Catastrophic process of coherence degradation

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Abstract: We predict a catastrophic process of coherence degradation characterized by a virtually unlimited spectral broadening of the waves. This effect is described by self-similar solutions of the kinetic equations inherent to the wave turbulence theory.
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1. Introduction

It is well-known that the generic evolution of a conservative (Hamiltonian) system of nonlinear waves is characterized by the spontaneous formation of coherent structures such as, e.g., solitons in the focusing nonlinear regime. This process originally termed "soliton turbulence" [1,2] is exemplified by different regimes of incoherent supercontinuum generation in photonic crystal fibers [2,3]. This effect can be interpreted on the basis of nonequilibrium thermodynamic considerations: It is thermodynamically advantageous for the system to generate coherent soliton structures, because this allows the system to increase the amount of disorder (entropy) in the wave system. From this point of view, the spectral broadening due to supercontinuum generation can be interpreted as an increase of the amount of disorder (incoherence) carried by the optical waves [2].

Our aim in this communication is to show that an increase of disorder in a random wave system does not necessarily require the generation of a coherent (soliton) structure. We illustrate this idea by considering the example of two waves that propagate with opposite dispersion coefficients, i.e. in the normal and anomalous dispersion regimes of an optical fiber. We show that this system exhibits, as a rule, a virtually unlimited spectral broadening of the waves, in which the increase of kinetic energy in one component is exactly compensated by an opposite reduction of energy in the other component [4]. At variance with the expected soliton turbulence scenario, this catastrophic process of spectral broadening occurs unconditionally, even by considering fully coherent initial states of the waves. The coherence degradation process is described in detail by nonequilibrium self-similar solutions of the wave turbulence kinetic equations [4]. From a broader perspective, this work sheds new light on the originating mechanisms of spectral broadening of random waves, a feature that can be exploited to overcome the traditional constraints of limited spectral broadening during SC generation.

2. Self-similar solutions of the wave turbulence kinetic equations

We consider a system of two coupled nonlinear Schrödinger equations (NLSE):

$$i\partial_z u = -\partial_{tt} u + (|u|^2 + \kappa |v|^2)u, \quad (1)$$

$$i\partial_z v = -\eta\partial_{tt} v + (|v|^2 + \kappa |u|^2)v, \quad (2)$$

We have normalized the problem with respect to the nonlinear length, $L_{nl} = 1/(\gamma N)$, and the healing time, $\tau_0 = (\beta_u L_{nl})^{1/2}$, where γ is the nonlinear coefficient, β_u (resp. β_v) is the dispersion coefficient of u (resp. v) and $\eta = \beta_v/\beta_u$. The kinetic energies of the waves are $E_u = \int |u_t|^2 dt$, and $E_v = \eta\int |v_t|^2 dt$. On the basis of the wave turbulence theory [1,2,5], we derive the irreversible kinetic equations governing the evolutions of the averaged spectra $n_{u,v}(\omega, z)$:

$$\partial_z n_u(\omega,z) = \frac{\kappa^2}{4\pi|\eta|}\int \frac{n_u(x)n_v(\omega+f-x)n_v(f)n_u(\omega)}{|x-\omega|}\left[\frac{1}{n_u(\omega)} + \frac{1}{n_v(f)} - \frac{1}{n_v(\omega+f-x)} - \frac{1}{n_u(x)}\right]dx, \quad (3)$$

$$\partial_z n_v(\omega,z) = \frac{\kappa^2}{4\pi}\int \frac{n_v(x)n_u(\omega+g-x)n_u(g)n_v(\omega)}{|x-\omega|}\left[\frac{1}{n_v(\omega)} + \frac{1}{n_u(g)} - \frac{1}{n_u(\omega+g-x)} - \frac{1}{n_v(x)}\right]dx, \quad (4)$$

where the frequencies read $f = (1+\eta)x/(2\eta) + (1-\eta)\omega/(2\eta)$, $g = (1+\eta)x/(2) + (\eta-1)\omega/(\eta)$. The kinetic equations have been solved numerically (see blue lines in Fig. 1(a-b)), and a remarkable quantitative agreement has been obtained with NLSE simulations (gray lines in Fig. 1(a-b).

As a remarkable result, we have found non-equilibrium self-similar solutions of the kinetic Eqs.(3-4), which describe the self-similar process of unlimited spectral broadening [4]: $\Omega(z) \sim z^{1/2}$, where Ω is the spectral width of u and v. As a consequence, the kinetic energies of the two waves grow linearly with the propagation length: $|E_{u,v}| \sim z$. These predictions are confirmed by the numerical simulations, as remarkably illustrated in Fig. 1(c-d).

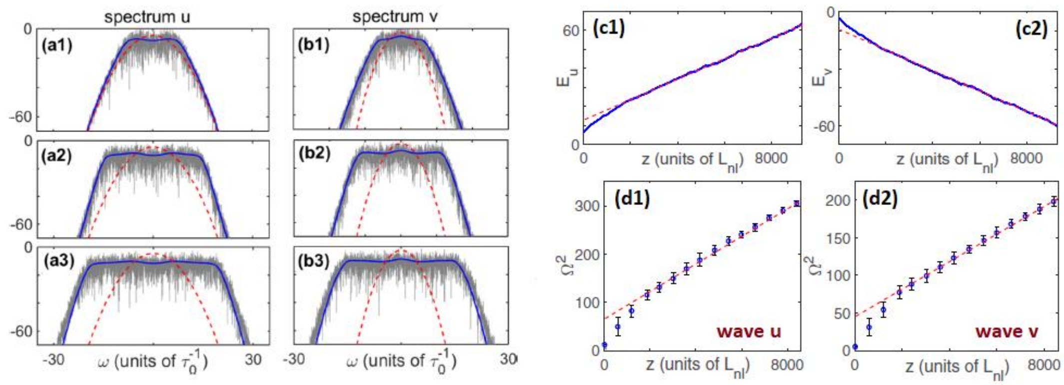

Fig. 1. (a-b) Numerical simulations of the VNLS, Eqs. (1-2) (light gray), and WT kinetic Eqs. (3-4) (blue line), showing the evolution of the spectra of the waves *u* (a) and *v* (b) in 10log10 scale at the propagation length $z = 600$ (a1, b1), $z = 4200$ (a2, b2), $z = 9000$ (a3, b3). The dashed-red lines denote initial conditions. (c) Self-similar process of spectral broadening: Evolutions of the kinetic energies E_u and E_v in blue lines: the energies exhibit linear and opposite evolutions with the propagation length *z*, as predicted by the self-similar behavior $E_{u,v} \sim z$ (dashed red lines). (d) Evolutions of the squares of the spectral bandwidths, $\Omega^2(z)$, for the waves u (a), and v (b) obtained by VNLS simulations and the WT kinetic equations (circles). ($\eta = -1.28$, $\kappa = 0.5$).

3. Solitonic initial condition

To complete our study, we discuss the process of unlimited spectral broadening by considering a solitonic initial condition (i.e., a "cold" initial state). We have considered as initial condition a pair of vector bright-dark solitons for the *v-u* components. The interaction between them is reported in Fig. 2: In the first stage ($z < 25$), the two solitons propagate against each other with a constant velocity and in an apparent stable fashion, see Fig. 2(a-b). The solitons then exhibit an inelastic collision by emission of radiation, and then enter the incoherent erratic regime, which is characterized by an unconstrained spectral broadening: The kinetic energies compensate each other and tend to follow a linear behavior with *z*, as predicted by the kinetic self-similar solution, see Fig. 2(c-d).

4. Unconstrained thermalization

In the long term evolution, the waves unexpectedly relax toward a thermodynamic equilibrium spectrum that is not constrained by energy conservation: the equilibrium spectrum is constant $n_{u,v}^{eq}(\omega) = \text{const}$, which reflects the equilibrium property of *power equipartition*, instead of *energy equipartition* [4]. Notice that a similar unexpected process of unconstrained thermalization has been recently discussed for weakly dispersive waves in Ref.[6].

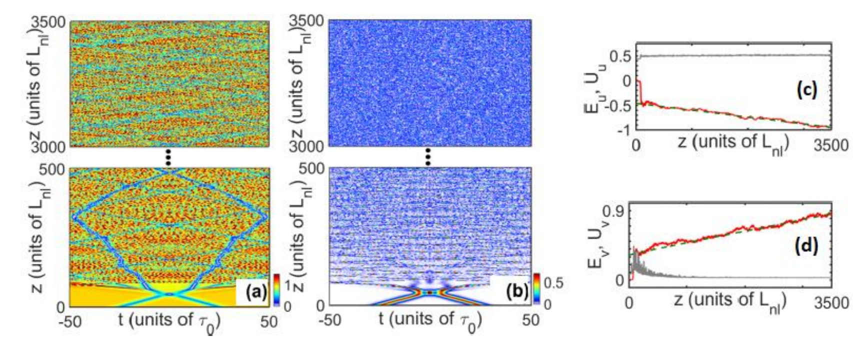

Fig. 2. Simulation of VNLS Eqs. (1-2) starting from a pair of dark-bright vector solitons for the *u-v* components. [(a), (b)] Spatiotemporal evolutions of $|u|^2(t,z)$ (e) and $|v|^2(t,z)$ (f): Despite the weak interaction between the waves imposed by the initial solitonic condition ($N_v/N_u \sim 0.084$), in the long-term evolution the system tends to evolve toward the strongly incoherent regime that inhibits the generation of robust soliton structures. [(c), (d)] Evolutions during the propagation of the linear energies E_u (a) and E_v (b), in red lines: They follow a linear behavior (dashed green lines) as predicted by the self-similar solution of the kinetic equations; The light grey lines report the corresponding evolutions of the nonlinear energies U_u (a) and U_v (b). Parameters: $\eta = -0.77$, $\kappa = 2/3$.

5. References

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