Inverse Problems and Imaging

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First lecture: friday, january 14, 2022, 9:00-12:00 (ENS, room 1N82).
Material on the course website.
Validation: project (notebook jupyter + oral presentation).

Sensor array imaging

• Sensor array imaging (echography in medical imaging, sonar, non-destructive testing, seismic exploration, radar, etc) has two steps:
- data acquisition: an unknown medium is probed with waves; waves are emitted by a source (or a source array) and recorded by a receiver array.
- data processing: the recorded signals are processed to identify the quantities of interest (reflector locations, etc).

• Example:
Ultrasound echography

$\longrightarrow$

• Standard imaging techniques require:
- good receivers,
- suitable conditions for wave propagation (ideally, the "target" is embedded in a homogeneous medium),
- controlled and known sources.

Sensor array imaging

- Goal: Propose and study imaging techniques that are robust with respect to:

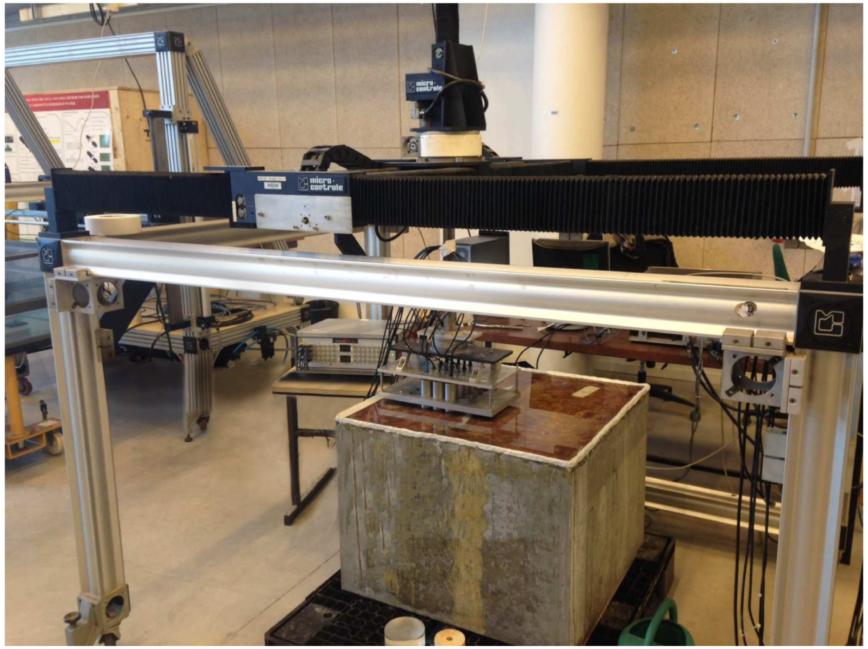
- measurement noise,
- the complexity of the medium (heterogeneous medium),
- the control and the knowledge of the sources.

- More generally: resolution of ill-posed inverse problems.

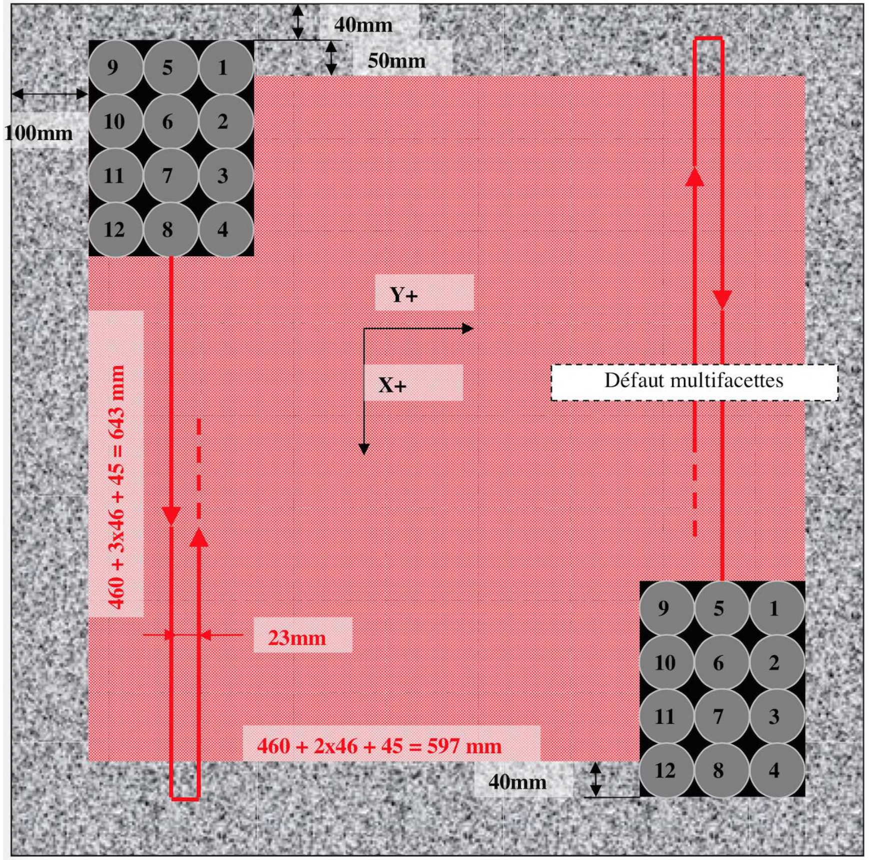
$\hookrightarrow$ Introduce probabilistic and statistical techniques:

- Bayesian analysis,
- Random matrix theory,
- Spectral theory for stationary processes,
- Gaussian processes.

Application 1: Ultrasound echography in concrete

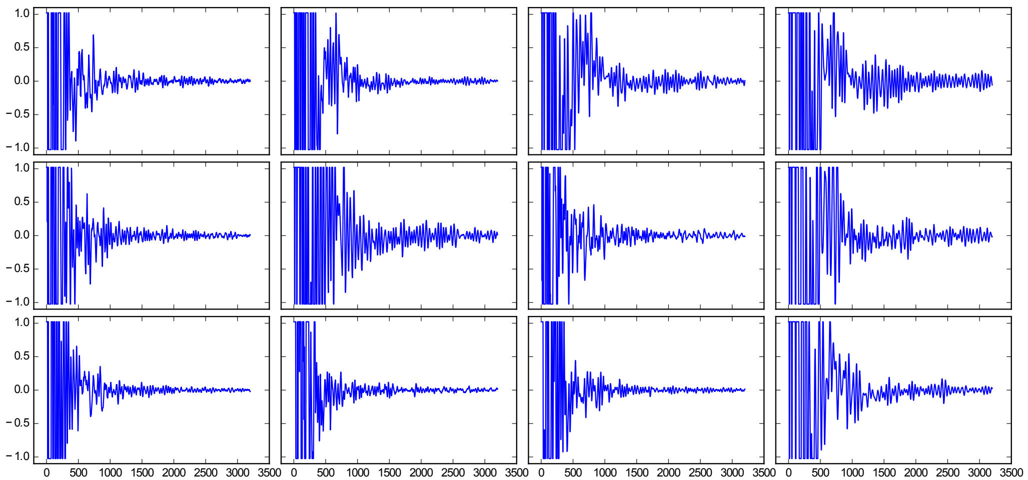


Experimental configuration



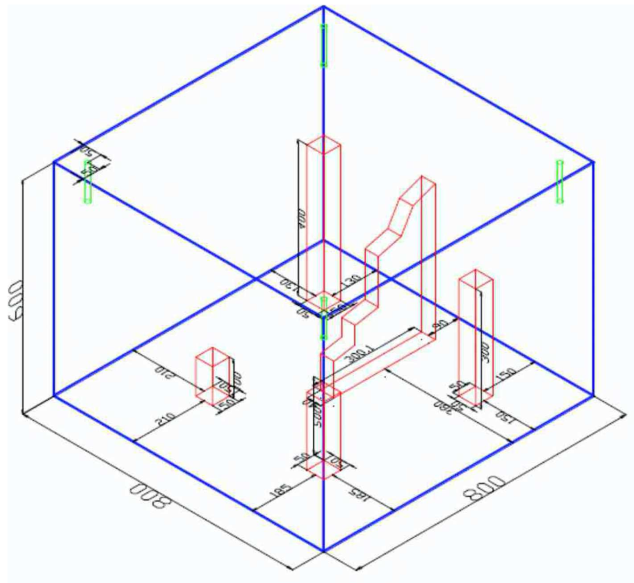
Top view of the acquisition geometry

Application 1: Ultrasound echography in concrete



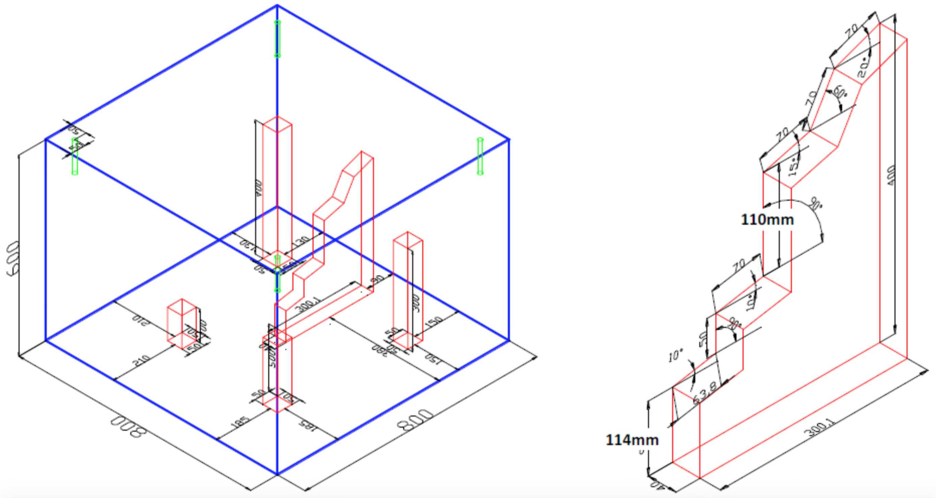
Data

$\overset{?}{\longrightarrow}$

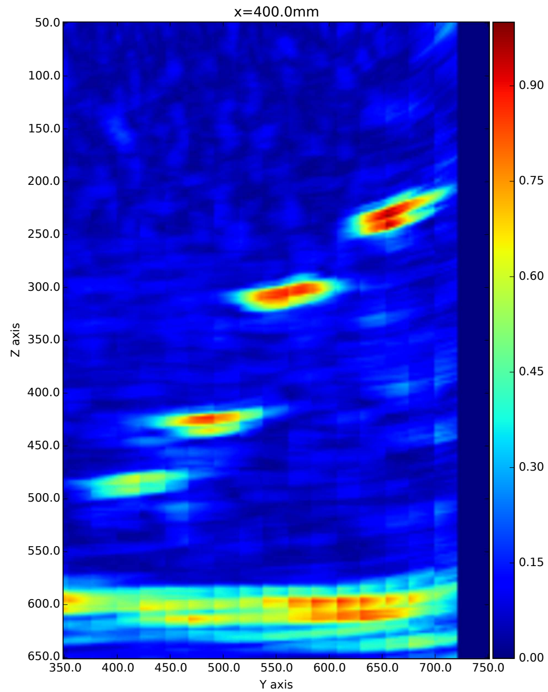


Real configuration

Application 1: Ultrasound echography in concrete

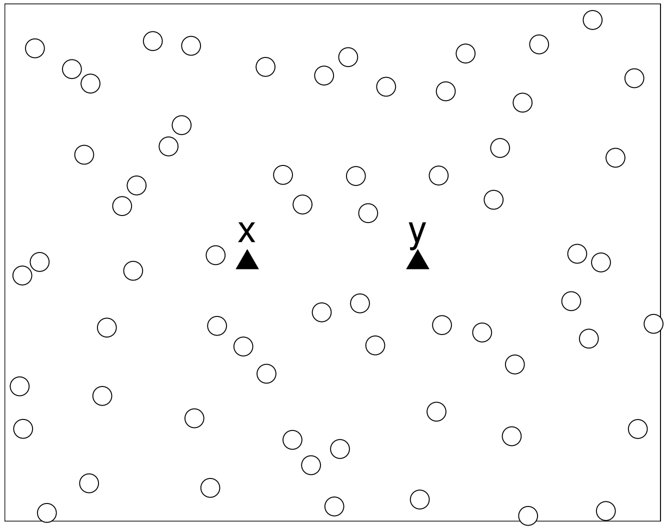


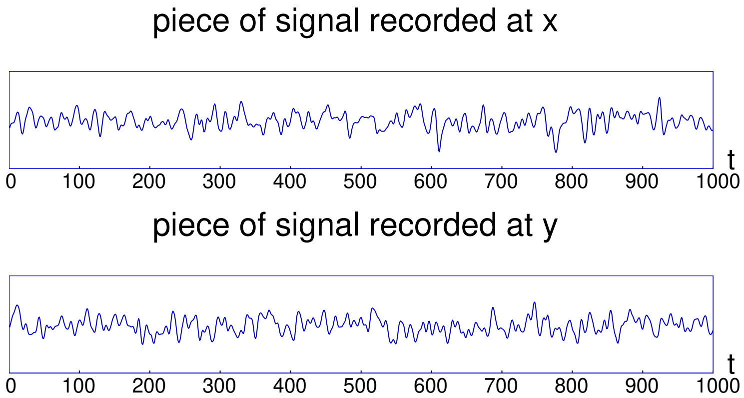
Real configuration



2D Image (along the complex defect plane)

Theory: Cross correlation of signals transmitted by noise sources





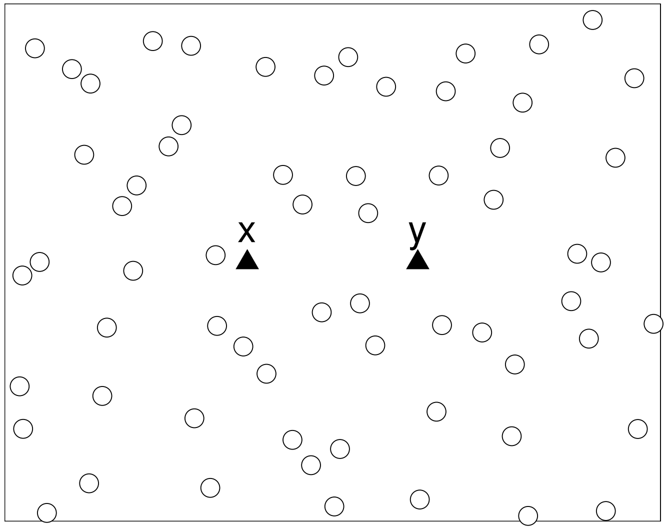
Numerical simulation of wave propagation

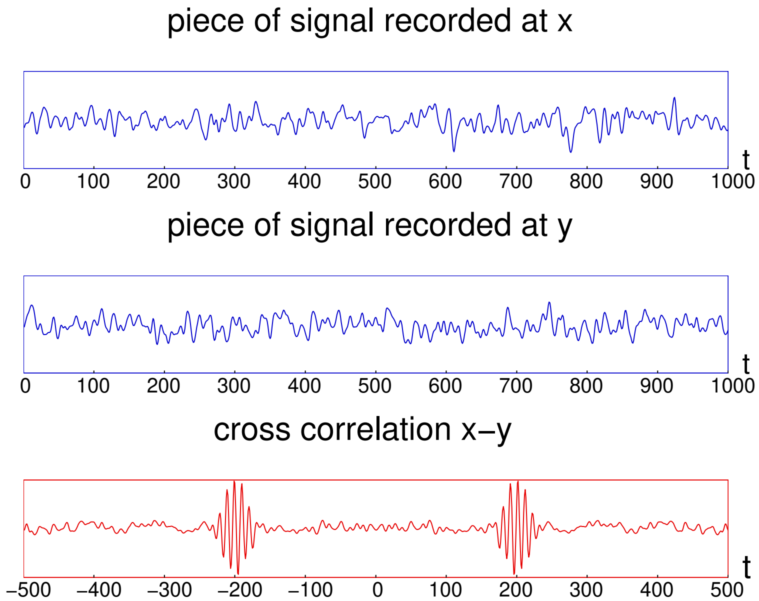
with many noise sources (○) and two receivers at $\boldsymbol{x}$ and $\boldsymbol{y}$ (▲)

How to extract information from the recorded signals $u_{\boldsymbol{x}}(t)$ and $u_{\boldsymbol{y}}(t)$?

These signals are just noise !

Theory: Cross correlation of signals transmitted by noise sources





Numerical simulation of wave propagation

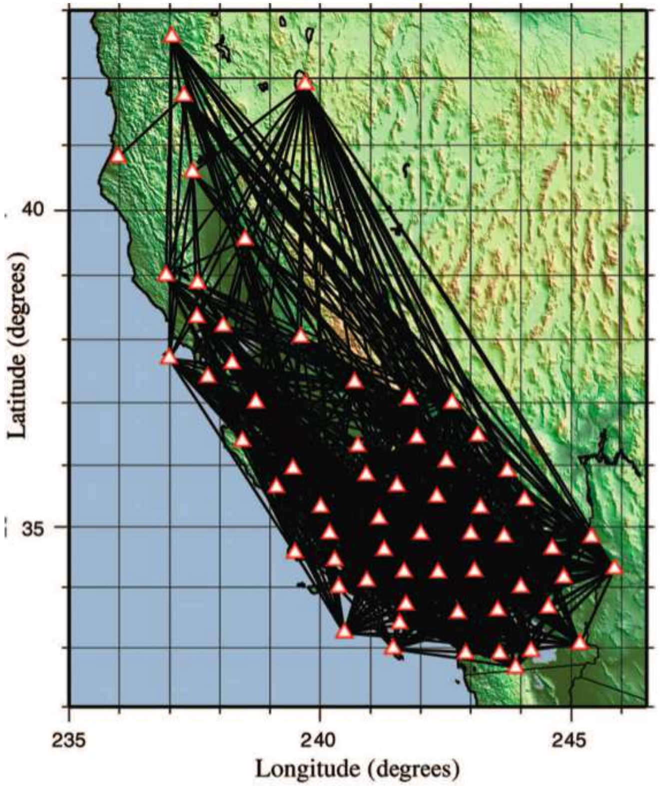
with many noise sources ($\circ$) and two receivers at $\boldsymbol{x}$ and $\boldsymbol{y}$ ($\blacktriangle$)

$\hookrightarrow$ Compute the cross correlation of the recorded signals

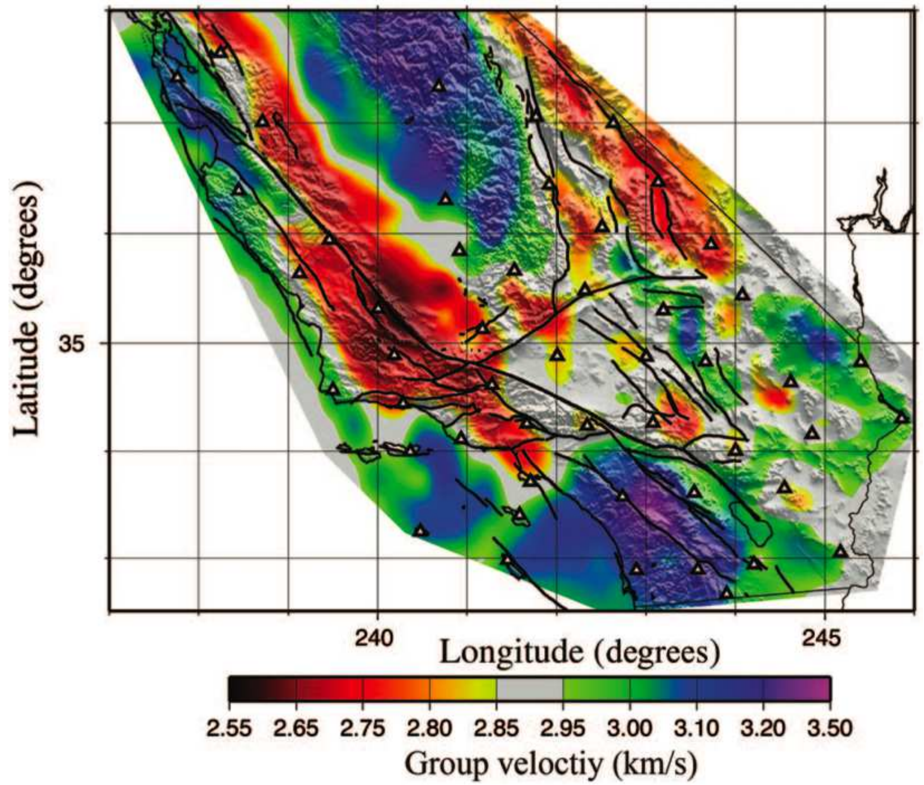
$$C^T_{\boldsymbol{x},\boldsymbol{y}}(t) = \frac{1}{T}\int_0^T u_{\boldsymbol{x}}(s)u_{\boldsymbol{y}}(s+t)ds$$

and extract the travel time between the receivers at $\boldsymbol{x}$ and $\boldsymbol{y}$.

Application 2: Seismic interferometry

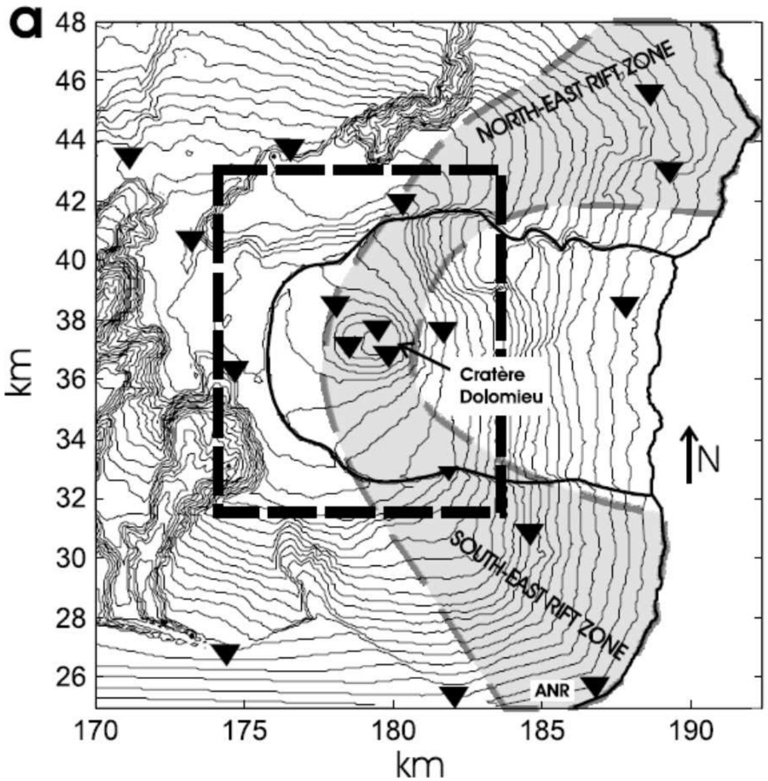


Travel time estimation

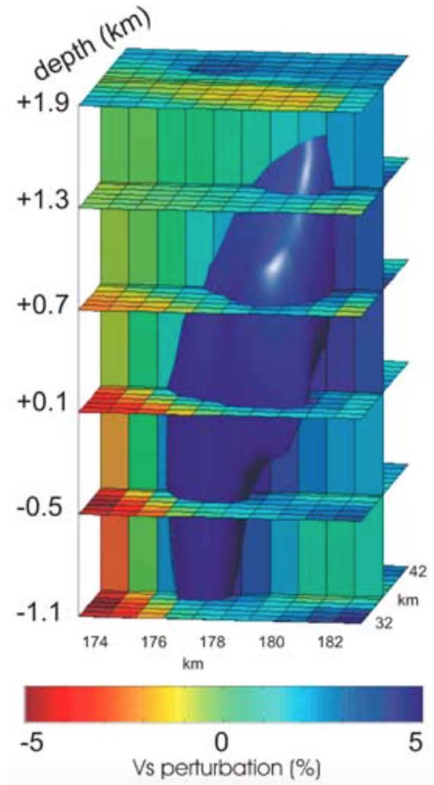


Background velocity estimation

Application 2: Seismic interferometry



Piton de la Fournaise



Eruption warning system