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Beaming Through Turbulence

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Abstract: When (laser) beams propagate through atmospheric turbulence in the scintillation regime the beam exhibits anomalous spreading and gradually becomes incoherent due to scattering and eventually forms a speckle pattern. We characterize here the scintillation scaling regime for beams and describe the beam transformation via a moment theory. © 2019 The Author(s)

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1. Introduction

In the context of imaging and communication through the turbulent atmosphere the system performance is affected by scattering. The evaluation and design of systems with turbulence compensation schemes require a quantitative description of the effects of the scattering process [7]. When the source is coherent and time-harmonic we describe here the theory that gives the important statistics of the beam in the regime in which it gradually starts to scintillate. We refer to [1] and the references therein for background on beam propagation in turbulence.

2. The Beam Scintillation Regime

We consider a primary scaling regime which is characterized by the scaling relations: $\lambda_0 \ll l_0 \ll L$, where λ_0 is the wavelength, l_0 is the inner scale of the turbulence (below which energy is dissipated) and L is the range or beam propagation distance. This leads to the Itô-Schrödinger equation that describes the evolution of the beam in terms of an initial value problem. This equation was analyzed mathematically in [2] while it was derived from the Helmholtz equation in [3]. Two primary effects of the turbulence can be differentiated, first beam wander (spot dancing) and second beam scintillation [4]. The relative standard deviation of the fluctuations of the intensity approaches one because of the second effect. This occurs for a relatively wide beam so that $r_0 \gg L_0$ where r_0 is the beam waist and L_0 is the outer scale of the turbulence (size of largest eddies in the turbulence cascade). The scaling assumptions are made precise in [4, 5].

3. Modified von Kármán Turbulence

We use a common model for atmospheric turbulence, the modified von Kármán model [1, 6] where the spectrum (Fourier transform of the covariance function C) of the fluctuations in the index of refraction is modeled by

$$\widehat{C}(\vec{\kappa}) = \frac{A(\alpha)\tilde{C}_n^2}{2\pi} \frac{\exp(-|\vec{\kappa}|^2/\kappa_m^2)}{(|\vec{\kappa}|^2+\kappa_0^2)^{\alpha/2}}.$$

Here, $\vec{\kappa}$ is the spatial wave vector, $l_0 = 2\pi/\kappa_m$, $L_0 = 2\pi/\kappa_0$, and $A(\alpha)$ chosen so that $\tilde{C}_n^2$ describes the "strength" of the turbulence (its structure function at unit lag). The parameter α is the spectral exponent of the turbulence which governs the power law decay of the spectrum and with classic Kolmogorov turbulence corresponding to $\alpha = 11/3$. Here, we consider the more general case with $\alpha \in (3,4)$ corresponding to relative rough index fluctuations motivated by observations that the spectral exponent may indeed vary [8].

4. The Transmitted Intensity Covariance Function

We consider the covariance function of the intensity at range L and as function of two cross-ranges. We let R_x describe the support of this function in central cross-range (that is the intensity spreading scale) and ρ_y its support in the cross-range offset (that is the intensity decorrelation scale). We then have the following fractional scaling behavior of these parameters when also $l_0 \ll \lambda_0 L/r_0 \ll L_0$ based on the general theory in [5]:

$$R_x \propto \left(\frac{L^{\alpha-1}\tilde{C}_n^2}{\lambda_0^{4-\alpha}}\right)^{\frac{1}{\alpha-2}}, \quad \rho_y \propto \left(\frac{\lambda_0^2}{L\tilde{C}_n^2}\right)^{\frac{1}{\alpha-2}}.$$

Observe that both of these scales exhibit a fractional dependence on range and wavelength. Indeed relatively strong and rough turbulence gives a relatively strong spreading and rapid decorrelation, and thus also a relatively rough speckle pattern.

5. The Scintillation Index

A fundamental parameter used to describe the effect of the turbulence is the scintillation index I or relative variance of the intensity along the main propagation axis at range L [1]. We find with $l_0 = 0$:

$$I = 1 - 4/\Big|\int_0^\infty \exp\Big(\frac{2L}{Z_{\rm sca}}\int_0^1 \tilde{V}(uLs/Z_c)ds - \frac{u^2}{4}\Big)udu\Big|^2, \quad \tilde{V}(x) = \frac{(\alpha-2)(\pi x)^{\alpha/2-1}}{\Gamma(\alpha/2)}K_{\alpha/2-1}(2\pi x),$$

for $K_{\alpha/2-1}$ the modified Bessel function of the second kind and of order $\alpha/2-1$. Here, the scattering mean free path is $Z_{\rm sca}{=}\lambda_0^2/(\tilde{C}_n^2 L_0^{\alpha-2})$ and characterizes the range scale at which the coherent wave decays, while the diffraction length is $Z_{\rm c}{=}r_0 L_0/\lambda_0$, and characterizes the range scale at which diffraction effects become important. Then we find that the scintillation index approaches unity for large range. However, in the case of rough turbulence and small diffraction length the scintillation index may be significantly below one even though the range is larger than the scattering mean free path.

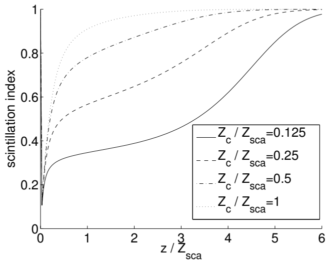

Fig. 1. Scintillation index as function of scattering mean free path and diffraction length.

6. Conclusions

We have analyzed from first principles propagation of beams in general non-Kolmogorov turbulence in the scintillation regime. We have found a very interesting anomalous scaling of both the spreading and decorrelation for the beam intensity profile. These scales show a fractional dependence on the range and the wavelength with the exponents depending on the spectral exponent for the turbulence. We have, moreover, identified how the scintillation index depends on the turbulence and its universal form that depends only on two effective parameters: the scattering mean free path and the diffraction length. Here we considered homogeneous and isotropic turbulence in the wide beam regime. More general models for the turbulence as well as the spot-dancing regime with beam wander will be considered elsewhere.

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