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Identification of Green's Functions Singularities by Cross Correlation of Ambient Noise Signals

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Abstract

In this paper we consider the problem of estimating the singular support of the Green's function of the wave equation by using ambient noise signals recorded by passive sensors. We assume that noise sources emit stationary random signals into the medium which are recorded by sensors. We explain how the cross correlation of the signals recorded by two sensors is related to the Green's function between the sensors. By looking at the singular support of the cross correlation we can obtain an estimate of the travel time between them. We consider different situations, such as when the support of the noise distribution extends over all space or is spatially limited, the medium is open or bounded, homogeneous or inhomogeneous, dissipative or not. We identify the configurations under which travel time estimation by cross correlation is possible. We show that iterated cross correlations using auxiliary sensors can be efficient for travel time estimation when the support of the noise sources is spatially limited.

1 Introduction

In this paper we consider the estimation of the Green's function of the wave equation in an inhomogeneous medium by cross correlation of noisy signals. We assume that noise sources with unknown spatial support emit stationary random signals, that propagate into the medium and are recorded at observation points. The cross correlation of the recorded signals has been shown to provide a reliable estimate of the Green's function and the travel time between the observation points in geophysics [18]. The travel time estimates can then be used for background velocity estimation. Indeed tomographic travel time velocity analysis, based on cross correlations, was applied successfully for surface-wave velocity estimation in Southern California [23], in Tibet [30], and in the Alps [27].

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The idea of using the cross correlation of noisy signals to retrieve information about travel times was used previously in helioseismology and seismology [12, 21]. It is now applied to seismic data from regional to local scales [18, 25, 17], volcano monitoring [8] and petroleum prospecting [11]. When the noise sources are uncorrelated and have support that extends over all space, the derivative of the cross correlation of the recorded signals can be shown to be proportional to the symmetrized Green's function between the observation points [22]. This property also holds when the source distribution has limited spatial support provided the waves propagate in an ergodic cavity [10, 3]. At the physical level this result has been established in other configurations provided that the noisy field is equipartitioned [19, 26, 20]. In an open environment this means that the recorded signals are an uncorrelated and isotropic superposition of plane waves in all directions. In a closed environment it means that the recorded signals are superpositions of normal modes with random amplitudes that are statistically uncorrelated and identically distributed. In this paper we introduce a mathematical formulation in which travel time estimation by cross correlation of noisy signals is possible when there is enough noise source diversity.

In many realistic environments the noise source distribution is spatially limited and the field is not equipartitioned. As a result, the waves recorded by the observation points are dominated by the flux coming from the direction of the noise sources, which results in an azimuthal dependence of the quality of the Green's function estimation, with poor results for some azimuths [27]. To overcome this problem, Campillo and Stehly [28] have proposed the use higher-order cross correlations. In this paper, we explain why the usual cross correlation technique fails when the noise sources have limited spatial support. We also show that iterated cross correlations using auxiliary observation points can exploit the enhanced directional diversity of the waves scattered by the heterogeneities of the medium. We analyze a special fourth-order cross correlation function that can provide acceptable travel time estimates even when the support of the noise sources is spatially limited.

The paper is organized as follows. In Section 2 we describe the physical principles for travel time estimation by cross correlation of noisy signals. In Section 3 we present a mathematical formulation of the estimation problem. In Section 4 we give a simple proof of the relation between the cross correlation and the Green's function when the sources are distributed all over space. In Section 5 we present the Helmholtz-Kirchhoff theorem and its application to cross correlations when the noise sources completely surround the region under investigation. In Sections 4-5 it is sufficient to assume that the recording time is much larger than the coherence time of the sources and then the full Green's function can be estimated. When the noise sources have a spatially limited distribution, the singular (high-frequency) component of the Green's function can still be estimated provided that some additional conditions are fulfilled. As a result travel time estimation is still possible provided that the typical travel time is much larger than the coherence time of the noise sources. In Section 6 we give conditions under which travel time estimation by cross correlations is possible in an open medium when the noise sources are spatially localized, using stationary phase analysis. In Section 7 we show that travel time estimation in an ergodic cavity is possible even when the sources are spatially localized, using semi-classical analysis. In Section 8 we study properties of iterated cross correlations, which requires the analysis of fluctuations of cross correlations due to heterogeneities in the medium, and we show that travel time estimation can be done with iterated cross correlations even when the spatial support of the noise source distribution is limited.

2 Travel time estimation with cross correlations

In this section we present the physical context that motivates travel time estimation with cross correlations, and discuss the limitations of this approach.

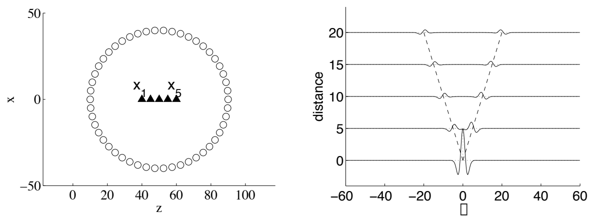
The problem is to reconstruct the background velocity of the earth's crust. The usual technique for this is to wait for an earthquake to occur, which plays the role of a seismic source, and to record the signals (seismograms) at various observation points. Travel time estimation is done using the recorded direct arrivals and then, if the observation points cover the region of interest, it is possible to estimate the map of the background velocity tomographically [4]. The direct arrivals correspond to ballistic waves that propagate along rays from the sources to the observation points. After the direct arrivals, the seismograms are long oscillatory signals with decreasing amplitude but still above the noise level. These signals correspond to coda waves that are scattered by the heterogeneities of the earth crust. Coda waves have been analyzed because they contain information about the medium [2, 24]. It was understood only very recently that the background noise (the stationary, noisy signals recorded during the long time intervals between earthquakes) also contains information about the medium. The issue is then how to extract this information, which is not as easy as travel time estimation from direct arrivals.

The noise signals recorded over time intervals between earthquakes have components due to surface waves generated from the interaction of the ocean swell with the coast [27]. The medium in which the waves propagate has a slowly varying background velocity profile, which determines the travel times that we want to estimate, as well as heterogeneities that are responsible for wave scattering. It was proposed in [12, 21] to compute the cross correlation (in time) of the noisy signals $u(t, \boldsymbol{x}_1)$ and $u(t, \boldsymbol{x}_2)$ recorded at two observation points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$:

$$C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{T} \int_0^T u(t, \boldsymbol{x}_1) u(t+\tau, \boldsymbol{x}_2) dt \,. \tag{1}$$

When the medium is homogeneous and the source of the waves is a space-time stationary random field that is also delta correlated in space and in time, then it can been shown [26, 22] that

$$\frac{\partial}{\partial \tau} C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \simeq G(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) - G(-\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \,, \tag{2}$$



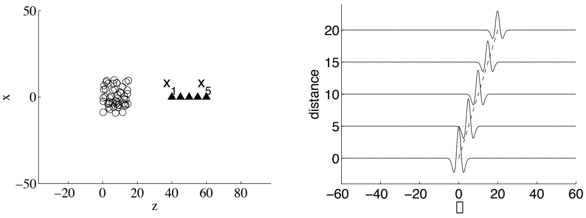
a) Configuration **b)** $C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$

Figure 1: When the noise sources (circles) completely surround the region of interest then the cross correlation function is symmetric. Its positive and negative parts correspond to the Green's function between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ and its anti-causal form, respectively. The configuration is shown in Figure a: the circles are the noise sources and the triangles are the sensors. Figure b shows the cross correlation C_T between the pairs of sensors $(\boldsymbol{x}_1, \boldsymbol{x}_j)$, $j = 1, \ldots, 5$, versus the distance $|\boldsymbol{x}_j - \boldsymbol{x}_1|$ and versus the time lag τ.

where G is the time-domain Green's function of the wave propagation process. This approximate equality (up to a multiplicative constant) holds for T sufficiently large, provided some limiting absorption is introduced to regularize the integral. When the medium is homogeneous, a mathematical analysis of (2) is given in Section 4. When the medium is inhomogeneous and the sources surround the inhomogeneous region of interest, then (2) still holds, as can be shown by the Kirchhoff-Helmholtz theorem that we present in Section 5. The main point here is that the time-symmetrized Green's function can be obtained from the cross correlation if there is enough source diversity. In this case the wave field at any sensor is essentially equipartitioned, in the sense that it is a superposition of uncorrelated plane waves in all directions. The travel time between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ can then be obtained from the singular support of the cross correlation.

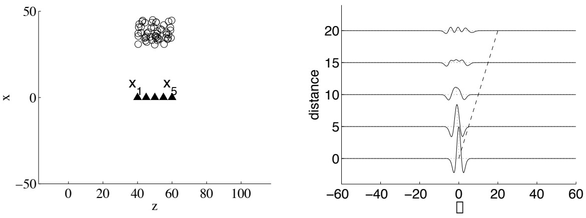
The configuration (Figure 1) in which the noise sources completely surround the region of interest is rarely encountered in applications. Significant departures from this ideal situation occur when limited spatial diversity of the sources introduces directivity into the recorded fields, which affects the quality of the estimate of the Green's function. If, in particular, the source distribution is spatially localized, then the flux of wave energy is not isotropic, and the cross correlation function is not symmetric (Figure 2). In some situations it may be impossible to distinguish the coherent part of the cross correlation function, which contains information about the travel time (Figure 3). A mathematical analysis using the stationary phase method that explains the dependence of the travel time estimate on the source distribution is given in Section 6.

In the case of a spatially localized distribution of noise sources, directional diversity of the recorded fields can be enhanced if there is sufficient scattering in the medium. An ergodic cavity with a homogeneous or inhomogeneous interior



a) Configuration **b)** $C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$

Figure 2: When the distribution of noise sources is spatially localized then the cross correlation function is not symmetric.

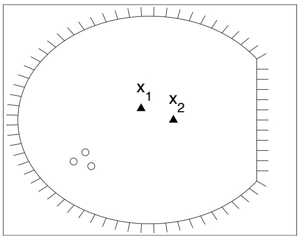


a) Configuration **b)** $C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$

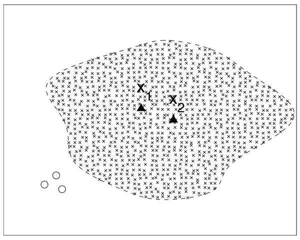
Figure 3: When the distribution of noise sources is spatially localized then the cross correlation function does not seem to provide information on the travel time between the two sensors if the axis formed by the sensors is perpendicular to the main direction of energy flux from the noise sources.

is a good example (Figure 4, left): Even with a source distribution that has very limited spatial support, the reverberations of the waves in the cavity generate interior fields with high directional diversity [10, 3]. We analyze this situation in Section 7. Multiple scattering of waves by random inhomogeneities can also lead to wave field equipartition if the transport mean free path is short compared to the distance from the sources to the sensors [18, 14, 13]. The transport mean free path is the propagation distance over which wave energy transport in a scattering medium is effectively isotropic. In such a scattering medium (Figure 4, right), the inhomogeneities can be viewed as secondary sources in the vicinity of the sensors. In Section 8 we describe how to exploit the enhanced directional diversity of the scattered waves.

The role of scattering in a random medium for travel time estimation depends on the transport mean free path. We have just seen that directional diversity is enhanced provided that the transport mean free path is short compared to the distance between the sources and the sensors. If the transport mean free path is also short compared to the distance between the sensors, then the cross correlation function gives an acceptable estimate of the Green's function,



a) Ergodic cavity



b) Scattering medium

Figure 4: Configurations in which wave fields have directional diversity. An ergodic cavity (a) and a randomly inhomogeneous medium (b).

but it is random because of the medium and the coherent part of the Green's function that has information about the travel time is essentially unobservable. Therefore, when the noise sources are spatially limited then the travel time can be estimated in a random medium provided that (i) the transport mean free path is short compared to the distance between the sources and the sensors, and (ii) it is long compared to the distance between the sensors. This is the physical situation in which the random inhomogeneities actually enhance the estimation process.

3 The empirical cross correlation and the statistical cross correlation

We consider the solution u of the wave equation in a d-dimensional inhomogeneous medium:

$$\frac{1}{c^2(\boldsymbol{x})} \frac{\partial^2 u}{\partial t^2}(t, \boldsymbol{x}) - \Delta_{\boldsymbol{x}} u(t, \boldsymbol{x}) = n^\varepsilon(t, \boldsymbol{x}) . \tag{3}$$

The domain can be bounded, with prescribed boundary conditions at the boundary, or unbounded, in which case the support of the inhomogeneous region is assumed to be compactly supported. The term $n^\varepsilon(t, \boldsymbol{x})$ models a random distribution of noise sources. It is a zero-mean stationary (in time) Gaussian process with autocovariance function

$$\langle n^\varepsilon(t_1, \boldsymbol{y}_1) n^\varepsilon(t_2, \boldsymbol{y}_2) \rangle = F^\varepsilon(t_2 - t_1) \Gamma^\varepsilon(\boldsymbol{y}_1, \boldsymbol{y}_2) . \tag{4}$$

Here $\langle \cdot \rangle$ stands for statistical average with respect to the distribution of the noise sources. The parameter ε denotes the ratio of the decoherence time of the noise sources (i.e. the width of the time covariance function F^ε) over the typical travel time between sensors. In the first sections of this paper, ε can be arbitrary. In Sections 6-8 we assume that ε is small and carry out an asymptotic analysis using this hypothesis. We can then write the time correlation function F^ε in the form

$$F^\varepsilon(t_2 - t_1) = F\Big(\frac{t_2 - t_1}{\varepsilon}\Big)\,, \tag{5}$$

where t_1 and t_2 are scaled relative to typical sensor travel times. The Fourier transform $\widehat{F}^\varepsilon$ of the time correlation function is a nonnegative, even real-valued function. It is proportional to the power spectral density of the sources:

$$\widehat{F}^\varepsilon(\omega) = \varepsilon \widehat{F}(\varepsilon\omega)\,, \tag{6}$$

with the Fourier transform defined by

$$\widehat{F}(\omega) = \int F(t) e^{i\omega t} dt\,. \tag{7}$$

The spatial distribution of the noise sources is characterized by the auto-covariance function Γ^ε. It is the kernel of a symmetric nonnegative definite operator. For simplicity, we assume in the first sections of this paper that the process n^ε is delta-correlated in space:

$$\Gamma^\varepsilon(\boldsymbol{y}_1, \boldsymbol{y}_2) = \Gamma_0(\boldsymbol{y}_1)\delta(\boldsymbol{y}_1 - \boldsymbol{y}_2)\,, \tag{8}$$

where Γ_0 characterizes the spatial support of the sources. One can consider a more general form for the spatial auto-covariance function as is done in Section 7. This requires the use of semiclassical analysis.

The stationary solution of the wave equation has the integral representation

$$u(t, \boldsymbol{x}) = \int d\boldsymbol{y} \int ds G(s, \boldsymbol{x}, \boldsymbol{y}) n^\varepsilon(t - s, \boldsymbol{y})\,, \tag{9}$$

where $G(t, \boldsymbol{x}, \boldsymbol{y})$ is the time-dependent Green's function. It is the fundamental solution of the wave equation

$$\frac{1}{c^2(\boldsymbol{x})}\frac{\partial^2 G}{\partial t^2}(t, \boldsymbol{x}, \boldsymbol{y}) - \Delta_{\boldsymbol{x}} G(t, \boldsymbol{x}, \boldsymbol{y}) = \delta(t)\delta(\boldsymbol{x} - \boldsymbol{y})\,, \tag{10}$$

starting from $G(0, \boldsymbol{x}, \boldsymbol{y}) = \partial_t G(0, \boldsymbol{x}, \boldsymbol{y}) = 0$ (and extended to the negative time axis by $G(t, \boldsymbol{x}, \boldsymbol{y}) = 0\ \forall t \leq 0$). The Fourier transform is the outgoing time-harmonic Green's function $\widehat{G}$. It is the solution of the Helmholtz equation

$$\Delta_{\boldsymbol{x}} \widehat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) + \frac{\omega^2}{c^2(\boldsymbol{x})}\widehat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) = -\delta(\boldsymbol{x} - \boldsymbol{y})\,, \tag{11}$$

and it satisfies the Sommerfeld radiation condition ($c(\boldsymbol{x}) = c_0$ at infinity)

$$\lim_{|\boldsymbol{x}|\to\infty} |\boldsymbol{x}|^{\frac{d-1}{2}} \Big(\frac{\boldsymbol{x}}{|\boldsymbol{x}|}\cdot\nabla_{\boldsymbol{x}} - i\frac{\omega}{c_0}\Big)\widehat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) = 0\,,$$

uniformly in $\boldsymbol{x}/|\boldsymbol{x}|$.

The empirical cross correlation of the signals recorded at $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ for an integration time T is defined by (1). It is a statistically stable quantity, in the sense that for a large integration time T, C_T is independent of the realization of the noise sources. This is stated in the following proposition proved in [15]. The statistical stability is analyzed in detail in [16].

Proposition 3.1 *1. The expectation of C_T (with respect to the noise source distribution) is independent of T:*

$$\langle C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)\rangle = C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)\,, \tag{12}$$

where $C^{(1)}$ is given by

$$C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \int d\boldsymbol{y} \iint ds ds' G(s, \boldsymbol{x}_1, \boldsymbol{y}) G(\tau + s + s', \boldsymbol{x}_2, \boldsymbol{y}) F^\varepsilon(s') \Gamma_0(\boldsymbol{y})\,, \tag{13}$$

or equivalently by

$$C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \int d\boldsymbol{y} \int d\omega \overline{\widehat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{y}) \widehat{G}(\omega, \boldsymbol{x}_2, \boldsymbol{y}) \widehat{F}^\varepsilon(\omega) e^{-i\omega\tau} \Gamma_0(\boldsymbol{y})\,. \tag{14}$$

2. The empirical cross correlation C_T is a self-averaging quantity:

$$C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \overset{T\to\infty}{\longrightarrow} C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)\,, \tag{15}$$

in probability with respect to the distribution of the sources. More precisely, the fluctuations of C_T around its mean value $C^{(1)}$ are of order $T^{-1/2}$.

In this paper, we shall always assume that the integration time T is large enough so that the empirical cross correlation C_T can be considered as equal to the statistical cross correlation $C^{(1)}$. This allows us to focus our attention on the properties of the statistical cross correlation.

4 Emergence of the Green's function for an extended distribution of sources in a homogeneous medium

In this section we give an elementary proof of the relation between the cross correlation and the Green's function when the medium is homogeneous and open with background velocity c_0, and the source distribution extends over all space, i.e. $\Gamma_0 \equiv 1$, as in Figure 1. In this case the signal amplitude diverges because the contributions from the noise sources far away from the sensors are not damped. For a well-posed formulation we need to introduce some dissipation, so we consider the solution u of the damped wave equation:

$$\frac{1}{c_0^2}\Big(\frac{1}{T_a} + \frac{\partial}{\partial t}\Big)^2 u(t, \boldsymbol{x}) - \Delta_{\boldsymbol{x}} u(t, \boldsymbol{x}) = n^\varepsilon(t, \boldsymbol{x})\,. \tag{16}$$

The following proposition can be found in [15]. A somewhat different form, with delta-correlated in time sources and with a different definition of dissipation, can be found in [22].

Proposition 4.1 *In a three-dimensional open medium with dissipation, if the source distribution extends over all space* $\Gamma_0 \equiv 1$*, then*

$$\frac{\partial}{\partial \tau} C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = -\frac{c_0^2 T_a}{4} e^{-\frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}{c_0 T_a}} \Big[F^{\varepsilon} * G(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) - F^{\varepsilon} * G(-\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \Big] , \tag{17}$$

where $*$ *stands for the convolution in* τ *and* G *is the Green's function of the homogeneous medium without dissipation:*

$$G(t, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{4\pi |\boldsymbol{x}_1 - \boldsymbol{x}_2|} \delta\Big(t - \frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}{c_0} \Big) .$$

If the decoherence time of the sources is much shorter than the travel time (i.e., $\varepsilon \ll 1$), then F^{ε} behaves like a Dirac distribution in (17) and we have

$$\frac{\partial}{\partial \tau} C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \simeq e^{-\frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}{c_0 T_a}} \Big[G(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) - G(-\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \Big] ,$$

up to a multiplicative constant. It is therefore possible to estimate the travel time $\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2) = |\boldsymbol{x}_1 - \boldsymbol{x}_2| / c_0$ between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ from the cross correlation, with an accuracy of the order of the decoherence time of the noise sources.

5 Emergence of the Green's function for an extended distribution of sources in an inhomogeneous medium

The cross correlation function is closely related to the symmetrized Green's function from $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$ not only for a homogeneous medium but also for an inhomogeneous medium, as discussed in the introduction. Here we give a simple and rigorous proof for an open inhomogeneous medium in the case in which the noise sources are located on the surface of a sphere that encloses both the inhomogeneous region and the sensors, located at $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ (Figure 1). The proof is based on an approximate identity that follows from the second Green's identity and the Sommerfeld radiation condition. This approximate identity can be viewed as a version of the Helmholtz-Kirchhoff integral theorem (known in acoustics [5, p. 473] and in optics [7, p. 419]) and it is also presented in [29, 15].

Proposition 5.1 *Let us assume that the medium is homogeneous with background velocity* c_0 *outside the ball* $B(\mathbf{0}, D)$ *with center* $\mathbf{0}$ *and radius* D*. Then, for any* $\boldsymbol{x}_1, \boldsymbol{x}_2 \in B(\mathbf{0}, D)$ *we have for* $L \gg D$*:*

$$\widehat{G}(\omega, \boldsymbol{x}_1, \boldsymbol{x}_2) - \overline{\widehat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{2i\omega}{c_0} \int_{\partial B(\mathbf{0}, L)} \overline{\widehat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{y}) \widehat{G}(\omega, \boldsymbol{x}_2, \boldsymbol{y}) dS(\boldsymbol{y}) . \tag{18}$$

The right side of the Helmholtz-Kirchhoff identity (18) is related to the representation (14) of the cross correlation function $C^{(1)}$ in the Fourier domain. Therefore, by substituting (18) into (14) we get the following corollary.

Corollary 5.1 *We assume that*
1) the medium is homogeneous outside the ball $B(\mathbf{0}, D)$ with center $\mathbf{0}$ and radius D,
2) the sources are localized with a uniform density on the sphere $\partial B(\mathbf{0}, L)$ with center $\mathbf{0}$ and radius L.
If $L \gg D$, then for any $\boldsymbol{x}_1, \boldsymbol{x}_2 \in B(\mathbf{0}, D)$, we have (up to a multiplicative factor)

$$\frac{\partial}{\partial \tau} C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = F^\varepsilon * G(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) - F^\varepsilon * G(-\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) . \tag{19}$$

If in addition we have $\varepsilon \ll 1$, then F^ε behaves approximately like a delta distribution acting on the Green's function and we get (2).

6 Travel time estimation with spatially localized noise sources in an open medium

We study in this section the cross correlation function when the support of the sources is spatially limited in an open non-dissipative medium. We assume in this section that the fluctuations of the medium parameters are modeled by a smooth background velocity profile $c(\boldsymbol{x})$. The outgoing time-harmonic Green's function $\widehat{G}$ of the medium is the solution of (11) along with the radiation condition at infinity. When the background is homogeneous with constant wave speed c_0 then the homogeneous outgoing time-harmonic Green's function is

$$\widehat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) = \frac{e^{i\omega \frac{|\boldsymbol{y}-\boldsymbol{x}|}{c_0}}}{4\pi |\boldsymbol{y} - \boldsymbol{x}|} \tag{20}$$

in three-dimensional space, and

$$\widehat{G}(\omega, \boldsymbol{x}, \boldsymbol{y}) = \frac{i}{4} H_0^{(1)}\Big(\omega \frac{|\boldsymbol{y} - \boldsymbol{x}|}{c_0}\Big) \tag{21}$$

in two-dimensional space. Here $H_0^{(1)}$ is the zeroth order Hankel function of the first kind. Using the asymptotic form of the Hankel function [1, formula 9.2.3], we see that the high-frequency behavior of the Green's function is related to the homogeneous medium travel time $|\boldsymbol{x} - \boldsymbol{y}|/c_0$:

$$\widehat{G}\Big(\frac{\omega}{\varepsilon}, \boldsymbol{x}, \boldsymbol{y}\Big) \sim \frac{1}{|\boldsymbol{x} - \boldsymbol{y}|^{(d-1)/2}} e^{i \frac{\omega}{\varepsilon} \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_0}} .$$

For a general smoothly varying background with propagation speed $c(\boldsymbol{x})$, the high-frequency behavior of the Green's function is also related to the travel time and it is given by the WKB (Wentzel-Kramers-Brillouin) approximation [6]

$$\widehat{G}\Big(\frac{\omega}{\varepsilon}, \boldsymbol{x}, \boldsymbol{y}\Big) \sim \mathcal{A}(\boldsymbol{x}, \boldsymbol{y}) e^{i \frac{\omega}{\varepsilon} \mathcal{T}(\boldsymbol{x}, \boldsymbol{y})} , \tag{22}$$

which is valid when $\varepsilon \ll 1$. Here the coefficients $\mathcal{A}(\boldsymbol{x},\boldsymbol{y})$ and $\mathcal{T}(\boldsymbol{x},\boldsymbol{y})$ are smooth except at $\boldsymbol{x}=\boldsymbol{y}$. The amplitude $\mathcal{A}(\boldsymbol{x},\boldsymbol{y})$ satisfies a transport equation and the travel time $\mathcal{T}(\boldsymbol{x},\boldsymbol{y})$ satisfies the eikonal equation. It is a symmetric function $\mathcal{T}(\boldsymbol{x},\boldsymbol{y})=\mathcal{T}(\boldsymbol{y},\boldsymbol{x})$ and it can be obtained from Fermat's principle

$$\mathcal{T}(\boldsymbol{x},\boldsymbol{y}) = \inf\left\{ T \ \text{ s.t.} \, \exists \, (\boldsymbol{X}_t)_{t\in[0,T]} \in \mathcal{C}^1 \, , \, \boldsymbol{X}_0=\boldsymbol{x} \, , \, \boldsymbol{X}_T=\boldsymbol{y} \, , \, \big|\frac{d\boldsymbol{X}_t}{dt}\big| = c(\boldsymbol{X}_t) \right\} . \tag{23}$$

A curve $(\boldsymbol{X}_t)_{t\in[0,T]}$ that produces the minimum in (23) is a ray and it satisfies Hamilton's equations (28-28).

For simplicity we assume that the background speed $c(\boldsymbol{x})$ is such that there is a unique ray joining any pair of points $(\boldsymbol{x},\boldsymbol{y})$ in the region of interest. We can then describe the behavior of the cross correlation function between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ when ε is small, with and without directional energy flux from the sources.

Proposition 6.1 *As ε tends to zero, the cross correlation $C^{(1)}(\tau,\boldsymbol{x}_1,\boldsymbol{x}_2)$ has singular components if and only if the ray going through $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ reaches into the source region, that is, into the support of the function Γ_0. In this case there are either one or two singular components at $\tau=\pm\mathcal{T}(\boldsymbol{x}_1,\boldsymbol{x}_2)$.*

More precisely, any ray going from the source region to $\boldsymbol{x}_2$ and then to $\boldsymbol{x}_1$ gives rise to a singular component at $\tau=-\mathcal{T}(\boldsymbol{x}_1,\boldsymbol{x}_2)$. Rays going from the source region to $\boldsymbol{x}_1$ and then to $\boldsymbol{x}_2$ give rise to a singular component at $\tau=\mathcal{T}(\boldsymbol{x}_1,\boldsymbol{x}_2)$.

This proposition explains why travel time estimation is bad when the ray joining $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ is roughly perpendicular to the direction of the energy flux from the noise sources, as in Figure 3. Its proof is given in [15] and it is based on the use of the high-frequency asymptotic expression (22) of the Green's function and a stationary phase argument.

7 Emergence of the Green's function for a localized distribution of sources in an ergodic cavity

In the case of a spatially localized distribution of noise sources, directional diversity of the recorded fields can be enhanced if there is sufficient scattering in the medium. An ergodic cavity with a homogeneous or inhomogeneous interior is a good example (Figure 4, left): Even with a source distribution that has very limited spatial support, the reverberations of the waves in the cavity generate interior fields with high directional diversity [10, 3]. In this section we consider the damped wave equation

$$\Big(\frac{1}{T_a}+\frac{\partial}{\partial t}\Big)^2 u(t,\boldsymbol{x}) - \nabla_{\boldsymbol{x}}\cdot\big[c^2(\boldsymbol{x})\nabla_{\boldsymbol{x}}\big]u(t,\boldsymbol{x}) = c^2(\boldsymbol{x})n^\varepsilon(t,\boldsymbol{x})\,, \tag{24}$$

in a bounded domain Ω with Dirichlet boundary conditions on $\partial\Omega$. Semiclassical analysis is a very efficient tool to study wave propagation in an ergodic cavity with a smoothly varying background velocity $c(\boldsymbol{x})$. Note that a wave equation with a self-adjoint operator is considered in (24) in order to simplify the algebra, but the result could be extended to more general wave equations. In this section we assume that the source distribution is not delta-correlated but has spatial correlation, because semi-classical analysis allows us to study the role of the spatial correlation of the noise sources. Therefore, we assume that the spatial covariance function has the form

$$\Gamma^\varepsilon(\boldsymbol{x},\boldsymbol{y}) = \Gamma\Big(\frac{\boldsymbol{x}+\boldsymbol{y}}{2},\frac{\boldsymbol{x}-\boldsymbol{y}}{\varepsilon}\Big)\,. \tag{25}$$

Here the spatial correlation radius of the noise sources is assumed to be of the same order as the decoherence time (ε), which is the regime in which time and space noise correlations contribute to the Green's function estimation at the same order of magnitude.

The covariance operator $\Theta^\varepsilon : L^2(\Omega) \to L^2(\Omega)$ defined by

$$\Theta^\varepsilon \psi(\boldsymbol{x}) = \int \Gamma^\varepsilon(\boldsymbol{x},\boldsymbol{y})\psi(\boldsymbol{y})dy \tag{26}$$

is a zero-order pseudodifferential operator with symbol $\widehat{\Gamma}(\boldsymbol{x},\boldsymbol{\xi})$

$$\Theta^\varepsilon = \mathrm{Op}^\varepsilon\big[\widehat{\Gamma}(\boldsymbol{x},\boldsymbol{\xi})\big]\,,$$

where the Fourier transform $\widehat{\Gamma}(\boldsymbol{x},\boldsymbol{\xi})$ of the function $\boldsymbol{z}\mapsto\Gamma(\boldsymbol{x},\boldsymbol{z})$ is

$$\widehat{\Gamma}(\boldsymbol{x},\boldsymbol{\xi}) = \int \Gamma(\boldsymbol{x},\boldsymbol{z})e^{-i\boldsymbol{\xi}\cdot\boldsymbol{z}}d\boldsymbol{z}\,,$$

and we have used the Weyl quantization Op^ε defined by

$$\mathrm{Op}^\varepsilon\big[\widehat{\Gamma}(\boldsymbol{x},\boldsymbol{\xi})\big]\psi(\boldsymbol{x}) = \frac{1}{(2\pi)^d}\iint \widehat{\Gamma}\Big(\frac{\boldsymbol{x}+\boldsymbol{y}}{2},\boldsymbol{\xi}\Big)e^{\frac{i}{\varepsilon}\boldsymbol{\xi}\cdot(\boldsymbol{x}-\boldsymbol{y})}\psi(\boldsymbol{y})d\boldsymbol{y}d\boldsymbol{\xi}\,. \tag{27}$$

The main result of the papers [10, 3] is that it is possible to reconstruct the singular components of the Green's function in the ergodic case, up to a smoothing operator that depends on Γ^ε and F^ε. There are two ingredients that are used in this result:
1) Approximation of full wave propagation by classical ray dynamics (Egorov theorem): the singular (high-frequency) components propagate along the rays $(\boldsymbol{X}_t,\boldsymbol{\xi}_t)$ of geometric optics (Hamiltonian flow $h(\boldsymbol{x},\boldsymbol{\xi}) = c(\boldsymbol{x})|\boldsymbol{\xi}|$) defined by

$$\begin{aligned}
\frac{d\boldsymbol{X}_t}{dt} &= c(\boldsymbol{X}_t)\frac{\boldsymbol{\xi}_t}{|\boldsymbol{\xi}_t|}\,, \qquad \boldsymbol{X}_0(\boldsymbol{x},\boldsymbol{\xi}) = \boldsymbol{x}\,,\\
\frac{d\boldsymbol{\xi}_t}{dt} &= -\nabla c(\boldsymbol{X}_t)|\boldsymbol{\xi}_t|\,, \qquad \boldsymbol{\xi}_0(\boldsymbol{x},\boldsymbol{\xi}) = \boldsymbol{\xi}\,,
\end{aligned}$$

and with specular reflection at the boundary $\partial\Omega$.
2) Ergodicity of the ray dynamics in the cavity Ω: starting from almost any point

$\boldsymbol{x}$ and almost any direction $\boldsymbol{\xi}$, the ray $(\boldsymbol{X}_t, \boldsymbol{\xi}_t)$ visits all the energy surface. For any $f \in L^\infty(S^*(\Omega))$ and for $(\boldsymbol{x}, \boldsymbol{\xi})$ in a subset of full measure of $S^*(\Omega)$,

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t f\big(\boldsymbol{X}_s(\boldsymbol{x}, \boldsymbol{\xi}), \boldsymbol{\xi}_s(\boldsymbol{x}, \boldsymbol{\xi})\big) ds = \frac{1}{\mu(S^*(\Omega))} \int_{S^*(\Omega)} f(\boldsymbol{m}) d\mu(\boldsymbol{m}) ,$$

where $S^*(\Omega)$ is the cotangent spherical bundle (energy surface)

$$S^*(\Omega) = \{(\boldsymbol{x}, \boldsymbol{\xi}) \in T^*\Omega \, , \, c(\boldsymbol{x})|\boldsymbol{\xi}| = 1\} ,$$

and μ is the Liouville measure on $S^*(\Omega)$.

Proposition 7.1 *If $c \in W^{4,\infty}(\Omega)$, $\widehat{\Gamma}$ is smooth, bounded, and integrable, then $\partial_\tau C^{(1)}(\tau, \boldsymbol{x}, \boldsymbol{y})$ is the kernel of the operator*

$$e^{-\frac{\tau}{T_a}} \mathcal{K}_\Gamma^\varepsilon \mathcal{F}_F^\varepsilon [G(\tau) - G(-\tau)] + R^\varepsilon(\tau) + R_{T_a}(\tau) , \tag{28}$$

for any $\tau > 0$, where
- $G(\tau)$ is the Green's function operator with kernel $G(\tau, \boldsymbol{x}, \boldsymbol{y})$.
- $\mathcal{F}_F^\varepsilon$ is the convolution operator in τ (due to the time correlations of the sources):

$$\mathcal{F}_F^\varepsilon G(\tau) = \int F^\varepsilon(s) G(\tau - s) ds .$$

- $\mathcal{K}_\Gamma^\varepsilon$ is the smoothing operator (due to the spatial correlations of the sources):

$$\mathcal{K}_\Gamma^\varepsilon = \mathrm{Op}^\varepsilon \left[\widehat{k}_\Gamma\big(c(\boldsymbol{x})\boldsymbol{\xi}\big) \right] ,$$

with

$$\widehat{k}_\Gamma(\tilde{\boldsymbol{\xi}}) = \frac{\int_\Omega d\boldsymbol{z} c(\boldsymbol{z})^{-d} \int_{\partial B(\mathbf{0},1)} dS(\boldsymbol{\eta}) \widehat{\Gamma}\big(\boldsymbol{z}, |\tilde{\boldsymbol{\xi}}| \frac{\boldsymbol{\eta}}{c(\boldsymbol{z})}\big)}{\int_\Omega d\boldsymbol{z} c(\boldsymbol{z})^{-d} \int_{\partial B(\mathbf{0},1)} dS(\boldsymbol{\eta})} . \tag{29}$$

The remainder $R^\varepsilon(\tau)$ is determined by the error in the semiclassical approximation and it is small if ε is small (Egorov theorem).
The remainder $R_{T_a}(\tau)$ is determined by the rate of convergence of the ergodic theorem for the function $\widehat{\Gamma}$ of the classical Hamiltonian flow. If T_{erg} is the characteristic convergence time of $\frac{1}{t} \int_0^t \widehat{\Gamma}(\boldsymbol{X}_s, \boldsymbol{\xi}_s) ds$ to its ergodic limit $\widehat{k}_\Gamma(c(\boldsymbol{x})\boldsymbol{\xi})$, then $R_{T_a}(\tau)$ is small if $T_a \gg T_{\mathrm{erg}}$.

The symbol of the smoothing operator $\mathcal{K}_\Gamma^\varepsilon$ is $\widehat{k}_\Gamma(c(\boldsymbol{x})\boldsymbol{\xi})$. The form of the symbol of $\mathcal{K}_\Gamma^\varepsilon$ is obtained by averaging the symbol $\widehat{\Gamma}(\boldsymbol{x}, \boldsymbol{\xi})$ of the covariance operator Θ^ε over the Liouville measure on surfaces of constant energy. This makes sense intuitively since, in the semiclassical limit, we can expect the symbol of $\mathcal{K}_\Gamma^\varepsilon$ to be close to the one of Θ^ε transported by the classical Hamiltonian flow, and this converges to (29) by the ergodic theorem. This shows that the support of the smoothing operator $\mathcal{K}_\Gamma^\varepsilon$ has an effective radius that is of the order of the correlation radius of the sources. To summarize, if $T_{\mathrm{erg}} \ll T_a$ and $\varepsilon \ll 1$, then detecting the first peak of $\tau \mapsto C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$ gives an estimate of the travel time from $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$. The accuracy of this estimate depends on the correlation radius and the decoherence time of the noise sources.

8 Iterated cross correlations for travel time estimation in a weakly scattering medium

For travel time tomography to be successful it is necessary to have good estimates of the travel times between pairs of sensors that cover well the region of interest. When the noise sources are spatially localized and there is a strong directional energy flux at the sensors, then travel time estimates will be poor for sensor pairs with axis in directions perpendicular to this flux. In this section we show that it is possible to exploit scattering from random inhomogeneities so as to enhance travel time estimation.

We consider distributions of noise sources that are spatially localized and media with scattering that is not strong enough for equipartition of the fields at the sensors [27]. Therefore, even with scattering, the signals depend strongly on the spatial localization of the noise sources, which affects the quality of travel time estimation. However, the coda (i.e. the tails) of the cross correlations are generated by scattered waves, which have more directional diversity than the direct waves from the noise sources. By cross correlating the coda of the cross correlations, which produces special fourth-order cross correlations, it is possible to exploit scattered waves and their enhanced directional diversity. Campillo and Stehly [28] suggest a way to estimate the Green's function between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ as follows.

1) Calculate the cross correlations between $\boldsymbol{x}_1$ and $\boldsymbol{x}_{\mathrm{a},k}$ and between $\boldsymbol{x}_2$ and $\boldsymbol{x}_{\mathrm{a},k}$ for each auxiliary sensor $\boldsymbol{x}_{\mathrm{a},k}$:

$$C_T(\tau, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_l) = \frac{1}{T}\int_0^T u(t, \boldsymbol{x}_{\mathrm{a},k}) u(t+\tau, \boldsymbol{x}_l) dt, \qquad l = 1, 2, \quad k = 1, \ldots, N_a.$$

2) Calculate the coda (i.e. the tails) of these cross correlations:

$$C_{T,\mathrm{coda}}(\tau, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_l) = C_T(\tau, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_l) \mathbf{1}_{[T_{\mathrm{c}1}, T_{\mathrm{c}2}]}(|\tau|), \qquad l = 1, 2, \quad k = 1, \ldots, N_a.$$

3) Cross correlate the tails of the cross correlations and sum them over all auxiliary sensors to form the coda cross correlation between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$:

$$C_T^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \sum_{k=1}^{N_{\mathrm{a}}} \int C_{T,\mathrm{coda}}(\tau', \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_1) C_{T,\mathrm{coda}}(\tau' + \tau, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_2) d\tau'. \quad (30)$$

This algorithm depends on three important time parameters:

1) The time T is the integration time and it should be large so as to ensure statistical stability with respect to the distribution of the noise sources.

2) The time $T_{\mathrm{c}1}$ is chosen so that the parts of the Green's functions $t \mapsto G(t, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_1)$ and $t \mapsto G(t, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_2)$ limited to $[T_{\mathrm{c}1}, T_{\mathrm{c}2}]$ do not contain the contributions of the direct waves. This means that $T_{\mathrm{c}1}$ depends on the position of the auxiliary sensor $\boldsymbol{x}_{\mathrm{a},k}$ and should be a little bit larger than the travel time between the auxiliary sensor and the sensors $\max(\mathcal{T}(\boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_1), \mathcal{T}(\boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_2))$.

3) The time $T_{\mathrm{c}2}$ should be large enough so that the parts of the Green's functions $t \mapsto G(t, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_1)$ and $t \mapsto G(t, \boldsymbol{x}_{\mathrm{a},k}, \boldsymbol{x}_2)$ limited to $[T_{\mathrm{c}1}, T_{\mathrm{c}2}]$ contain the

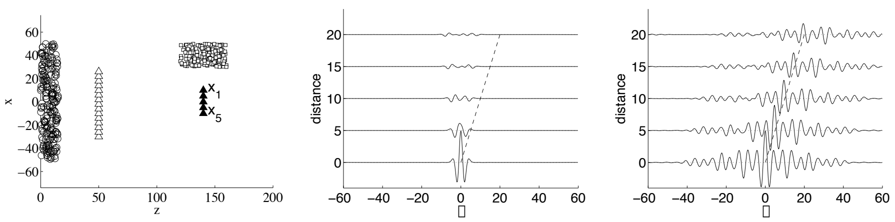
contributions of the incoherent scattered waves. This means that $T_{\rm c2}$ should be of the order of the power delay spread.

Using the stationary phase method it can be shown [15, 16] that the algorithm proposed by Campillo and Stehly succeeds in exploiting the enhanced directivity of scattered waves. In particular the empirical coda cross correlation $C_T^{(3)}$ defined by (30) is a self-averaging quantity and it is equal to the statistical coda cross correlation $C^{(3)}$ as $T \to \infty$:

$$\begin{aligned} C^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) &= \sum_{k=1}^{N_a} \int \overline{\widehat{C}^{(1)}_{\rm coda}}(\omega, \boldsymbol{x}_{{\rm a},k}, \boldsymbol{x}_1) \widehat{C}^{(1)}_{\rm coda}(\omega, \boldsymbol{x}_{{\rm a},k}, \boldsymbol{x}_2) e^{-i\omega\tau} d\omega \, , \\ C^{(1)}_{\rm coda}(\tau, \boldsymbol{x}_{{\rm a},k}, \boldsymbol{x}_l) &= C^{(1)}(\tau, \boldsymbol{x}_{{\rm a},k}, \boldsymbol{x}_l) \mathbf{1}_{[T_{\rm c1}, T_{\rm c2}]}(|\tau|) \, . \end{aligned}$$

The statistical coda cross correlation has singular components at the travel time between the sensors even in the unfavorable case in which the ray joining $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ does not reach into the source region. This result is presented in Proposition 8.1 below. Its proof requires to specify a model for the inhomogeneous medium. A simple, single-scattering model is sufficient for this purpose [15, 16].

Proposition 8.1 *There are two (and only two) singular components in $C^{(3)}$, at times $\tau = \pm \mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$, if the two following conditions hold:*
1) The ray going through $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ (excluding the segment between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$) reaches into the scattering region. The scatterers along this ray are the primary ones for enhanced travel time estimation.
2) Rays going from the source region to the primary scatterers reach into the observation region.



a) Configuration **b)** $C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$ **c)** $C_T^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$

Figure 5: The configuration is shown in Figure a: the circles are the noise sources, the squares are the scatterers, and the triangles are the sensors. Figure b shows the cross correlation $C^{(1)}$ between the pairs of sensors $(\boldsymbol{x}_1, \boldsymbol{x}_j)$, $j = 1, \dots, 5$, versus the distance $|\boldsymbol{x}_j - \boldsymbol{x}_1|$. Figure c shows the coda cross correlation $C^{(3)}$ between the pairs of sensors $(\boldsymbol{x}_1, \boldsymbol{x}_j)$, $j = 1, \dots, 5$, which shows the singular peak at lag time equal to the travel time $\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_j)$ in the coda cross correlation $C^{(3)}$, because the ray going through $\boldsymbol{x}_1$ and $\boldsymbol{x}_j$ intersects the scattering region.

The statistical coda cross correlation $C^{(3)}$ differs from the statistical cross correlation $C^{(1)}$ in that the contributions of the direct waves are eliminated and only the contributions of the scattered waves are taken into account (note that some of the contributions of scattered waves are also eliminated, but only those which correspond to small additional travel times, which are also those which induce small directional diversity). Since scattered waves have more directional diversity than the direct waves when the noise sources are spatially localized, the coda cross correlation $C^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$ usually exhibits a stronger peak at lag time equal to the inter-sensor travel time $\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$ (see Figure 5). In particular, in contrast with the cross correlation $C^{(1)}$, the existence of a singular component at lag time equal to the travel time $\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$ does not require that the ray joining $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ reaches into the source region, but only into the scattering region.

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