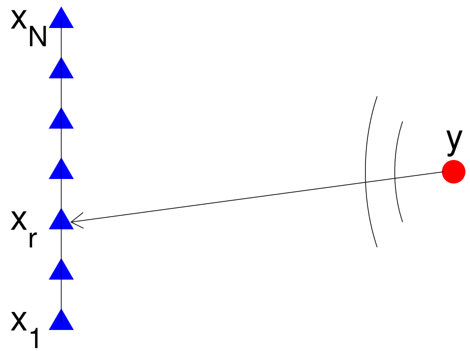
Source imaging

Source imaging: data acquisition



Passive array $\Longleftrightarrow$ The sensors are receivers.

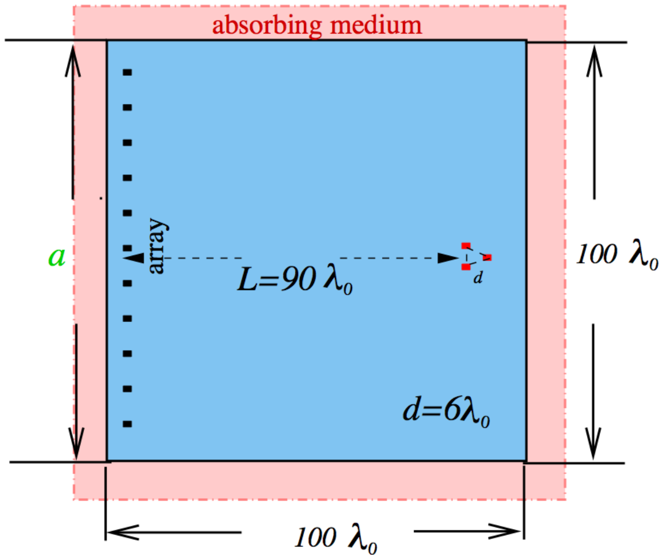
The source $\boldsymbol{y}$ emits a short pulse.

The sensors $(\boldsymbol{x}_r)_{r=1,\dots,N}$ record the waves.

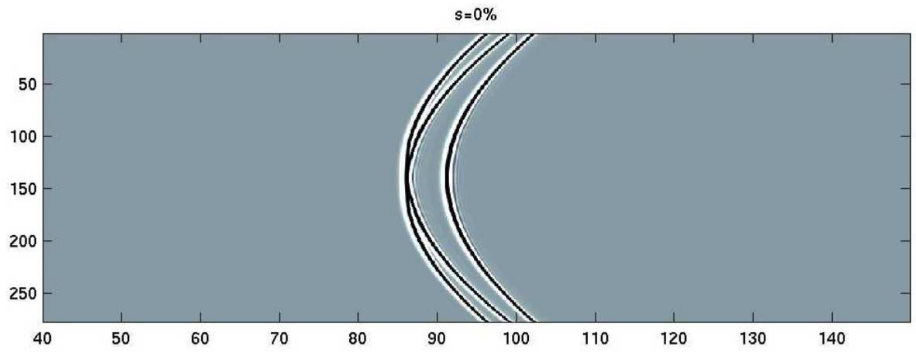
The data set is $\big(u(t,\boldsymbol{x}_r)\big)_{t\in\mathbb{R},r=1,\dots,N}$.

Goal: find the source position $\boldsymbol{y}$ (more generally, find the *source* region).

Source imaging: simulation

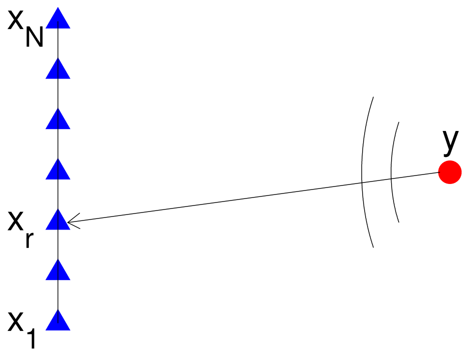


Configuration

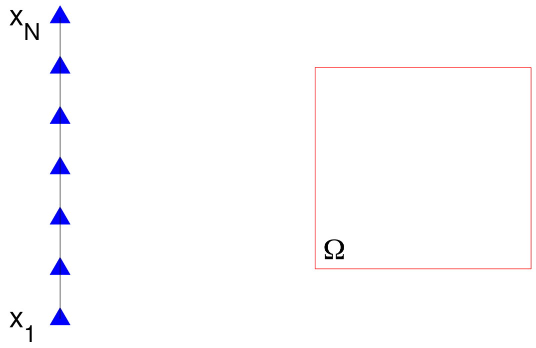


Data set $(u(t,\boldsymbol{x}_r))_{80\leq t\leq 200, r=1,\ldots,270}$

Source imaging: imaging function



Data acquisition



Search region

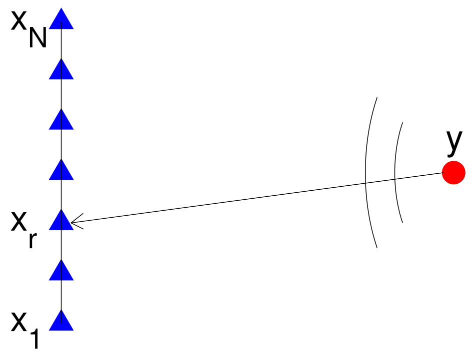
Goal: find the source point $\boldsymbol{y}$ (more generally, find the *source* region).

The data set is $\big(u(t, \boldsymbol{x}_r)\big)_{t \in \mathbb{R}, r=1,\ldots,N}$.

$\hookrightarrow$ Given the data set, build an imaging function in the search region $\Omega \subset \mathbb{R}^3$:

$$\mathcal{I} : \left| \begin{array}{l} \Omega \to \mathbb{R}^+ \\ \boldsymbol{y}^S \mapsto \mathcal{I}(\boldsymbol{y}^S) \end{array} \right.$$ which plots an image of the search region.

Source imaging - the linear forward operator



The source term is of the form $n(t, \boldsymbol{y}) = \rho_{\rm real}(\boldsymbol{y})\delta(t)$.

Goal: find the source function $\rho_{\rm real}$.

Here: the Green's function is known.

The data set is $\hat{\boldsymbol{u}} = \big(\hat{u}(\omega, \boldsymbol{x}_r)\big)_{\omega \in \mathcal{B}, r=1,\dots,N}$ with $\hat{u}(\omega, \boldsymbol{x}_r) = \int_\Omega \hat{G}(\omega, \boldsymbol{x}_r, \boldsymbol{y})\rho_{\rm real}(\boldsymbol{y})d\boldsymbol{y}$.

We define

$$[\hat{\mathbf{A}}\rho](\omega, \boldsymbol{x}_r) = \int_\Omega \hat{G}(\omega, \boldsymbol{x}_r, \boldsymbol{y})\rho(\boldsymbol{y})d\boldsymbol{y}.$$

$\hat{\mathbf{A}}$ is the linear operator that maps the source function to the array data $\hat{\boldsymbol{u}}$:

$$\hat{\boldsymbol{u}} = \hat{\mathbf{A}}\rho_{\rm real}$$

Source imaging - the adjoint problem

The forward operator $\hat{\mathbf{A}}$ is the linear operator from $L^2(\Omega)$ to $L^2(\mathcal{B} \times \{1, \dots, N\})$:

$$[\hat{\mathbf{A}}\rho](\omega, \boldsymbol{x}_r) = \int_\Omega \hat{G}(\omega, \boldsymbol{x}_r, \boldsymbol{y})\rho(\boldsymbol{y})d\boldsymbol{y}.$$

The adjoint operator is such that, for all test functions $\hat{u}(\omega, \boldsymbol{x}_r)$ and $\rho(\boldsymbol{y})$:

$$\sum_{r=1}^N \int_\mathcal{B} \overline{\hat{u}(\omega, \boldsymbol{x}_r)}[\hat{\mathbf{A}}\rho](\omega, \boldsymbol{x}_r)d\omega = \int_\Omega \overline{[\hat{\mathbf{A}}^*\hat{\boldsymbol{u}}](\boldsymbol{y})}\rho(\boldsymbol{y})d\boldsymbol{y}$$

(Existence and uniqueness guaranteed by Riesz representation theorem)

The adjoint operator is

$$[\hat{\mathbf{A}}^*\hat{\boldsymbol{u}}](\boldsymbol{y}) = \sum_{r=1}^N \int_\mathcal{B} \overline{\hat{G}(\omega, \boldsymbol{y}, \boldsymbol{x}_r)}\hat{u}(\omega, \boldsymbol{x}_r)d\omega$$

The normal operator $\hat{\mathbf{A}}^*\hat{\mathbf{A}}$ is the operator with kernel

$$[\hat{\mathbf{A}}^*\hat{\mathbf{A}}\rho](\boldsymbol{y}) = \int_\Omega a(\boldsymbol{y}, \boldsymbol{y}')\rho(\boldsymbol{y}')d\boldsymbol{y}'$$

$$a(\boldsymbol{y}, \boldsymbol{y}') = \sum_{r=1}^N \int_\mathcal{B} \overline{\hat{G}(\omega, \boldsymbol{y}, \boldsymbol{x}_r)}\hat{G}(\omega, \boldsymbol{y}', \boldsymbol{x}_r)d\omega$$

Source imaging - the inverse problem

The least squares inverse problem is to minimize $J[\rho]$ where

$$J[\rho] = \int d\omega \sum_r \left| \hat{u}(\omega, \boldsymbol{x}_r) - [\hat{\mathbf{A}}(\omega)\rho](\boldsymbol{x}_r) \right|^2$$

Solution:

$$\rho_{\rm LS} = \big(\hat{\mathbf{A}}^* \hat{\mathbf{A}}\big)^{-1} \big(\hat{\mathbf{A}}^* \hat{\boldsymbol{u}}\big)$$

(Mercer's theorem gives an explicit representation of the kernel of the normal operator $\hat{\mathbf{A}}^* \hat{\mathbf{A}}$, which then gives an expression of the -regularized- inverse).

The adjoint operator is

$$[\hat{\mathbf{A}}^* \hat{\boldsymbol{u}}](\boldsymbol{y}) = \sum_r \int \overline{\hat{G}(\omega, \boldsymbol{y}, \boldsymbol{x}_r)} \hat{u}(\omega, \boldsymbol{x}_r) d\omega$$

Note: complex conjugation = time reversal.

Adjoint operator = backpropagation to the test point $\boldsymbol{y}$.

Source imaging - the reverse-time imaging function

• Least-squares imaging function:

$$\mathcal{I}_{\rm LS}(\boldsymbol{y}^S) = \Big[\big(\hat{\mathbf{A}}^*\hat{\mathbf{A}}\big)^{-1}\big(\hat{\mathbf{A}}^*\hat{\boldsymbol{u}}\big)\Big](\boldsymbol{y}^S)$$

with

$$\big[\hat{\mathbf{A}}^*\hat{\boldsymbol{u}}\big](\boldsymbol{y}) = \sum_r \int \overline{\hat{G}(\omega,\boldsymbol{y},\boldsymbol{x}_r)}\hat{u}(\omega,\boldsymbol{x}_r)d\omega$$

• The operator $\hat{\mathbf{A}}^*\hat{\mathbf{A}}$ is approximately proportional to the identity operator (related to time-reversal refocusing property) $\to$ drop the normalizing factor in the LS function.

$\hookrightarrow$ Reverse Time imaging function for the search point $\boldsymbol{y}^S$:

$$\mathcal{I}_{\rm RT}(\boldsymbol{y}^S) = \frac{1}{2\pi}[\hat{\mathbf{A}}^*\hat{\boldsymbol{u}}](\boldsymbol{y}^S) = \frac{1}{2\pi}\int d\omega \sum_r \overline{\hat{G}(\omega,\boldsymbol{y}^S,\boldsymbol{x}_r)}\hat{u}(\omega,\boldsymbol{x}_r)$$

Kirchhoff Migration (or travel-time migration) for source imaging

- Reverse Time imaging function for the search point $\boldsymbol{y}^S$:

$$\mathcal{I}_{\mathrm{RT}}(\boldsymbol{y}^S) = \frac{1}{2\pi}\int d\omega \sum_r \overline{\hat{G}(\omega,\boldsymbol{y}^S,\boldsymbol{x}_r)}\hat{u}(\omega,\boldsymbol{x}_r)$$

- If we take $\hat{G}(\omega,\boldsymbol{x},\boldsymbol{y}) \simeq \exp[i\omega\mathcal{T}(\boldsymbol{x},\boldsymbol{y})]$, where $\mathcal{T}(\boldsymbol{x},\boldsymbol{y})$ is the travel time from $\boldsymbol{x}$ to $\boldsymbol{y}$, then we get the Kirchhoff Migration imaging function:

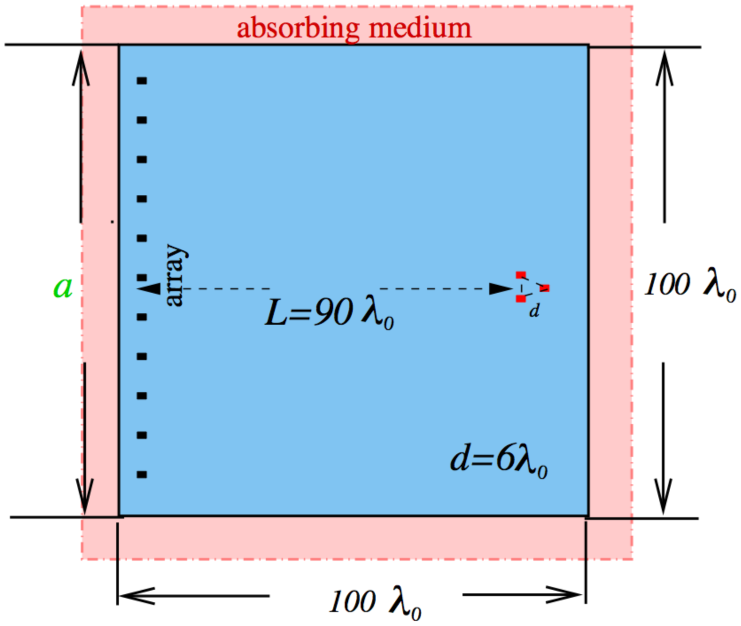
$$\begin{aligned}\mathcal{I}_{\mathrm{KM}}(\boldsymbol{y}^S) &= \frac{1}{2\pi}\int d\omega \sum_r \exp[-i\omega\mathcal{T}(\boldsymbol{x}_r,\boldsymbol{y}^S)]\hat{u}(\omega,\boldsymbol{x}_r) \\ &= \sum_r u\big(\mathcal{T}(\boldsymbol{x}_r,\boldsymbol{y}^S),\boldsymbol{x}_r\big)\end{aligned}$$

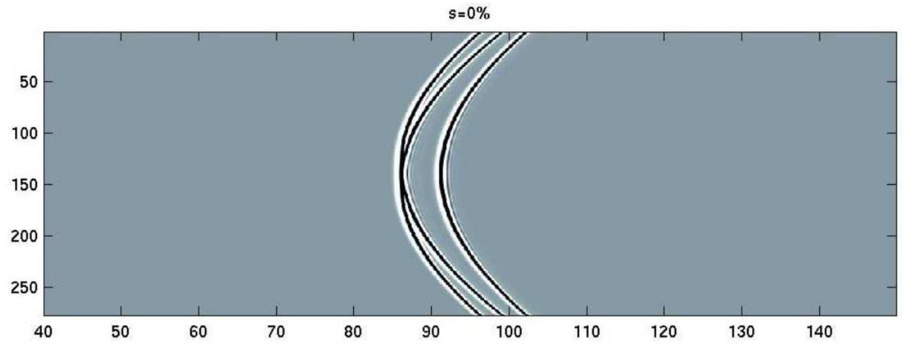
If the background medium is homogeneous, then $\mathcal{T}(\boldsymbol{x},\boldsymbol{y}) = |\boldsymbol{x}-\boldsymbol{y}|/c_o$.

Kirchhoff Migration (or travel time migration) has been analyzed in detail and is used extensively in practice.

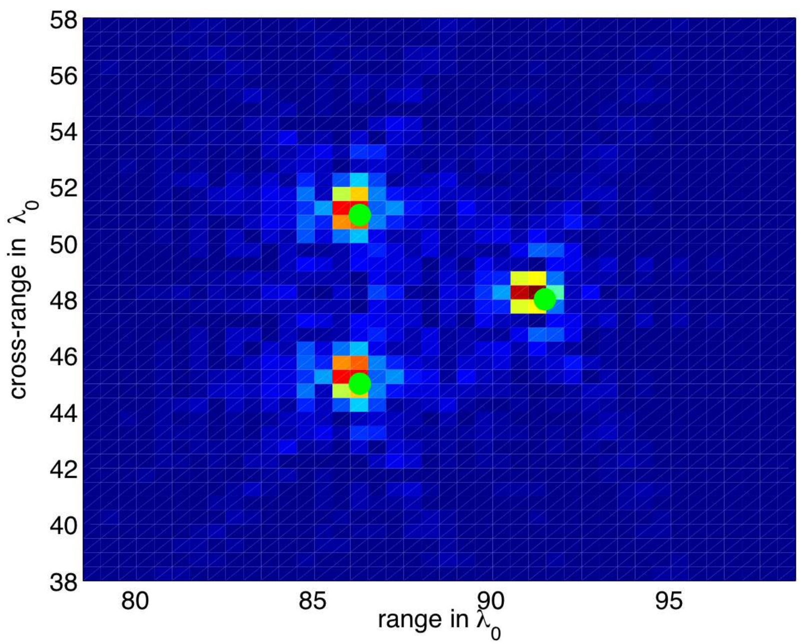
Cf: N. Bleistein, J. K. Cohen, and J. W. Stockwell Jr, Mathematics of multidimensional seismic imaging, migration, and inversion, Springer, 2001.

Kirchhoff migration for source imaging: simulation





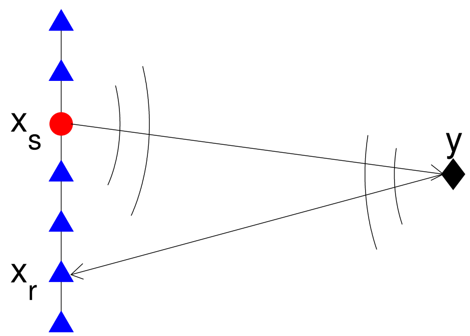
Data $\big(u(t, \boldsymbol{x}_r)\big)_{t \in [40,150], r=1,\dots,264}$



KM imaging function $\mathcal{I}_{\rm KM}(\boldsymbol{y}^S)$

Reflector imaging

Reflector imaging: data acquisition



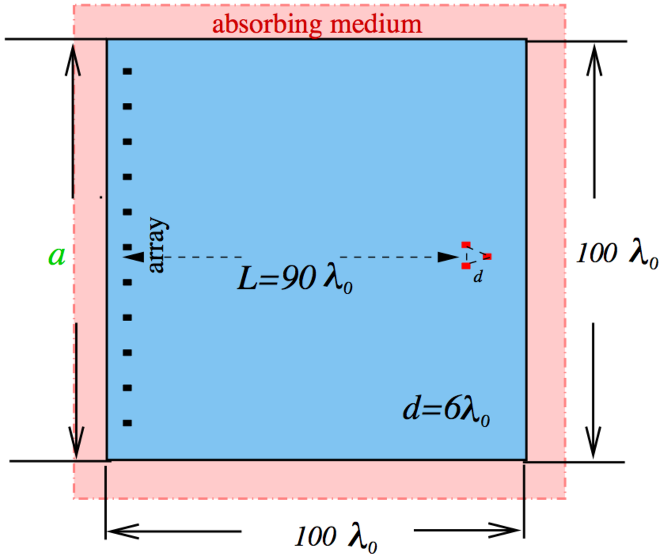
Active array $\Longleftrightarrow$ The sensors can be used as sources and/or as receivers.

For each $s = 1, \dots, N$:
- The source $\boldsymbol{x}_s$ emits a short pulse.
- The sensors $(\boldsymbol{x}_r)_{r=1,\dots,N}$ record the waves.

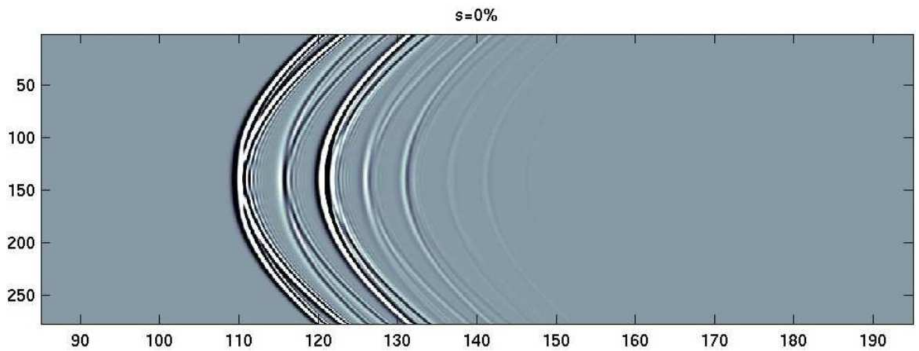
The data set is $(u(t, \boldsymbol{x}_r, \boldsymbol{x}_s))_{t \in \mathbb{R}, r,s=1,\dots,N}$ (the time-dependent response matrix).

Goal: find the reflector position $\boldsymbol{y}$ (more generally, find the reflectivity function of the *medium*).

Reflector imaging: simulation

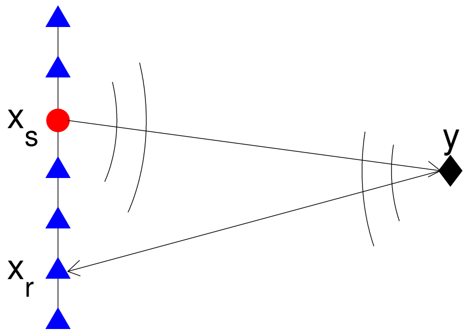


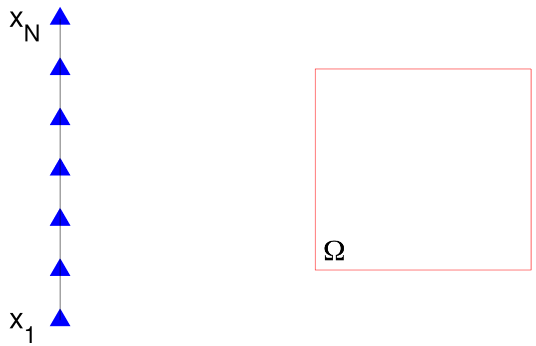
Configuration



Data set $(u(t, \boldsymbol{x}_r, \boldsymbol{x}_{135}))_{80 \leq t \leq 200, r=1,\ldots,270}$
(traces recorded for a central illumination)

Reflector imaging: imaging function





Data acquisition Search region

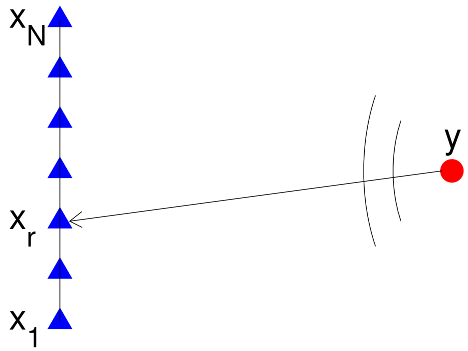
Goal: find the reflector position $\boldsymbol{y}$ (more generally, find the reflectivity function of the *medium*).

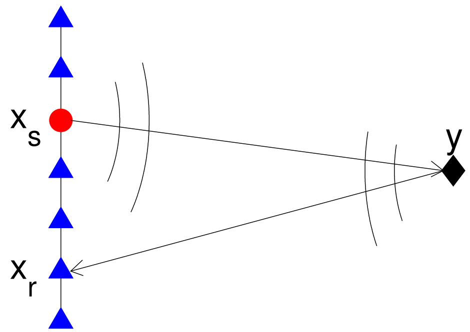
The data set is $(u(t, \boldsymbol{x}_r, \boldsymbol{x}_s))_{t\in\mathbb{R}, r,s=1,\ldots,N}$.

$\hookrightarrow$ Given the data set, build an imaging function in the search region $\Omega \subset \mathbb{R}^3$:

$$\mathcal{I} : \left| \begin{array}{l} \Omega \to \mathbb{R}^+ \\ \boldsymbol{y}^S \mapsto \mathcal{I}(\boldsymbol{y}^S) \end{array} \right. \quad \text{which plots an image of the search region.}$$

Source and reflector imaging: comparison





Passive array of sensors

The sensors $(\boldsymbol{x}_r)_{r=1,\ldots,N}$ record

$\boldsymbol{y}$ is a source

Data: $\big(u(t, \boldsymbol{x}_r)\big)_{t\in\mathbb{R}, r=1,\ldots,N}$

Active array of sensors/sources

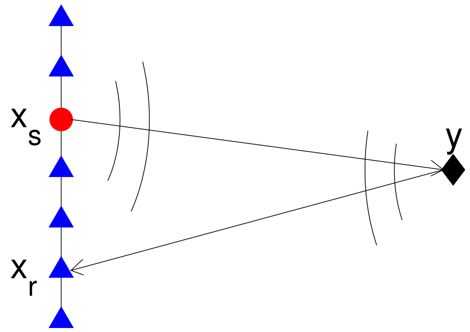
The sensors $(\boldsymbol{x}_r)_{r=1,\ldots,N}$ record

$\boldsymbol{y}$ is a reflector, $\boldsymbol{x}_s$ is a source

Data: $\big(u(t, \boldsymbol{x}_r, \boldsymbol{x}_s)\big)_{t\in\mathbb{R}, r,s=1,\ldots,N}$

Goal: process the data to find $\boldsymbol{y}$.

Reflector imaging - modeling



Goal: find the propagation speed $c_{\text{real}}(\boldsymbol{x})$.

The time Fourier transform of the data $\hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s)$ is

$$\hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) = \hat{G}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s; c_{\text{real}})$$

$\hat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}; c)$ is the Green's function that solves the Helmholtz equation

$$\Delta \hat{G} + \frac{\omega^2}{c^2(\boldsymbol{x})} \hat{G} = -\delta(\boldsymbol{x} - \boldsymbol{y}),$$

with the Sommerfeld radiation condition. It depends on the velocity $c(\boldsymbol{x})$.

Reflector imaging - nonlinear inversion

Data:

$$\hat{\mathbf{u}} = \big(\hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s)\big)_{\omega \in \mathcal{B}, r,s=1,\dots,N} = \big(\hat{G}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s, c_{\mathrm{real}})\big)_{\omega \in \mathcal{B}, r,s=1,\dots,N}$$

Goal: find the propagation speed $c_{\mathrm{real}}(\boldsymbol{x})$.

The inverse problem, the array least squares problem, is:

Minimize

$$J[c] \,\big(+ \alpha \|c\|^2_{\mathrm{REG}} \big),$$

where

$$J[c] = \int d\omega \sum_{r,s} \big| \hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) - \hat{G}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s; c) \big|^2$$

(and α is a strength of regularization parameter).

$\rightarrow$ this is a nonlinear problem for the unknown propagation speed $c(\boldsymbol{x})$.

Reflector imaging - linearization

$$\frac{1}{c^2(\boldsymbol{x})} = \frac{1}{c_o^2}\big(n_0^2(\boldsymbol{x}) + \rho(\boldsymbol{x})\big)$$

where

- c_o is a reference speed,
- $n_0(\boldsymbol{x})$ is a smooth background index of refraction (known, typically constant),
- $\rho(\boldsymbol{x})$ is the target reflectivity (unknown but small).

The Green's function satisfies:

$$\Delta \hat{G} + \frac{\omega^2}{c_o^2}\big(n_0^2(\boldsymbol{x}) + \rho(\boldsymbol{x})\big)\hat{G} = -\delta(\boldsymbol{x} - \boldsymbol{y})$$

The background Green's function is defined by:

$$\Delta \hat{G}_0 + \frac{\omega^2}{c_o^2} n_0^2(\boldsymbol{x})\hat{G}_0 = -\delta(\boldsymbol{x} - \boldsymbol{y})$$

Born approximation:

$$\hat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) = \hat{G}_0(\omega, \boldsymbol{x}, \boldsymbol{y}) + \frac{\omega^2}{c_o^2}\int \hat{G}_0(\omega, \boldsymbol{x}, \boldsymbol{z})\rho(\boldsymbol{z})\hat{G}_0(\omega, \boldsymbol{z}, \boldsymbol{y}) d\boldsymbol{z}$$

First term: direct waves.

Second term: single-scattered waves $\boldsymbol{y} \to \boldsymbol{z} \to \boldsymbol{x}$.

Proof of the Born approximation (1/2)

Consider

$$\begin{aligned}\Delta_{\boldsymbol{z}}\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x})+\frac{\omega^2}{c_o^2}n_0^2(\boldsymbol{z})\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x}) &= -\frac{\omega^2}{c_o^2}\rho(\boldsymbol{z})\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x})-\delta(\boldsymbol{z}-\boldsymbol{x})\\ \Delta_{\boldsymbol{z}}\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})+\frac{\omega^2}{c_o^2}n_0^2(\boldsymbol{z})\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y}) &= -\delta(\boldsymbol{z}-\boldsymbol{y})\end{aligned}$$

We multiply the first equation by $\hat{G}_0(\omega,\boldsymbol{x},\boldsymbol{y})$ and subtract the second equation multiplied by $\hat{G}(\omega,\boldsymbol{x},\boldsymbol{z})$:

$$\begin{aligned}&\nabla_{\boldsymbol{z}}\cdot\Big[\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})\nabla_{\boldsymbol{z}}\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x})-\hat{G}(\omega,\boldsymbol{z},\boldsymbol{y})\nabla_{\boldsymbol{z}}\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{x})\Big]\\ &=-\frac{\omega^2}{c_o^2}\rho(\boldsymbol{z})\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x})\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})-\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})\delta(\boldsymbol{z}-\boldsymbol{x})+\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x})\delta(\boldsymbol{z}-\boldsymbol{y})\\ &=-\frac{\omega^2}{c_o^2}\rho(\boldsymbol{z})\hat{G}(\omega,\boldsymbol{z},\boldsymbol{x})\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})-\hat{G}_0(\omega,\boldsymbol{x},\boldsymbol{y})\delta(\boldsymbol{z}-\boldsymbol{x})+\hat{G}(\omega,\boldsymbol{y},\boldsymbol{x})\delta(\boldsymbol{z}-\boldsymbol{y})\\ &\overset{reciprocity}{=}-\frac{\omega^2}{c_o^2}\rho(\boldsymbol{z})\hat{G}(\omega,\boldsymbol{x},\boldsymbol{z})\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})-\hat{G}_0(\omega,\boldsymbol{x},\boldsymbol{y})\delta(\boldsymbol{z}-\boldsymbol{x})+\hat{G}(\omega,\boldsymbol{x},\boldsymbol{y})\delta(\boldsymbol{z}-\boldsymbol{y})\end{aligned}$$

We integrate over $B(\mathbf{0},L)$ (with L large enough so that it encloses the support of ρ):

$$0=-\frac{\omega^2}{c_o^2}\int\hat{G}(\omega,\boldsymbol{x},\boldsymbol{z})\rho(\boldsymbol{z})\hat{G}_0(\omega,\boldsymbol{z},\boldsymbol{y})d\boldsymbol{z}-\hat{G}_0(\omega,\boldsymbol{x},\boldsymbol{y})+\hat{G}(\omega,\boldsymbol{x},\boldsymbol{y})$$

Proof of the Born approximation (2/2)

- Lippmann-Schwinger equation (exact):

$$\hat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) = \hat{G}_0(\omega, \boldsymbol{x}, \boldsymbol{y}) + \frac{\omega^2}{c_o^2} \int \hat{G}(\omega, \boldsymbol{x}, \boldsymbol{z}) \rho(\boldsymbol{z}) \hat{G}_0(\omega, \boldsymbol{z}, \boldsymbol{y}) d\boldsymbol{z}$$

- Born approximation (approximate):

$$\hat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) \simeq \hat{G}_0(\omega, \boldsymbol{x}, \boldsymbol{y}) + \frac{\omega^2}{c_o^2} \int \hat{G}_0(\omega, \boldsymbol{x}, \boldsymbol{z}) \rho(\boldsymbol{z}) \hat{G}_0(\omega, \boldsymbol{z}, \boldsymbol{y}) d\boldsymbol{z}$$

Reflector imaging - the linearized forward operator

The data set is modeled by:

$$\hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) = \frac{\omega^2}{c_o^2} \int \hat{G}_0(\omega, \boldsymbol{x}_r, \boldsymbol{z})\rho(\boldsymbol{z})\hat{G}_0(\omega, \boldsymbol{z}, \boldsymbol{x}_s) d\boldsymbol{z}$$

(remove $\hat{G}_0(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s)$).

We define

$$[\hat{\mathbf{A}}\rho](\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) = \int \hat{G}_0(\omega, \boldsymbol{x}_r, \boldsymbol{z})\rho(\boldsymbol{z})\hat{G}_0(\omega, \boldsymbol{z}, \boldsymbol{x}_s) d\boldsymbol{z}$$

It is the linear operator that maps the reflectivity function to the array data.

Reflector imaging - inversion

The least squares linearized inverse problem is to minimize $J_{\rm LS}[\rho]$ where

$$J_{\rm LS}[\rho] = \int d\omega \sum_{r,s} \big| \hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) - [\hat{\mathbf{A}}\rho](\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) \big|^2$$

Solution:

$$\rho_{\rm LS} = \big(\hat{\mathbf{A}}^* \hat{\mathbf{A}}\big)^{-1} \big(\hat{\mathbf{A}}^* \hat{\mathbf{u}}\big)$$

The adjoint operator is

$$[\hat{\mathbf{A}}^* \hat{\mathbf{u}}](\boldsymbol{y}) = \sum_{r,s} \int \overline{\hat{G}_0(\omega, \boldsymbol{y}, \boldsymbol{x}_r) \hat{G}_0(\omega, \boldsymbol{x}_s, \boldsymbol{y})} \hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) d\omega$$

Remember: complex conjugation = time reversal.

Adjoint operator = Backpropagation both from $\boldsymbol{x}_r$ and from $\boldsymbol{x}_s$ to the test point $\boldsymbol{y}$.

Kirchhoff migration (or travel-time migration) for reflector imaging

• Reverse-time migration (using the full Green's function for migration).

Imaging function for the search point $\boldsymbol{y}^S$:

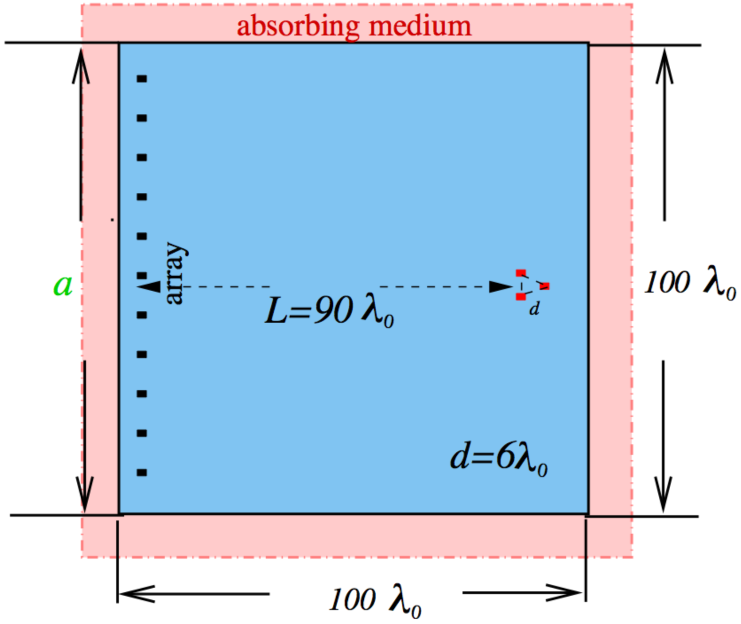
$$\mathcal{I}_{\rm RT}(\boldsymbol{y}^S) = \frac{1}{2\pi}\int d\omega \sum_{r,s} \overline{\hat{G}_0(\omega, \boldsymbol{y}^S, \boldsymbol{x}_r)\hat{G}_0(\omega, \boldsymbol{x}_s, \boldsymbol{y}^S)}\hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s)$$

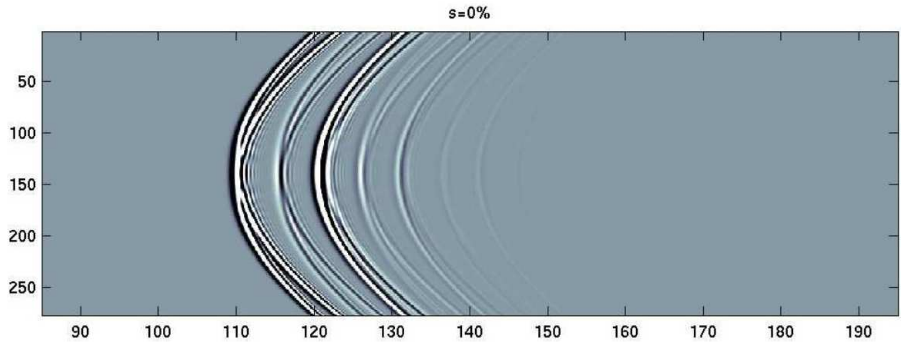
• Kirchhoff migration (or travel time migration).

If we take $\hat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) \simeq \exp[i\omega\mathcal{T}(\boldsymbol{x}, \boldsymbol{y})]$, where $\mathcal{T}(\boldsymbol{x}, \boldsymbol{y})$ is the travel time from $\boldsymbol{x}$ to $\boldsymbol{y}$, then we get the KM imaging function:

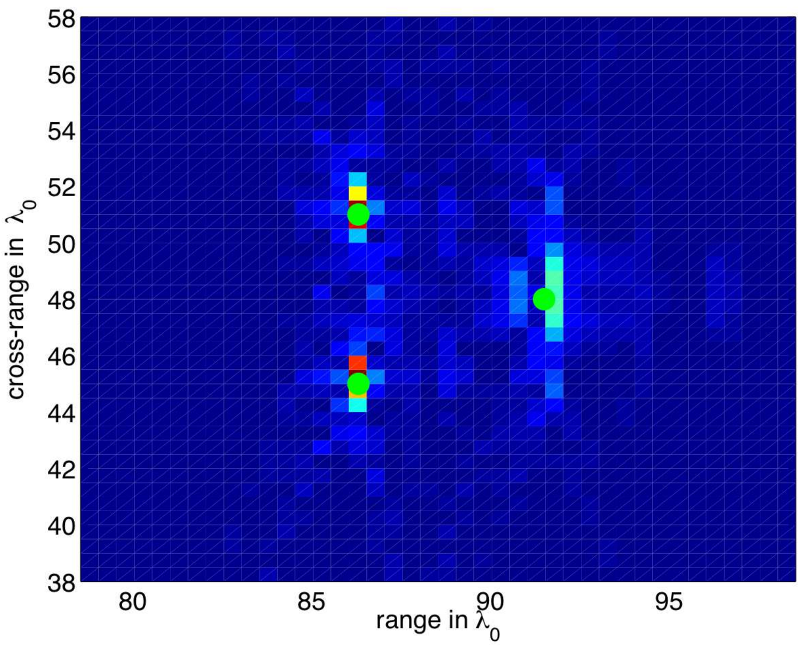
$$\begin{aligned}
\mathcal{I}_{\rm KM}(\boldsymbol{y}^S) &= \frac{1}{2\pi}\int d\omega \sum_{r,s} \exp[-i\omega(\mathcal{T}(\boldsymbol{x}_r, \boldsymbol{y}^S) + \mathcal{T}(\boldsymbol{x}_s, \boldsymbol{y}^S))]\hat{u}(\omega, \boldsymbol{x}_r, \boldsymbol{x}_s) \\
&= \sum_{r,s} u\big(\mathcal{T}(\boldsymbol{x}_r, \boldsymbol{y}^S) + \mathcal{T}(\boldsymbol{x}_s, \boldsymbol{y}^S), \boldsymbol{x}_r, \boldsymbol{x}_s\big)
\end{aligned}$$

Kirchhoff migration for reflector imaging: simulation





Data $\left(u(t, \boldsymbol{x}_r, \boldsymbol{x}_{128})\right)_{t \in [80,200], r=1,\dots,264}$



KM imaging function
$\mathcal{I}_{\rm KM}(\boldsymbol{y}^S)$