of 0.05 for the value of the capacity factor for different singleton distributions.

Risk Aversion Parameter Since there is no literature specifying realistic values for investors' decisions, we considered in an ad hoc way risk aversion parameter β ranging from 0.5 to 1 in our experiments. We will focus on specific values of 0.5 and 0.9 in the analysis. The expected shortfall level is fixed at 0.05, following classical values found in literature [7, 40].

Costs, Discount Rate and Depreciation Rate Cost evolution assumptions are taken from RTE [49]. Ground solar panels costs are assumed to fall from 750 EUR/MW in 2020 to 550EUR/MW in 2050. Onshore wind turbines costs are assumed to fall from 1300 EUR/MW in 2020 to 900 EUR/MW.

Parameter $\tilde{c}$ is calibrated as follows. When total capacity is considered, the quadratic component in investment costs scales as N, so that $C_t(q) = c_t q + N\tilde{c}q^2$. For a classical power plant size, we assume that the two components are equivalent: $c_t q \sim N\tilde{c}q^2$. Assuming an average installation rate of 10 GW per time step of 5 years [49] and an average cost of $C = 12{,}000$ EUR/GW, this yields $N\tilde{c} = \frac{C}{10/N}$.

Therefore, $\tilde{c} \sim 12{,}000$ for wind turbines. We obtain similarly that $\tilde{c} \sim 5500$ for solar panels.

Following RTE [49], we consider that solar and wind power plants have an average lifetime of 30 years, corresponding to 6 time stamps of 5 years. We model lifetime as a geometric random variable, with average of 30 years. The devaluation rate ν is therefore taken equal to $\frac{1}{6}$. Following Aïd et al. [4], we take a yearly discount rate of $r = 8.6\%$. The RTE study also specifies such range of values when private investors' decisions are considered. (A normative approach would on the other hand rely on a lower discount rate.)

4.2 Interpretation

We perform 100 iterations of the algorithm as described in Sect. 3.3, and we monitor the gain increase from switching to the best response for renewable producers, by summing gain increase for solar producers and onshore wind producers. It can be seen in Fig. 1 that this quantity converges to zero.

In the following, the average installed capacity refers to the quantity $\frac{Q}{\Gamma}$ with notations from Sect. 2, while the average installed capacity with capacity factor refers to the quantity Q. The former relates to actual investment decisions, while the latter relates to the average production (product of investment decision and capacity factor). It is the latter which impacts the electricity spot price.

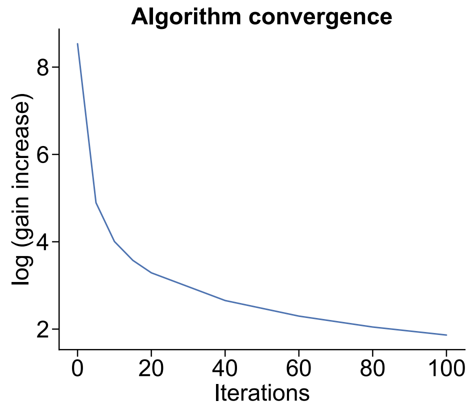

Fig. 1 Evolution of gain increase over algorithm iterations

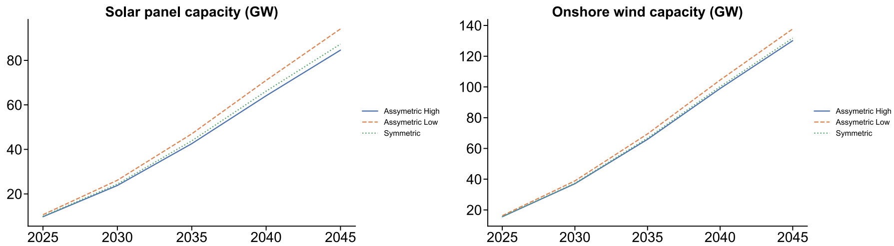

Fig. 2 Impact of capacity factor distribution for three different distributions: a symmetric two Dirac distributions and two asymmetric two Dirac distributions. Left: average installed capacity in solar panels. Right: average installed capacity in onshore wind turbines

When not specified, evolution of installed capacities in figures is taken by simulating the "Reference" demand scenario trajectory.

The impact of the choice of distribution for the capacity factor is illustrated in Fig. 2. When an asymmetric distribution is considered, we obtain a larger average installed capacity when the asymmetry is biased toward lower capacity factors. Intuition for this finding goes as follows: deriving the optimal control against initial invested capacity using Equation (24) for a smaller capacity factor results in reduced out-of-equilibrium production. In other words, the optimal control does not align with the distribution it was originally optimized against. Consequently, the fixed point condition defining the equilibrium is displaced, leading to an increase in investment for smaller capacity factor. Figure 2 highlights the need to correctly specify the distribution of the heterogeneity among agents when using such a prospective model.

The impact of the risk aversion parameter is illustrated in Fig. 3. A lower risk parameter β corresponds to higher risk aversion. When risk aversion increases, invested capacity decreases by up to 6%. Indeed, higher risk aversion places more emphasis on bad outcomes, resulting in reduced investment. As explained previously, the fixed point condition characterizing the equilibrium is again displaced, mitigating the total reduction in investments due to higher risk aversion. Such a result is aligned with the literature [31, 41].

Figure 4 shows the impact of the heterogeneity in terms of distribution of the installed capacity. We observe that as time goes by and average installed renewable capacity increases, the spread of installed capacity increases across the continuum of agents with different capacity factors. This illustrates the cannibalization effect, and how such effect can impact

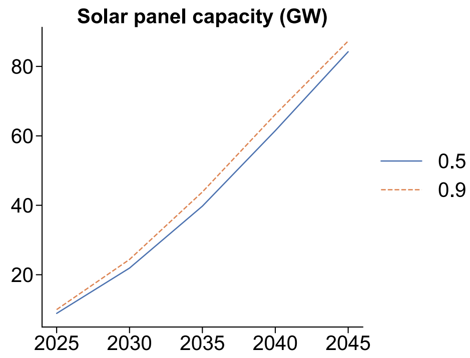

Fig. 3 Impact of risk aversion parameter β for two different values of 0.5 and 0.9

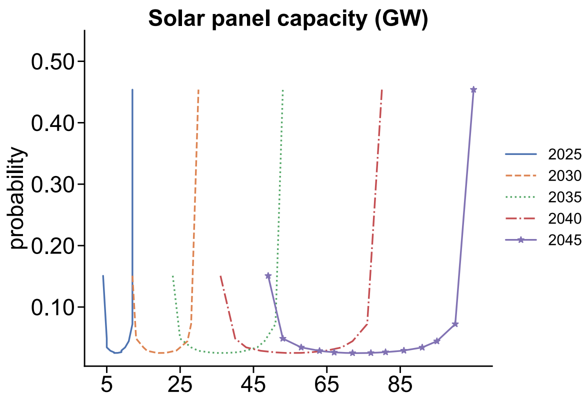

Fig. 4 Evolution of the distribution of installed capacity across the considered time range. The x-axis represents the value for the solar panel capacity, while the y-axis represents the probability with which this value is attained. The probability corresponds to the distribution for the capacity factor parameter

differently heterogeneous producers (e.g., in terms of geographical localizations). This points out that a result of private risk-averse agents taking investment decisions may be a higher concentration of renewable energy technologies in the most productive areas, which could lead to congestion, increased costs and decreased energy supply security, because of increased production correlation. Such an insight also highlights the interest for practitioners to work with models designed to handle heterogeneity, as a representative agent model would not allow to analyze such an impact.

Finally, we explore whether the results of the model with heterogeneity can be reproduced with a representative agent model relying on a unique parameter Γ representing homogenized weather condition instead of a whole distribution. Remark 4 already highlighted that from a theoretical point of view, there exists many capacity factor distributions for which the output of the model cannot be reproduced with a representative agent model. In the following, we compare the average installed capacity and the average installed capacity weighted by capacity factor for different distributions. Those two quantities jointly characterize the Nash equilibrium. Specifically, we consider a beta binomial distribution with parameter $a = 0.3$ and $b = 0.15$. We compare in Fig. 5 the results obtained by solving the game with the whole distribution with the results obtained by solving the game with singleton distributions. The relative difference with respect to the beta binomial distribution is represented. Singleton distributions with capacity factor of 1.3 and 1.25 are the closest to the installed capacity weighted by capacity factor, within a 2% difference. However, the same singleton distributions yield an installed capacity which differs by almost 10% from the capacity for the beta binomial distribution. The same can be said when we try to select the singleton distributions yielding

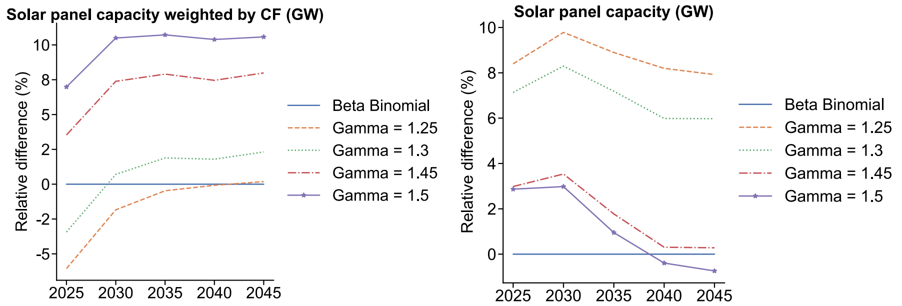

Fig. 5 Comparison of model outcomes for different capacity factor distributions. Left: average capacity weighted by the capacity factor. This is the quantity that impacts the spot price and therefore agents' profits. Right: average capacity. This is the quantity actually invested by agents, and derived from the average capacity weighted by the capacity factor

the closest results to the beta binomial distribution in terms of installed capacity: in that case, the selected singleton distributions yield an installed capacity weighted by capacity factor which differs by almost 10% from the one obtained with the beta binomial distribution. This indicates that a singleton distribution cannot approximate similarly well both the installed capacity and the installed capacity weighted by the capacity factor. Therefore, the Nash equilibrium corresponding to the beta binomial distribution cannot be obtained by solving a game with a singleton distribution. When solving the limiting game, the solution actually depends on the whole distribution and not just on a representative parameter Γ. Once again, this prompts practitioners to use models handling explicitly heterogeneity, and to pay attention to the use of representative agent's model in settings where the heterogeneous parameter may impact the results.

5 Proofs

5.1 Proof of Proposition 7

We first prove that $\mathcal{J}^i(q^i, .)$ is strongly convex. The linearity of the dynamics (3) and the quadratic/convex functions in the definition (16) of $L^\beta(t, Q, q, m, D, D', \boldsymbol{Y})$ give us that

$$\begin{aligned}\mathcal{J}^i(\lambda q^1 + (1-\lambda)q^2, \boldsymbol{q}^{-i}) &= \lambda \mathcal{J}^i(q^1, \boldsymbol{q}^{-i}) + (1-\lambda)\mathcal{J}^i(q^2, \boldsymbol{q}^{-i}) \\ &\quad + \mathbb{E}\left[\sum_{t=1}^{T-1} e^{-rt}\tilde{c}\left(\left(\lambda q_t^1 + (1-\lambda)q_t^2\right)^2 - \lambda (q_t^1)^2 - (1-\lambda)(q_t^2)^2\right)\right] \\ &\leq \lambda \mathcal{J}^i(q^1, \boldsymbol{q}^{-i}) + (1-\lambda)\mathcal{J}^i(q^2, \boldsymbol{q}^{-i}) - e^{-rT}\tilde{c}\lambda(1-\lambda)\left\|q^1 - q^2\right\|_2^2.\end{aligned}$$

We now define $q^{i,*}$ and H_t as given in Equation (18). Let $\Delta q \in \boldsymbol{A}$ such that for all $t \in \mathcal{T}$, Δq_t takes arbitrary values in $\left[-q_t^{i,*}, +\infty\right)$, and define an admissible control $\bar{q}^i \in \boldsymbol{A}$ such that

$$\bar{q}_t^i = q_t^{i,*} + \Delta q_t.$$

Let us compute $\Delta\mathcal{J}^i(q^{i,*}, \boldsymbol{q}^{-i}) = \mathcal{J}^i(\bar{q}^i, \boldsymbol{q}^{-i}) - \mathcal{J}^i(q^{i,*}, \boldsymbol{q}^{-i})$. Denote $\Delta_{t'} = t' - t$, and $x = e^{-r}(1-\nu)$.

We have that

$$\begin{aligned}
&\Delta\mathcal{J}^i(q^{i,*},\boldsymbol{q}^{-i})\\
&=\sum_{t=1}^{T-1}e^{-rt}\mathbb{E}\left[\sum_{t'\geq t}x^{\Delta_{t'}}\Gamma^i f^\beta(t',m_{t'}^{N,i},D_{t'},D_{t'-1},\boldsymbol{Y}_{t'})\Delta q_t+\Delta q_t c_t+\tilde{c}\left(\left(\bar{q}_t^i\right)^2-\left(q_t^{i,*}\right)^2\right)\right]\\
&+\sum_{t=1}^{T-1}e^{-rt}\mathbb{E}\left[x^{\Delta_T}\Gamma^i\Delta q_t g(m_T^{N,i},D_{T-1},(\boldsymbol{Y}_t)_{T\leq t\leq T'})\right]\\
&=\sum_{t=1}^{T-1}e^{-rt}\mathbb{E}\left[\Delta q_t\mathbb{E}\left[\sum_{t'\geq t}x^{\Delta_{t'}}\Gamma^i f^\beta(t',m_{t'}^{N,i},D_{t'},D_{t'-1},\boldsymbol{Y}_{t'})+c_t\mid\mathcal{F}_t^N\right]+\tilde{c}\left(\left(\bar{q}_t^i\right)^2-\left(q_t^{i,*}\right)^2\right)\right]\\
&+\sum_{t=1}^{T-1}e^{-rt}\mathbb{E}\left[\Delta q_t\mathbb{E}\left[x^{\Delta_T}\Gamma^i g(m_T^{N,i},D_{T-1},(\boldsymbol{Y}_t)_{T\leq t\leq T'})\mid\mathcal{F}_t^N\right]\right]\\
&=\sum_{t=1}^{T-1}e^{-rt}\mathbb{E}\left[-\Delta q_t H_t+\tilde{c}\left(\left(\bar{q}_t^i\right)^2-\left(q_t^{i,*}\right)^2\right)\right],
\end{aligned}$$

where the third equality comes from definition of H_t.
Let us analyze the sign of the quantity $-\Delta q_t H_t+\tilde{c}\left(\left(\bar{q}_t^i\right)^2-\left(q_t^{i,*}\right)^2\right)$ inside of the expectation. If $q_t^{i,*}=0$, then $H_t\leq 0$ and $\Delta q_t\geq 0$: $-\Delta q_t H_t+\tilde{c}\left(\left(\bar{q}_t^i\right)^2-\left(q_t^{i,*}\right)^2\right)=-\Delta q_t H_t+\tilde{c}\left(\left(\bar{q}_t^i\right)^2\right)\geq 0$. Otherwise, $H_t=2\tilde{c}q_t^{i,*}$ and then $-\Delta q_t H_t+\tilde{c}\left(\left(\bar{q}_t^i\right)^2-\left(q_t^{i,*}\right)^2\right)=\tilde{c}(\Delta q_t)^2\geq 0$.

Therefore, we obtain that for all admissible control $q^{i,*}+\Delta q$, we have

$$\Delta\mathcal{J}^i(q^{i,*},\boldsymbol{q}^{-i})\geq e^{-rT}\tilde{c}\,\mathbb{E}\left[\sum_{t=1}^{T-1}(\Delta q_t)^2\right]\geq 0. \tag{31}$$

This proves that $q^{i,*}$ is a global minimum.

Moreover, we proved previously that $\mathcal{J}^i(q^i,.)$ is strongly convex. As $\boldsymbol{A}$ is a convex set, we conclude that $q^{i,*}$ is actually the unique global minimum. □

5.2 Proof of Proposition 9

The proof of the first point directly follows from Lemma 25. Indeed, since $\bar{q}$ is an ϵ-solution, we have:

$$\bar{\mathcal{J}}^i\left(\hat{q}^i,\boldsymbol{q}\right)\leq\inf_{q^i\in\boldsymbol{A}}\bar{\mathcal{J}}^i\left(q^i,\boldsymbol{q}\right)+\epsilon. \tag{32}$$

Therefore, there exists a constant $C_1>0$ independent of N such that $\bar{q}^i\in\boldsymbol{A}_{C_1}$. We also have by Corollary 8 that $q^{i,*}$ belongs to $\boldsymbol{A}_C$. We obtain the result by taking $C=\max[C_1,C]$.

The proof of the second point is a consequence of Bonnans and Shapiro, [12, Proposition 4.32]. We define $\Delta\mathcal{J}^i(q^i):=\mathcal{J}^i(q^i,\boldsymbol{q}^{-i})-\bar{\mathcal{J}}^i(q^i,\boldsymbol{q})$. Following the above reference, we will prove i) that

$$\mathcal{J}^i(q^i,\boldsymbol{q}^{-i})-\mathcal{J}^i(q^{i,*},\boldsymbol{q}^{-i})\geq e^{-rT}\tilde{c}\left\|q^i-q^{i,*}\right\|_2^2$$

and that ii) there exists $\kappa > 0$ such that

$$\left|\Delta\mathcal{J}^i(q^{i,1}) - \Delta\mathcal{J}^i(q^{i,2})\right| \leq \frac{\kappa}{N} \|\Delta q\|_2 .$$

Details are given in the supplementary material Escribe et al., [27, Section 5.3]. The proof is complete. □

5.3 Proof of Theorem 13

Step 1. By Proposition 11, there exists $C > 0$ such that the minimizer of ($\mathcal{P}$) belongs to $\boldsymbol{A}_C$. Therefore, the map Ψ maps $\boldsymbol{A}_C$ into itself. Moreover, by Assumption 3, the support of filtration $\mathbb{F}$ is finite. Therefore, $\boldsymbol{A}$ is of finite dimension, and by Riesz theorem, $\boldsymbol{A}_C$ is compact. Moreover, it is also convex.

Step 2. We now check that Ψ is continuous on $\boldsymbol{A}_C$. Let $\hat{q}^1, \hat{q}^2 \in \boldsymbol{A}_C$. Denote Q^1 and Q^2 the associated installed capacities, m^{q^1} and m^{q^2} the associated random vectors of probability measures, and $q^1 = \Psi(\hat{q}^1)$ and $q^2 = \Psi(\hat{q}^2)$. We also denote

$$\begin{aligned}
\Delta f^\beta(t, m_t^{\hat{q}^1}, m_t^{\hat{q}^2}) &= f^\beta(t, m_t^{\hat{q}^1}, D_t, D_{t-1}, \boldsymbol{Y}_t) - f^\beta(t, m_t^{\hat{q}^2}, D_t, D_{t-1}, \boldsymbol{Y}_t), \\
\Delta g(m_T^{q^1}, m_T^{\hat{q}^2}) &= g(m_T^{\hat{q}^1}, D_{T-1}, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) - g(m_T^{\hat{q}^2}, D_{T-1}, (\boldsymbol{Y}_t)_{T \leq t \leq T'}).
\end{aligned}$$

We write $x = e^{-r}(1-\nu)$. Then,

$$\begin{aligned}
\|q^1 - q^2\|_2^2 &\leq \frac{1}{(2\tilde{c})^2} \sum_{t=1}^{T-1} \mathbb{E}\left[|H_t^1 - H_t^2|^2\right] \\
&\leq \frac{1}{2\tilde{c}^2} \sum_{t=1}^{T-1} \mathbb{E}\left[\mathbb{E}\left[(T-t)\sum_{t'=t}^{T-1} x^{2(t'-t)}\Gamma^2 \Delta f^\beta(t', m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2})^2 + x^{2(T-t)}\Gamma^2 \Delta g(m_T^{\hat{q}^1}, m_T^{\hat{q}^2})^2 \mid \mathcal{F}_t\right]\right]
\end{aligned}$$

where we used the definition of q in (24), Jensen and Cauchy–Schwarz inequalities.

We note that after conditioning on $\mathcal{F}_{t'}$, $m_{t'}^{q^1}$ and $m_{t'}^{q^2}$ are deterministic probability measures. Therefore, by Lemma 27,

$$\mathbb{E}\left[\Delta f^\beta(t', m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2})^2 \mid \mathcal{F}_{t'}\right] \leq L_f^2 \mathbb{E}\left[d_1\left(m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2}\right)^2 \mid \mathcal{F}_{t'}\right].$$

From Remark 1, it readily follows that

$$d_1\left(m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2}\right) = \sup_{\psi\ 1-Lip} \mathbb{E}\left[\psi(\hat{Q}_{t'}^1) - \psi(\hat{Q}_{t'}^2) \mid \mathcal{F}_{t'}^B\right] \leq \mathbb{E}\left[\left|\hat{Q}_{t'}^1 - \hat{Q}_{t'}^2\right| \mid \mathcal{F}_{t'}^B\right].$$

Therefore,

$$\begin{aligned}
\mathbb{E}\left[\Delta f^\beta(t', m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2})^2 \mid \mathcal{F}_{t'}\right] &\leq L_f^2 \mathbb{E}\left[\mathbb{E}\left[\left|\hat{Q}_{t'}^1 - \hat{Q}_{t'}^2\right| \mid \mathcal{F}_{t'}^B\right]^2 \mid \mathcal{F}_{t'}\right] \\
&\leq L_f^2 \mathbb{E}\left[\left(\hat{Q}_{t'}^1 - \hat{Q}_{t'}^2\right)^2 \mid \mathcal{F}_{t'}^B\right],
\end{aligned}$$

where we used Jensen inequality. Thus, for $t' \geq t$,

$$\begin{aligned}
\mathbb{E}\left[\Gamma^2 \Delta f^\beta(t', m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2})^2 \mid \mathcal{F}_t\right] &= \mathbb{E}\left[\Gamma^2 \mathbb{E}\left[\Delta f^\beta(t', m_{t'}^{\hat{q}^1}, m_{t'}^{\hat{q}^2})^2 \mid \mathcal{F}_{t'}\right] \mid \mathcal{F}_t\right] \\
&\leq L_{\Delta_f}^2 \mathbb{E}\left[\mathbb{E}\left[\left(\hat{Q}_{t'}^1 - \hat{Q}_{t'}^2\right)^2 \mid \mathcal{F}_{t'}^B\right] \mid \mathcal{F}_t\right],
\end{aligned} \tag{33}$$

with $L^2_{\Delta_f} := (\Gamma^{max})^2 L^2_f$. Similarly, with Lemma 28 and Remark 1, we obtain that

$$\mathbb{E}\left[\Gamma^2 \Delta g(m_T^{\hat{q}^1}, m_T^{\hat{q}^2})^2 \mid \mathcal{F}_{T-1}\right] \le L^2_{\Delta_g} \mathbb{E}\left[(\hat{Q}^1_{T-1} - \hat{Q}^2_{T-1})^2 \mid \mathcal{F}^B_{T-1}\right],$$

with $L^2_{\Delta_g} = (\Gamma^{max})^2 L^2_g (1-\nu)^2$. We used similarly Cauchy–Schwarz inequality and the fact that since the state dynamics are uncontrolled after time T, $m_{t'}^{q^1}$ and $m_{t'}^{q^2}$ are deterministic probability measures for all $t' \ge T$ after conditioning on $\mathcal{F}_{T-1}$. Therefore,

$$\mathbb{E}\left[\Gamma^2 \Delta g(m_T^{\hat{q}^1}, m_T^{\hat{q}^2})^2 \mid \mathcal{F}_t\right] \le L^2_{\Delta_g} \mathbb{E}\left[\mathbb{E}\left[\left(\hat{Q}^1_{T-1} - \hat{Q}^2_{T-1}\right)^2 \mid \mathcal{F}^B_{T-1}\right] \mid \mathcal{F}_t\right]. \tag{34}$$

The combination of (33) and (34) gives the existence of constant $L = \frac{1}{2\tilde{c}^2}\left((T-1)L^2_{\Delta_f} + L^2_{\Delta_g}\right)$ such that

$$\left\|q^1 - q^2\right\|_2^2 \le L \sum_{t=1}^{T-1}\sum_{t'=t}^{T-1} \mathbb{E}\left[\left(\hat{Q}^1_{t'} - \hat{Q}^2_{t'}\right)^2\right] \le L(T-1)\mathbb{E}\left[\sum_{t=1}^{T-1}\left(\hat{Q}^1_t - \hat{Q}^2_t\right)^2\right]. \tag{35}$$

With the state dynamics, we finally have obtain that there exists a constant $L > 0$ such that $\left\|q^1 - q^2\right\|_2^2 \le L \left\|\hat{q}^1 - \hat{q}^2\right\|_2^2$. We conclude that the map Ψ is continuous in q.
Step 3. We finally conclude by using Schauder's fixed point theorem on the map Ψ which is continuous on the compact convex set $\boldsymbol{A}_C$. □

5.4 Proof of Theorem 17

Let $q^1, q^2 \in \boldsymbol{A}_C$ two fixed points of the map Ψ. By Lemma 14, we have that almost surely, for all $t \in \mathcal{T}$, $\int x \mathrm{d}m_t^{q^1}(x) \le C'_e$, where m^{q^1} and m^{q^2} are the associated random vectors of probability measures. The same holds for m^{q^2}. As previously, we denote

$$\Delta f^\beta(t, m_t^{q^1}, m_t^{q^2}) = f^\beta(t, m_t^{q^1}, D_t, D_{t-1}, \boldsymbol{Y}_t) - f^\beta(t, m_t^{q^2}, D_t, D_{t-1}, \boldsymbol{Y}_t),$$
$$\Delta g(m_T^{q^1}, m_T^{q^2}) = g(m_T^{q^1}, D_{T-1}, (\boldsymbol{Y}_t)_{T\le t\le T'}) - g(m_T^{q^2}, D_{T-1}, (\boldsymbol{Y}_t)_{T\le t\le T'}).$$

As stated previously, fixed points of Ψ are minimizers of ($\mathcal{P}$), and therefore,

$$\mathcal{J}^{mfg}(q^1, m^{q^1}) \le \mathcal{J}^{mfg}(q^2, m^{q^1}), \quad \text{and} \quad \mathcal{J}^{mfg}(q^2, m^{q^2}) \le \mathcal{J}^{mfg}(q^1, m^{q^2}).$$

By summing the two previous inequalities, we get

$$\mathbb{E}\left[\sum_{t=1}^{T-1} e^{-rt}\left(Q^1_t - Q^2_t\right)\Delta f^\beta(t, m_t^{q^1}, m_t^{q^2}) + \left(Q^1_T - Q^2_T\right) e^{-rT}\Delta g(m_T^{q^1}, m_T^{q^2})\right] \le 0.$$

Conditioned on $\mathcal{F}^B_t$, $m_t^{q^1}$ and $m_t^{q^2}$ are deterministic probability measures. By Lemma 29,

$$\begin{aligned}\mathbb{E}\left[\left(Q^1_t - Q^2_t\right)\Delta f^\beta(t, m_t^{q^1}, m_t^{q^2})\right] &= \mathbb{E}\left[\mathbb{E}\left[\left(Q^1_t - Q^2_t\right)\Delta f^\beta(t, m_t^{q^1}, m_t^{q^2}) \mid \mathcal{F}^B_t\right]\right]\\ &= \mathbb{E}\left[\mathbb{E}\left[\Delta f^\beta(t, m_t^{q^1}, m_t^{q^2})\mathbb{E}\left[\left(Q^1_t - Q^2_t\right) \mid \mathcal{F}^B_t\right] \mid \mathcal{F}^B_t\right]\right]\\ &\ge K^\epsilon_{(A8)}\mathbb{E}\left[\left(\mathbb{E}\left[Q^1_t \mid \mathcal{F}^B_t\right] - \mathbb{E}\left[Q^2_t \mid \mathcal{F}^B_t\right]\right)^2\right].\end{aligned}$$

By Lemma 30, we obtain similarly that

$$\mathbb{E}\left[\left(Q_T^1 - Q_T^2\right) \Delta g(m_T^{q^1}, m_T^{q^2}) \mid \mathcal{F}_t\right] \geq K^{\epsilon}_{(A11)} \mathbb{E}\left[\left(\mathbb{E}\left[Q_T^1 \mid \mathcal{F}_T^B\right] - \mathbb{E}\left[Q_T^2 \mid \mathcal{F}_T^B\right]\right)^2\right].$$

Therefore, by combining the above inequalities, we get

$$0 \geq K \mathbb{E}\left[\sum_{t=1}^{T} e^{-rt} \left(\mathbb{E}\left[Q_t^1 \mid \mathcal{F}_t^B\right] - \mathbb{E}\left[Q_t^2 \mid \mathcal{F}_t^B\right]\right)^2\right]$$

where $K = \min\left[K^{\epsilon}_{(A8)}, K^{\epsilon}_{(A11)}\right]$. This implies that for all $t \in \mathcal{T}$,

$$\mathbb{E}\left[Q_t^1 \mid \mathcal{F}_t^B\right] = \mathbb{E}\left[Q_t^2 \mid \mathcal{F}_t^B\right].$$

We will now prove that the equality actually holds for the probability measure in addition to the expectation. By Proposition 11, for a given vector of random probability measures m, there exists a unique minimizer $q^* \in \boldsymbol{A}$. We have shown that if q^1, q^2 are two fixed points of the mapping Ψ, we have that for all $t \in \mathcal{T}$

$$\int x \mathrm{d}m_t^{q^1}(x) = \int x \mathrm{d}m_t^{q^2}(x).$$

Therefore, since the mapping $\mathcal{J}^{mfg}(., m^q)$ only depends on the quantity $\int x \mathrm{d}m_t^q(x)$, we can conclude that the minimizers $q^{1,*}$ and $q^{2,*}$ are also equal. This concludes that if q^1, q^2 are two Nash equilibria, we have that $q^1 = q^2$. □

5.5 Proof of Lemma 21

We abuse notation somewhat by assuming that $P_N \to P$, with the understanding that this is along a subsequence. We now check that P satisfies the four conditions of Lemma 18. Define $\rho := P \circ (D, \boldsymbol{Y}, \Gamma, m)^{-1}$. By definition of P_N in (30), we have that

$$P_N \circ (D, \boldsymbol{Y}, \Gamma)^{-1} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{P} \circ \left(D, \boldsymbol{Y}, \Gamma^i\right)^{-1} = \lambda_D \otimes \lambda_Y^{\otimes T'} \otimes \lambda_\Gamma.$$

Therefore, $P_N \circ (D, \boldsymbol{Y}, \Gamma)^{-1}$ satisfies the required law. Since P_N converges in distribution to P, we directly obtain that $\rho \circ (D, \boldsymbol{Y}, \Gamma)$ also satisfies the required law. Therefore, $\rho \in \mathcal{M}$. Since Γ^i and $D, \boldsymbol{Y}$ are independent under $\mathbb{P}$ for each i, it follows that Γ and $D, \boldsymbol{Y}$ are independent under P_N for each N. Thus, Γ, D and $\boldsymbol{Y}$ are independent under P.

Since $\left(q^{1,N}, \ldots, q^{N,N}\right)$ is a Nash equilibrium, we have by Corollary 8 that

$$\sup_N \frac{1}{N} \mathbb{E}^{\mathbb{P}}\left[\sum_{i=1}^{N} \sum_{t=1}^{T-1} (q_t^{i,N})^2\right] \leq C.$$

Moreover, all processes $q^{i,N}$ are uniformly bounded in N. Therefore, by taking the limit in the previous equation, we get that $\mathbb{E}^P\left[\sum_{t=1}^{T-1} (q_t)^2\right] \leq C$.

Moreover, since $\left(\Gamma^i, q^i, Q^i\right)$ verifies the state equation under $\mathbb{P}$, the canonical processes $(D, \boldsymbol{Y}, \Gamma, m, q, D)$ also verify the state equation under each P_N. Since all processes are uniformly bounded in N, we directly obtain that the state equation also holds under the limit P.

We now check the third condition: Consider $\Phi : E^{T-1} \to \mathbb{R}$ and $\Psi : \mathcal{D}^{T-1} \times \mathcal{Y}^{T'} \to \mathbb{R}$ two bounded and Lipschitz continuous mappings with corresponding Lipschitz constants L_Φ and L_Ψ. Note that by Remark 2 and by definition of E, we have $d_1(m^N, m^{N,i}) \leq \frac{2C'_e}{N}$ almost surely. Therefore,

$$\left|\mathbb{E}^{\mathbb{P}}\left[\Psi(D, \boldsymbol{Y})\frac{1}{N}\sum_{i=1}^{N}\int_E \Phi \mathrm{d}(m^N, m^{N,i})\right]\right| \leq \frac{2C'_e L_\Phi}{N}\mathbb{E}^{\mathbb{P}}\left[|\Psi(D, \boldsymbol{Y})|\right] \to 0.$$

We finally obtain that

$$\begin{aligned}\mathbb{E}^P\left[\Psi(D, \boldsymbol{Y})\Phi(Q)\right] &= \lim_{N\to\infty}\mathbb{E}^{\mathbb{P}}\left[\Psi(D, \boldsymbol{Y})\frac{1}{N}\sum_{i=1}^{N}\int_E \Phi \mathrm{d}m^{N,i}\right]\\ &= \mathbb{E}^P\left[\Psi(D, \boldsymbol{Y})\int_K \Phi \mathrm{d}m)\right].\end{aligned}$$

Finally, from the density of Lipschitz functions in the space of bounded uniformly continuous functions, we obtain that for all $\Phi : E^{T-1} \to \mathbb{R}$ and $\Psi : \mathcal{D}^{T-1} \times \mathcal{Y}^{T'} \to \mathbb{R}$ bounded and uniformly continuous mappings, $\mathbb{E}^P\left[\Psi(D, \boldsymbol{Y})\Phi(Q)\right] = \mathbb{E}^P\left[\Psi(D, \boldsymbol{Y})\int_K \Phi \mathrm{d}m\right]$. Therefore, we conclude that conditioned on the common noise, the law of Q under P is m.

We now check the final condition, which requires to prove that P is optimal, i.e., that $P \in \mathcal{RA}^*(\rho)$, and so that for all $\tilde{P} \in \mathcal{RA}(\rho)$, $J(\tilde{P}) \geq J(P)$. Let $\tilde{P} \in \mathcal{RA}(\rho)$. We have $\tilde{P} \circ (D, \boldsymbol{Y}, \Gamma, m)^{-1} = \rho$. Moreover, there exists $\tilde{q} \in \boldsymbol{A}$ such that $\tilde{P} \circ (D, \boldsymbol{Y}, \Gamma, m, q, Q)^{-1}$ denotes the joint law of the solution and the inputs to state equation (20) associated with control $\tilde{q}$. Denote $Q^{\tilde{q}}$ the associated state process.
For $1 \leq k \leq N$, let

$$\tilde{P}_{N,k} := \mathbb{P} \circ (D, \boldsymbol{Y}, \Gamma^k, m^{N,k}, \tilde{q}, Q^{\tilde{q}})^{-1}.$$

By definition of the P_N and their weak convergence toward P, and since

$$\tilde{P}_{N,k} \circ (D, \boldsymbol{Y}, \Gamma^k)^{-1} = P \circ (D, \boldsymbol{Y}, \Gamma^k)^{-1},$$

we obtain that $\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\tilde{P}_{N,k} = \tilde{P}$. It is fairly straightforward to verify that J is linear. The continuity of J of Lemma 19 implies

$$\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} J(\tilde{P}_{N,k}) = \lim_{N\to\infty} J(\frac{1}{N}\sum_{k=1}^{N}\tilde{P}_{N,k}) = J(\tilde{P}).$$

By Lemma 19, we have that $J(P) = \lim_{N\to\infty} J(P_N) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\mathcal{J}^k(q^k, \boldsymbol{q}^{-k})$. Moreover, by definition of $\tilde{P}_{N,k}$, we obtain that

$$\mathbb{E}^{\mathbb{P}}\left[\Lambda(D, \boldsymbol{Y}, \Gamma^k, m^{N,k}, \tilde{q}, Q^{\tilde{q}})\right] = J(\tilde{P}_{N,k}).$$

Finally, since all P_N are Nash equilibria, we have

$$J(P) \leq \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\mathcal{J}^k(\tilde{q}, \boldsymbol{q}^{-k}) \leq \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} J(\tilde{P}_{N,k}) = J(\tilde{P}).$$

This finally proves that $P \in \mathcal{RA}^*(\rho)$. ◻

Conclusion

We developed in this paper a new modeling framework which allows to study the long-term evolution of investment into renewable resources under uncertainty and risk aversion. We contribute to bridging the gap between the equilibrium models and agent-based models, and the mathematical approach of mean field games, by using the MFG framework to derive strong theoretical guarantees for games with large number of players, while striving to introduce a simple and interpretable model with closed-form solutions. We prove that both the N-player game and the corresponding MFG game admit an equilibrium solution, which we characterize through a closed-form solution. We also prove that any sequence of Nash equilibria to the N-player game converges to the unique solution of the MFG game. We develop a toy model applied to the specific case of France electricity market. Our numerical experiments highlight the importance of the risk aversion parameter and the analysis of invested capacity spread, allowed by the explicit modeling of heterogeneity. We show that our results derived in the MFG framework with heterogeneity regarding geographical localization cannot be reproduced with a representative agent's model. This highlights the importance of explicitly modeling heterogeneity in energy prospective models. This paper paves the way for future research into the impact of agents' heterogeneity on the prospective evolution of energy markets.

This paper constitutes a first step and calls for future research and improvements. In particular, the representation of the price mechanism could be improved from the current static representation through the merit-order curve to a more evolved representation capturing dynamic effects caused by storage and demand flexibility. Another future line of research could strive to modify the scope of the model by adding other types of producers, such as storage producers, in order to come closer to partial equilibrium models of the whole electricity market. It would also be interesting to include a time dependence for the weather parameter.

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Declarations

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Appendix A Technical lemmas

This section is concerned with proving some technical lemmas on the structure of our problem.

A.1 Some properties of the Expected Shortfall

Lemma 23 *Let A, B be two random variables on $(\Omega, \mathcal{F}, \mathbb{P})$, and let $\mathcal{F}'$ be a sub-σ-field of $\mathcal{F}$. Then,*

$$\left|\mathbf{ES}^{\alpha}_{\mathcal{F}'}(A) - \mathbf{ES}^{\alpha}_{\mathcal{F}'}(B)\right| \leq \frac{\mathbb{E}\left[|A-B| \mid \mathcal{F}'\right]}{\alpha}, \tag{A1}$$

$$\mathbf{ES}^{\alpha}_{\mathcal{F}'}(A) - \mathbf{ES}^{\alpha}_{\mathcal{F}'}(B) \geq \frac{1}{\alpha} \inf_{x\in\mathbb{R}} \mathbb{E}\left[(A-x)^+ - (B-x)^+ \mid \mathcal{F}'\right]. \tag{A2}$$

Proof The proof follows [2, p.14]. □

A.2 Lemmas for the *N*-player game

The following lemma uses the fact that the market price is bounded to derive some bounds on the coefficients in the cost function.

Lemma 24 *There exists $K > 0$ such that for all $m \in \mathcal{M}_2(\mathbb{R}^+)$, for all $D, D' \in \mathcal{D}$, and for all $\boldsymbol{Y} \in \mathcal{Y}$,*

$$|f^{\beta}(t, m, D, D', \boldsymbol{Y})| \leq K. \tag{A3}$$

Proof Definition of function f in Equation (9) and the fact that the price mapping is bounded directly yields that function f is bounded. Moreover, definition of expected shortfall (12) gives that $\mathbf{ES}^{\alpha}_{D'}(f(t, m, ., .))$ is also bounded. Finally, by definition of function f^{β} in Equation (15), we obtain that there exists $K > 0$ satisfying (A3). □

The following lemma is an estimate of the second-order moment of all suboptimal controls.

Lemma 25 *Let $\boldsymbol{q} \in \prod_{i\in\mathcal{N}} \boldsymbol{A}$ and $C_1 > 0$. There exists $C_2 > 0$ independent from N such that for any $i \in \mathcal{N}$,*

- *if $\hat{q}^i$ satisfies*

$$\mathcal{J}^i\left(\hat{q}^i, \boldsymbol{q}^{-i}\right) \leq \inf_{q^i\in\boldsymbol{A}} \mathcal{J}^i\left(q^i, \boldsymbol{q}^{-i}\right) + C_1, \tag{A4}$$

  and then, $\hat{q}^i \in \boldsymbol{A}_{C_2}$.
- *if $\hat{q}^i$ satisfies*

$$\bar{\mathcal{J}}^i\left(\hat{q}^i, \boldsymbol{q}\right) \leq \inf_{q^i\in\boldsymbol{A}} \bar{\mathcal{J}}^i\left(q^i, \boldsymbol{q}\right) + C_1, \tag{A5}$$

  then $\hat{q}^i \in \boldsymbol{A}_{C_2}$.

Proof We will prove the first point, as the second point directly follows from the same proof. Let $i \in \mathcal{N}$ and let $\hat{q}^i$ satisfy (A4). Then,

$$\mathcal{J}^i\left(\hat{q}^i, \boldsymbol{q}^{-i}\right) \leq \mathcal{J}^i\left(0, \boldsymbol{q}^{-i}\right) + C_1 = C_1. \tag{A6}$$

We need now to bound $\mathcal{J}^i\left(\hat{q}^i, \boldsymbol{q}^{-i}\right)$ from below. Lemma 24 directly gives us that there exists a constant $K > 0$ such that for all $m \in \mathcal{M}_2(\mathbb{R}_+)$, for all $D_t, D_{t-1} \in \mathcal{D}$, for all $\boldsymbol{Y}_t \in \mathcal{Y}$, $f^\beta(t, m, D_t, D_{t-1}, \boldsymbol{Y}_t) \geq -K$. Therefore, combining this previous result with the fact that $q_t^i \geq 0$ a.s., and that $c_t > 0$, we have that

$$L^\beta(t, Q_t^i, q_t^i, m, D_t, D_{t-1}, \boldsymbol{Y}_t) \geq e^{-rt}\left(-Q_t^i K + \tilde{c} q_t^i)^2\right) \quad \text{a.s..}$$

Therefore, we obtain by combining the definition of the state equation and Young's inequality that

$$\begin{aligned}
\mathcal{J}^i\left(\hat{q}^i, \boldsymbol{q}^{-i}\right) &\geq \mathbb{E}\left[\sum_{t=1}^{T-1}\left(\tilde{c}|q_t^i|^2 - Q_t^i K\right) - Q_T^i K\right] \\
&\geq \mathbb{E}\left[\sum_{t=1}^{T-1}\left(\tilde{c}|q_t^i|^2 - 2\sum_{t=1}^{T-1}\frac{KT}{\sqrt{\tilde{c}}}\sqrt{\tilde{c}}q_t^i\right)\right] \qquad \text{(A7)}\\
&\geq \frac{\tilde{c}}{2}\mathbb{E}\left[\sum_{t=1}^{T-1}|q_t^i|^2\right] - K'.
\end{aligned}$$

By combining (A6) and (A7), we finally obtain that $\mathbb{E}\left[\sum_{t=1}^{T-1}|q_t^i|^2\right] \leq \frac{2}{\tilde{c}}\left(C_1 + K'\right) =: C_2$ as announced. □

We now move to some lemmas corresponding to the limiting game. We first prove that the different mappings are Lipschitz with respect to the Kantorovich–Rubinstein distance.

Lemma 26 *Let $m_1, m_2 \in \mathcal{M}_2(\mathbb{R}^+)$, $D \in \mathcal{D}$ and $\boldsymbol{Y} \in \mathcal{Y}$. Then,*

$$\begin{aligned}
|R(h, m_1, D, \boldsymbol{Y}) - R(h, m_2, D, \boldsymbol{Y})| &\leq d_1(m_1, m_2), \\
|\phi(t, h, m_1, D, \boldsymbol{Y}) - \phi(t, h, m_2, D, \boldsymbol{Y})| &\leq L_F d_1(m_1, m_2).
\end{aligned}$$

Proof By definition (6) of R,

$$\begin{aligned}
&|R(h, m_1, D, \boldsymbol{Y}) - R(h, m_2, D, \boldsymbol{Y})| \\
&= \left|\left(D + \tilde{D} - \Gamma_h \int x \mathrm{d}m_1(x)\right)_+ - \left(D + \tilde{D} - \Gamma_h \int x \mathrm{d}m_2(x)\right)_+\right| \\
&\leq d_1(m_1, m_2)
\end{aligned}$$

where we used the fact that $\Gamma_h \in [0, 1]$ and the definition of the distance d_1 in (1). By definition (7) of ϕ,

$$\begin{aligned}
&|\phi(t, h, m_1, D, \boldsymbol{Y}) - \phi(t, h, m_2, D, \boldsymbol{Y})| \\
&= \left|F^{-1}(t, R(h, m_1, D, \boldsymbol{Y})) \wedge \bar{P} - F^{-1}(t, R(h, m_2, D, \boldsymbol{Y})) \wedge \bar{P}\right| \\
&\leq \left|F^{-1}(t, R(h, m_1, D, \boldsymbol{Y})) - F^{-1}(t, R(h, m_2, D, \boldsymbol{Y}))\right| \\
&\leq L_F\left|R(h, m_1, D, \boldsymbol{Y}) - R(h, m_2, D, \boldsymbol{Y})\right| \\
&\leq L_F d_1(m_1, m_2),
\end{aligned}$$

where line 4 comes from Assumption 6. □

Lemma 27 *Let $m_1, m_2 \in \mathcal{M}_2(\mathbb{R}^+)$, $D, D' \in \mathcal{D}$ and $\boldsymbol{Y} \in \mathcal{Y}$. Then, there exists a constant $L_f > 0$ such that*

$$\left| f^\beta(t, m_1, D, D', \boldsymbol{Y}) - f^\beta(t, m_2, D, D', \boldsymbol{Y}) \right| \leq L_f d_1(m_1, m_2).$$

Proof The proof directly follows from Lemma 26, relying on Lemma 23. Details are given in supplementary material [27, Lemma A.5]. □

Lemma 28 *Let $m_T^1, m_T^2 \in \mathcal{M}_2(\mathbb{R}^+)$, $D_{T-1} \in \mathcal{D}$ and $(\boldsymbol{Y}_t)_{T \leq t \leq T'} \in \mathcal{Y}^{T'-T+1}$. Then, there exists a constant $L_g > 0$ such that*

$$\left| g(m_T^1, D_{T-1}, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) - g(m_T^2, D_{T-1}, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) \right| \leq L_g d_1(m_T^1, m_T^2).$$

Proof See supplementary material [27, Lemma A.6]. □

The next lemma introduces a monotonicity condition similar in spirit to the ones introduced in Cardaliaguet et al. [17] or Ahuja [3].

Lemma 29 *Assume Assumption 16 and Assumption 15 hold, and fix $\epsilon > 0$ as defined in Assumption 16. Let $t \in \mathcal{T}$. There exists $K^\epsilon_{(A8)} > 0$ such that for all $\boldsymbol{Y} \in \mathcal{Y}$, for all $D, D' \in \mathcal{D}$, for all $m^1, m^2 \in \mathcal{M}_2(\mathbb{R}_+)$ such that $0 \leq \int x \mathrm{d}m^1 < \int x \mathrm{d}m^2 \leq C'_e$, we have*

$$f^\beta(t, m^1, D, D', \boldsymbol{Y}) - f^\beta(t, m^2, D, D', \boldsymbol{Y}) \leq K^\epsilon_{(A8)} \int x \mathrm{d}(m^1 - m^2) \leq 0. \tag{A8}$$

Proof Note that since $\int x \mathrm{d}m^1 < \int x \mathrm{d}m^2$,

$$F^{-1}(t, R(h, m^1, D, \boldsymbol{Y})) \geq F^{-1}(t, R(h, m^2, D, \boldsymbol{Y})),$$

for all $D \in \mathcal{D}$ and $\boldsymbol{Y} \in \mathcal{Y}$. By Assumption 16, for all $\boldsymbol{Y} \in \mathcal{Y}$, $\mathcal{H}_t^\epsilon \neq \emptyset$. Then, for all $D \in \mathcal{D}$ and $\boldsymbol{Y} \in \mathcal{Y}$,

$$\begin{aligned}
f(t, &m^1, D, \boldsymbol{Y}) - f(t, m^2, D, \boldsymbol{Y}) \\
&= - \sum_{h \in \mathcal{H}} \Gamma_{t,h} \left(F^{-1}(t, R(h, m^1, D, \boldsymbol{Y})) \wedge \bar{P} - F^{-1}(t, R(h, m^2, D, \boldsymbol{Y})) \wedge \bar{P} \right) \\
&\leq \sum_{h \in \mathcal{H}_t^\epsilon} \Gamma_{t,h} \left(F^{-1}(t, R(h, m^2, D, \boldsymbol{Y})) - F^{-1}(t, R(h, m^1, D, \boldsymbol{Y})) \right) \\
&\leq \sum_{h \in \mathcal{H}_t^\epsilon} \Gamma_{t,h} C_{(26)} \left(R(h, m^2, D, \boldsymbol{Y}) - R(t, m^1, D, \boldsymbol{Y}) \right) \\
&\leq \sum_{h \in \mathcal{H}_t^\epsilon} \Gamma_{t,h}^2 C_{(26)} \int x \mathrm{d}(m^1 - m^2) \\
&\leq \left| \mathcal{H}_t^\epsilon \right| C_{(26)} \epsilon^2 \int x \mathrm{d}(m^1 - m^2) \leq 0
\end{aligned} \tag{A9}$$

where third line comes from Assumption 15, and the rest follows from the definition of the subset $\mathcal{H}_t^\epsilon$.

Moreover, (A2) from Lemma 23 gives that

$$\begin{aligned}
&\mathbf{ES}^\alpha_{D'}(f(t, m^2, ., .)) - \mathbf{ES}^\alpha_{D'}(f(t, m^1, ., .)) \\
&\quad \geq \frac{1}{\alpha} \inf_{x \in \mathbb{R}} \mathbb{E}\left[(f(t, m^2, D, \boldsymbol{Y}) - x)^+ - (f(t, m^1, D, \boldsymbol{Y}) - x)^+ \mid D' \right] \geq 0
\end{aligned} \tag{A10}$$

using (A9) for the last inequality. Combining (A9) and (A10), we get that for all $D, D' \in \mathcal{D}$ and for all $\boldsymbol{Y} \in \mathcal{Y}$,

$$
\begin{aligned}
& f^{\beta}(t, m^1, D, D', \boldsymbol{Y}) - f^{\beta}(t, m^2, D, D', \boldsymbol{Y}) \\
& = \beta \left(f(t, m^1, D, \boldsymbol{Y}) - f(t, m^2, D, \boldsymbol{Y}) \right) + (1-\beta) \left(\mathbf{ES}^{\alpha}_{D'}(f(t, m^1, ., .)) \right. \\
& \quad \left. - \mathbf{ES}^{\alpha}_{D'}(f(t, m^2, ., .)) \right) \\
& \leq \beta \left| \mathcal{H}^{\epsilon}_t \right| C_{(26)} \epsilon^2 \int x \mathrm{d}(m^1 - m^2).
\end{aligned}
$$

We obtain the announced result with $K^{\epsilon}_{(A8)} = \beta \left| \mathcal{H}^{\epsilon}_t \right| C_{(26)} \epsilon^2$. □

Lemma 30 *Assume Assumption 16 and Assumption 15 hold, and fix $\epsilon > 0$ as defined in Assumption 16. Then, there exists $K^{\epsilon}_{(A11)} > 0$ such that for all $\boldsymbol{Y} \in \mathcal{Y}$, for all $D, D' \in \mathcal{D}$, for all $m^1, m^2 \in \mathcal{M}_2(\mathbb{R}_+)$ such that $0 \leq \int x \mathrm{d}m^1 < \int x \mathrm{d}m^2 \leq C'_e$,*

$$
g(m^1, D, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) - g(m^2, D, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) \leq K^{\epsilon}_{(A11)} \int x \mathrm{d}(m^1 - m^2). \tag{A11}
$$

Proof We mimic the proof of Lemma 29 with $\beta = 1$, which gives, in view of the definition (11) of g,

$$
\begin{aligned}
& g(m^1, D, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) - g(m^2, D, (\boldsymbol{Y}_t)_{T \leq t \leq T'}) \\
& \leq \sum_{t=T}^{T'} e^{-r(t-T)} (1-\nu)^{t-T} \left| \mathcal{H}^{\epsilon}_t \right| C_{(26)} \epsilon^2 \int (1-\nu)^{t-T} x \mathrm{d}(m^1 - m^2).
\end{aligned}
$$

We conclude by defining $K^{\epsilon}_{(A11)} = C_{(26)} \epsilon^2 \sum_{t=T}^{T'} \left| \mathcal{H}^{\epsilon}_t \right| > 0$. □

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