# Emergence of long-range phase coherence in nonlocal nonlinear media


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The emergence of long range phase coherence among random nonlinear waves is a fascinating effect that characterizes many fundamental phenomena. For instance, the condensation of classical waves [1,2] is an important example of self-organization process that generates lot of interest as a classical analogue of quantum Bose-Einstein condensation. Wave condensation is known to be characterized by the emergence of long-range order and phase-coherence, in the sense that the correlation function of the wave amplitude does not decay at infinity. This property of long range phase coherence is fundamental, for instance for the manifestation of superfluid behaviors, or the generation of Bogoliubov sound waves in optical wave systems [2].

Here, we report a previously unrecognized phenomenon of spontaneous emergence of long-range phase coherence in a system of conservative (Hamiltonian) random waves ruled by the nonlocal nonlinear Schrödinger equation (NLSE). Considering the highly nonlocal regime [3,4], we performed simulations starting from an initial incoherent speckle beam. We compare the weakly nonlinear regime where linear diffraction effects and nonlinear effects are of the same order (i.e., small correlation length of the initial speckle), with the strongly nonlinear propagation regime (i.e., large coherence length of the initial speckle). The simulations reveal that, in marked contrast to the weakly nonlinear regime where the system exhibits a random phase dynamics (Fig. 1(a)), the strong turbulence regime is responsible for the spontaneous emergence of a well structured spatial phase distribution (Fig. 1(b)). The phase coherence is characterized by almost radially symmetric spatial oscillations of the phase profile, whose period decreases with the radial coordinate (distance from the beam center, $r$) and with the propagation length ($z$). Our theoretical developments based on a hydrodynamic approach provide a detailed description of the phenomenon of spontaneous emergence of long-range phase coherence. In particular, the coherence length of intensity fluctuations is shown to increase in a dramatic way during the propagation, as $\sim \exp(z^2)$. The theory reveals that the emergence of long-range phase coherence constitutes a generic property of highly nonlocal random waves, which takes place both in the focusing or defocusing regimes.

The analysis is general and can be transposed to the temporal domain, in which the spatial nonlocal response is substituted by a noninstantaneous nonlinear response [4], which is known to be relevant to a multitude of systems involving radiation-matter interactions. In this framework, our study reveals that strongly nonlinear incoherent waves evolving in slowly responding nonlinear materials are characterized by the emergence of phase coherence over a time scale of the order of the nonlinear response time.

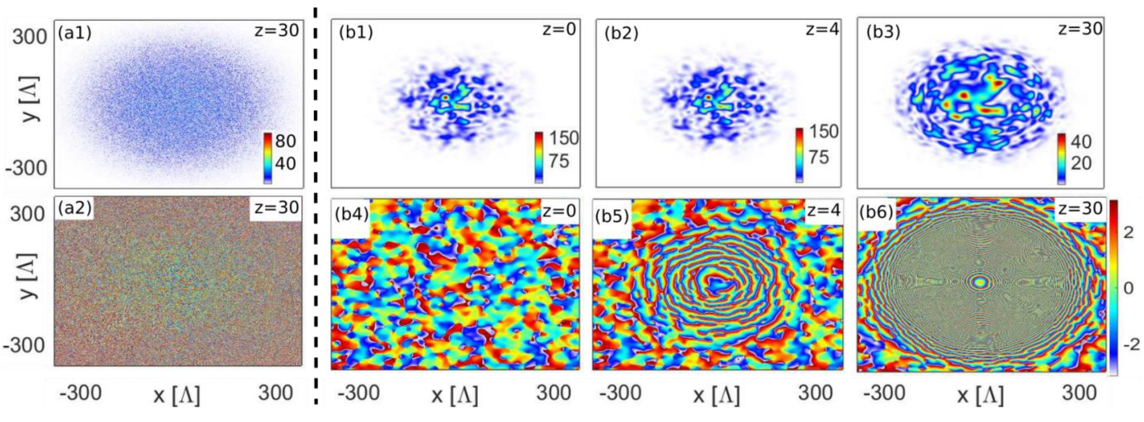


**Fig. 1** Simulations of the NLSE in the highly nonlocal regime showing the evolutions of the intensity (first line) and phase (second line), of the random wave: (a) Small initial coherence length of the initial speckle: No phase-coherence emerges in the system. (b) Large initial coherence length of the initial speckle: Long-range phase coherence emerges spontaneously from the initial random wave. The propagation length ($z$) is in units of the nonlinear length.

## References


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