Weakly Nonlinear Theory for the Ablative Rayleigh-Taylor Instability

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A weakly nonlinear model is proposed for the Rayleigh-Taylor instability in the presence of ablation and thermal transport. The second harmonic generation efficiency of a single-mode disturbance is computed, as well as the nonlinear correction to the exponential growth of the fundamental modulation. Mode coupling in the spectrum of a multimode disturbance is thoroughly analyzed. The ablative stabilization can be clearly discussed because the derived formulas for the evanescent ablation rate are in agreement with previously known results for incompressible, inviscid, irrotational, and immiscible fluids [S.W. Haan, Phys. Fluids B **3**, 2349 (1991); M. Berning and A. M. Rubenchik, Phys. Fluids **10**, 1564 (1998)].

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The Rayleigh-Taylor (RT) instability occurs when a fluid accelerates another fluid of higher density. This phenomenon may dramatically reduce the performance of inertial confinement fusion (ICF) experiments by degrading the symmetry of implosion [1]. In ICF targets the ablation process and the thermal transport are coming into play [2]. It has been shown by several authors that the ablative RT instability growth is stabilized relative to classical RT during the linear stage, where the growth is exponential in time. The self-consistent theory has shown a complicated dependence of the growth rate on the plasma parameters [3,4]. In this Letter a weakly nonlinear (WNL) theory is developed for the ablative RT instability. The first WNL analysis in a simplified framework was performed in Ref. [5]. We aim at deriving closed-form expressions for the most important physical quantities. In the single-mode case analytic formulas are derived for the first time for the second harmonic generation efficiency and the nonlinear correction to the exponential growth of the fundamental modulation for arbitrary Atwood numbers. Recently a WNL theory was presented in the single-mode case in the limit of a very large density ratio [6] and in the framework of a finite bandwidth [7] where the results can be integrated only numerically. In the multimode case we also give the expression of the interface elevation taking into account mode coupling. Our model belongs to the family of the sharp boundary models [8–10]. Our theory is based on a set of hypotheses that are stated at the very beginning, and the following steps of the study are rigorous at the mathematical level. We develop a third-order WNL model, but the theory can be extended to higher orders with the help of a computer algebra system (for instance, maple). We also point out that the basic hypotheses of our model have been proposed in Ref. [11] and that the perturbation techniques are based on general results about the stability of solutions of nonlinear hyperbolic systems of conservation laws [12].

In the standard case where the ablative flow is subsonic we can apply the isobaric approximation [8] so that the fluid dynamics is governed by the system of conservation laws for mass, x and $\mathbf{y}$ momenta, and energy

$$\begin{aligned} \partial_t\rho + \partial_x(\rho u) + \nabla_y \cdot (\rho \mathbf{v}) &= 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2 + p) + \nabla_y \cdot (\rho u \mathbf{v}) &= \rho g, \\ \partial_t(\rho \mathbf{v}) + \partial_x(\rho u \mathbf{v}) + \nabla_y \cdot (\rho \mathbf{v} \otimes \mathbf{v} + p) &= 0, \\ C_p[\partial_t(\rho T) + \partial_x(\rho u T) + \nabla_y \cdot (\rho \mathbf{v} T)] &= \partial_x q_x + \nabla_y \cdot q_{\mathbf{y}}, \end{aligned}$$

where ρ is the density, u (respectively, $\mathbf{v}$) is the fluid velocity in the x (respectively, $\mathbf{y}$) direction, p is the pressure, T is the temperature. $q_x = \lambda \partial_x T$ and $q_{\mathbf{y}} = \lambda \nabla_y T$ are the heat flows in the x (respectively, $\mathbf{y}$) direction, C_p is the specific heat per unit mass at constant pressure, and λ is the thermal conductivity. Gravity g points to the x direction.

Our analysis is based on a set of three hypotheses. First, we assume that the unperturbed flow is stationary and one dimensional. The ablation front is represented as a surface which separates a cool material with high density ρ_L on the left side and a blowoff plasma with low density ρ_R on the right side. Second, we assume that the density ρ and the thermal conductivity λ are constant on both sides of the ablation front. Finally, we assume that the thermal transport is small in the overdense region and that it is large in the blowoff region. This hypothesis reads as $d_L|\mathbf{k}| \ll 1 \ll d_R|\mathbf{k}|$, where $\mathbf{k}$ is the typical wave number of the interface modulation, $d_{L,R} = \lambda_{L,R}/(\dot{m}C_p)$ are

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the left and right thermal diffusion lengths, and $\dot{m}$ is the mass ablation rate. This model has been shown to reproduce accurately the linear growth of RT instabilities obtained by the self-consistent linear theory [3,4] if the jump density is calculated in a self-consistent way [11,13]. We show in this Letter that a WNL analysis of this model can be performed at any order without any more assumptions.

We study the growth of a small-amplitude perturbation of the interface whose displacement with respect to the unperturbed front $x = 0$ is described by $x = \eta(t, \mathbf{y})$. The typical amplitude δ of the initial disturbance is assumed to be small (i.e., smaller than its typical wavelength). The interface displacement η, the hydrodynamic variables $U = (u, \mathbf{v}, p)$, and the temperature T can be expanded in powers of δ as

$$\eta(t, \mathbf{y}) = \delta\eta_1(t, \mathbf{y}) + \delta^2\eta_2(t, \mathbf{y}) + \delta^3\eta_3(t, \mathbf{y}) + O(\delta^4),$$
$$U(t, x, \mathbf{y}) = U_0(x) + \delta U_1(t, x, \mathbf{y}) + \delta^2 U_2(t, x, \mathbf{y}) + \delta^3 U_3(t, x, \mathbf{y}) + O(\delta^4).$$

Substituting these *Ansätze* into the system of conservation laws and the jump conditions, and collecting the terms with the same powers in δ, we get for terms of order δ the system governing the linear stability of the unperturbed solution $(U_0, T_0, \eta_0 = 0)$. The terms of order δ^2 (respectively, δ^3) give the second-order (respectively, third-order) WNL corrections.

We first address the case of a single-mode initial disturbance with wave number $\mathbf{k}$: $\eta(t = 0, \mathbf{y}) = \delta\cos(\mathbf{k}\cdot\mathbf{y})$. It is convenient to take the Fourier transform with respect to the transverse spatial variable $\mathbf{y}$ and to consider the corresponding Fourier coefficients $\hat{\eta}$. The linear stability analysis establishes that in the early steps of the dynamics the interface modulation is single mode and grows exponentially in time with a characteristic growth rate $\gamma(\mathbf{k})$ [14]. The WNL analysis exhibits mode coupling that drives up harmonic modes [15]. After a short transition time terms such as $\exp[-\gamma(\mathbf{k})t]$ can be neglected, so that the problem is reduced to the identification of a finite set of Fourier coefficients

$$\hat{\eta}(t, \mathbf{k}) = \delta\hat{\eta}_1(\mathbf{k})\exp[\gamma(\mathbf{k})t] + \delta^3\hat{\eta}_3(\mathbf{k})\exp[3\gamma(\mathbf{k})t],$$
$$\hat{\eta}(t, 2\mathbf{k}) = \delta^2\hat{\eta}_2(2\mathbf{k})\exp[2\gamma(\mathbf{k})t],$$
$$\hat{\eta}(t, 3\mathbf{k}) = \delta^3\hat{\eta}_3(3\mathbf{k})\exp[3\gamma(\mathbf{k})t].$$

$\hat{\eta}_1$ and $\gamma(\mathbf{k})$ are imposed by the linear stability analysis. The term $\hat{\eta}_2(2\mathbf{k})$ originates from a second-order WNL effect (second harmonic generation). Calculations show that there is no excitation of a zeroth harmonic in the sense that $\hat{\eta}(t, 0\mathbf{k})$ is vanishing. The term $\hat{\eta}_3(3\mathbf{k})$ originates from a third-order WNL effect (third harmonic generation). The third-order WNL term $\hat{\eta}_3(\mathbf{k})$ gives the nonlinear correction to the exponential growth of the fundamental modulation.

The linear stability analysis demonstrates the existence of four possible linear modes for the hydrodynamic variables U_1 (one sonic mode in the overdense region, one sonic mode which merges with a thermal conduction mode, and two vorticity modes in the blowoff region) [9]. The amplitudes of the modes are four free parameters. They can be expressed in terms of the modulation interface η_1 by the four linearized jump conditions satisfied by the hydrodynamic variables at the interface. It is necessary to exhibit one more relation to fix the growth rate. This relation is obtained by studying the thermal flow. T_1 consists of five modes which depend on the hydrodynamic parameters and two more free parameters. The jump conditions for the temperature and the heat flow read as three complicated relations that can be dramatically simplified by using a separation of scales technique based on the assumption $d_L|\mathbf{k}| \ll 1 \ll d_R|\mathbf{k}|$. Accordingly, the two free parameters of T_1 are eliminated, and a new relation is established between the parameter of the sonic mode of the blowoff region and η_1. This provides a compatibility condition for the instability growth rate γ that does not depend on the values of the thermal diffusion lengths

$$\gamma(\mathbf{k}) = \sqrt{Ag|\mathbf{k}| - |\mathbf{k}|^2 u_L u_R A^2} - (1 + A)|\mathbf{k}|u_L, \qquad (1)$$

where $u_L = \dot{m}/\rho_L$ (respectively, $u_R = \dot{m}/\rho_R$) is the ablation velocity (respectively, blowoff velocity), and $A = (\rho_L - \rho_R)/(\rho_R + \rho_L) = (u_R - u_L)/(u_R + u_L)$ is the Atwood number. Note that the expression of the growth rate γ is in complete agreement with the one derived in Ref. [9]. γ is positive for any wave number below the cutoff wave number $k_c = g/(u_L u_R) \times (u_R - u_L)/(u_L + u_R)$. γ is maximal for a modulation with wave number

$$k_{\mathrm{opt}} = \frac{g}{2u_L u_R}\frac{\sqrt{u_R} - \sqrt{u_L}}{\sqrt{u_L} + \sqrt{u_R}}. \qquad (2)$$

If $|\mathbf{k}| \ll k_{\mathrm{opt}}$, then we get the classical RT growth rate $\gamma(\mathbf{k}) = \sqrt{Ag|\mathbf{k}|}$.

The second-order WNL analysis consists of collecting the terms of order δ^2 in the system of conservation laws and in the jump conditions for the hydrodynamic variables and the temperature. The system of conservation laws provides a system of linear equations for the hydrodynamic variables U_2 with source terms which are quadratic functions of the first-order perturbations U_1. The source terms are known from the linear stability analysis. We then derive the existence of five modes in the system, depending on four free parameters. There are two striking features. First, there is no vorticity mode in the overdense region. Second, vorticity plays an important role in the blowoff region, in particular, through the form of a hybrid mode that results from the interplay between a first-order sonic mode and a first-order vorticity mode.

The four free parameters of U_2 can be related to η_2 by the four jump conditions, which also read as linear relations between U_2 and η_2 with quadratic source terms in U_1 and η_1. T_2 consists of seven modes containing two free parameters. The jump conditions for the temperature and the heat flow read as three linear relations in terms of T_2 with source terms depending on T_1, U_1, and η_1. Accordingly, the two free parameters of T_2 can be eliminated, and a compatibility condition for η_2 can be obtained, which reads, after considerable simplification obtained from the hypothesis $d_L|\mathbf{k}| \ll 1 \ll d_R|\mathbf{k}|$,

$$\hat{\eta}_2(2\mathbf{k}) = \hat{\eta}_1^2\Bigg[|\mathbf{k}| - \frac{2u_L(\gamma + |\mathbf{k}|u_R)^3}{g(u_R - u_L)(\gamma + 3|\mathbf{k}|u_R)} - \frac{4|\mathbf{k}|^3 u_R u_L(u_R - u_L)}{g(\gamma + 3|\mathbf{k}|u_R)}\Bigg]. \quad (3)$$

If $|\mathbf{k}| \ll k_{\rm opt}$, then $\hat{\eta}_2(2\mathbf{k}) \simeq A|\mathbf{k}|\hat{\eta}_1^2$. This result is consistent with those obtained in the framework of incompressible, inviscid, irrotational, and immiscible fluids [15,16]. We can thus see that, as long as $|\mathbf{k}| \ll k_{\rm opt}$, the ablation process plays no role. If $|\mathbf{k}| = k_{\rm opt}$, then

$$\hat{\eta}_2(2k_{\rm opt}) = 2\hat{\eta}_1^2 k_{\rm opt}\frac{2\sqrt{1-A^2} - 1 + A}{\sqrt{1-A^2} + 3 + 3A}. \quad (4)$$

If $|\mathbf{k}|$ is of the order of $k_{\rm opt}$, then Eq. (1) shows that ablation reduces the linear growth rate, while Eq. (4) demonstrates that it also strongly modifies the second harmonic generation (SHG) efficiency. This feature is in dramatic contrast with the case of a classical RT system with surface tension. In that case the SHG efficiency is not affected by surface tension, and it is equal to the classical efficiency $A|\mathbf{k}|\hat{\eta}_1^2$ [16].

More generally, $\hat{\eta}_2(2\mathbf{k})/(\hat{\eta}_1^2|\mathbf{k}|)$ is a function of A and $|\mathbf{k}|/k_{\rm opt}$ only. If $A \simeq 1$, then Eq. (3) can be simplified into

$$\hat{\eta}_2(2\mathbf{k}) \simeq \hat{\eta}_1^2|\mathbf{k}|(1 - |\mathbf{k}|/k_{\rm opt}) \quad (5)$$

for $|\mathbf{k}| < k_c$. Note that $k_c \simeq 2k_{\rm opt}$ for $A \simeq 1$. The sign of the SHG efficiency changes when going from $|\mathbf{k}| < k_{\rm opt}$ to $|\mathbf{k}| > k_{\rm opt}$. This involves an inversion of the bubble-spike asymmetry. This inversion has recently been described in Ref. [6] in a self-consistent approach where ρ_R is substituted for a $|\mathbf{k}|$-dependent $\rho_{\mathbf{k}}$ that is assumed to be $\ll \rho_L$. Equation (3) also shows that, if $A < 1$, then the inversion occurs for some $|\mathbf{k}| > k_{\rm opt}$ [Fig. 1(b)].

The analysis of the second-order WNL regime is completed by adding that there is no zeroth harmonic generation for the modulation interface, but we have exhibited zeroth harmonic components for the pressure and the temperature. The third-order WNL analysis follows the same lines as the second-order analysis. We get linear systems for the third-order perturbations U_3, η_3, and T_3 in which source terms depending on the first-order and second-order perturbations are coming. On the one hand, it appears that the hydrodynamics is rather simple in the overdense region (no vorticity mode), while it is complicated in the blowoff region, as is consists of seven different modes. On the other hand, the thermal transport is simple in the blowoff region but complicated in the overdense region with the existence of a thin boundary layer. The evolution equations give expressions for U_3 and T_3 containing six free parameters, and we are able to write seven jump relations depending also on η_3. This consequently establishes the expression of η_3.

If $|\mathbf{k}| \ll k_{\rm opt}$, then $\hat{\eta}_3(\mathbf{k}) \simeq -[(1 + 3A^2)/4]|\mathbf{k}|^2\hat{\eta}_1^3$, which is the same expression as the one derived in Ref. [16] for the dynamics of the interface of two incompressible, inviscid, irrotational, and immiscible liquids. This observation, together with the corresponding one for SHG, demonstrates that our model could be useful to extend the saturation formulas of Haan [17] which are based on these simplified models.

If $|\mathbf{k}| = k_{\rm opt}$, then $\hat{\eta}_3(k_{\rm opt}) = \hat{\eta}_1^3 k_{\rm opt}^2 f(A)$. The function f is plotted in Fig. 2(a). For $A \simeq 1$ it can be expanded as $f(A) \simeq -(2/3)\sqrt{1-A}$. Note that f is negative valued, which shows that the nonlinear correction helps reducing the exponential growth of the fundamental modulation.

More generally, the expression of $\hat{\eta}_3(\mathbf{k})/(\hat{\eta}_1^3|\mathbf{k}|^2)$ is a function of A and $|\mathbf{k}|/k_{\rm opt}$. If $A \simeq 1$, then the expression of $\hat{\eta}_3(\mathbf{k})$ can be simplified into

$$\hat{\eta}_3(\mathbf{k}) \simeq -\hat{\eta}_1^3|\mathbf{k}|^2\frac{(4 - |\mathbf{k}|/k_{\rm opt})(1 - |\mathbf{k}|/k_{\rm opt})}{(4 - 2|\mathbf{k}|/k_{\rm opt})} \quad (6)$$

for $|\mathbf{k}| < k_c \simeq 2k_{\rm opt}$, which shows that the nonlinear correction reduces instability for $|\mathbf{k}| < k_{\rm opt}$, but enhances instability for $|\mathbf{k}| > k_{\rm opt}$ as observed in Ref. [6]. If $A < 1$, then the stabilizing effect is stronger as it concerns

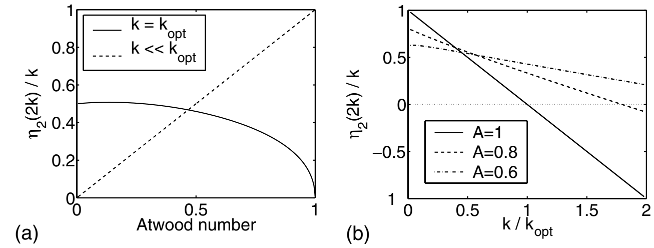

FIG. 1. SHG efficiency $\hat{\eta}_2(2\mathbf{k})$ divided by $\hat{\eta}_1^2|\mathbf{k}|$ as a function of A (a) and $|\mathbf{k}|/k_{\rm opt}$ (b).

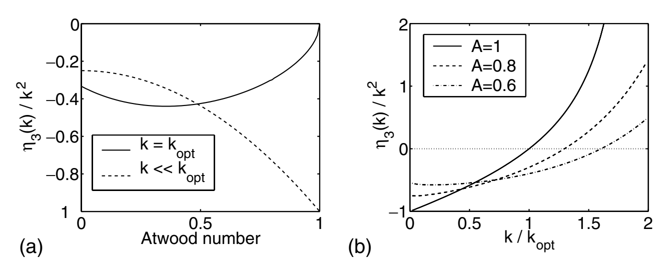

FIG. 2. Nonlinear correction $\hat{\eta}_3(\mathbf{k})$ divided by $\hat{\eta}_1^3|\mathbf{k}|^2$ as a function of A (a) and $|\mathbf{k}|/k_{\rm opt}$ (b).

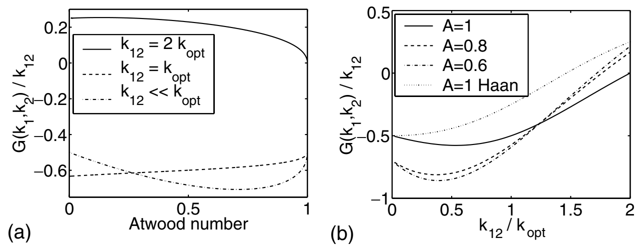

FIG. 3. SFG efficiency $\mathbf{k}_1 + \mathbf{k}_2 \mapsto \mathbf{k}_{12}$ for two wave vectors with wave numbers close to $k_{\rm opt}$. The dotted line labeled "$A = 1$ Haan" (b) is based on Haan's mode coupling formula.

a broader band of wave numbers including $k_{\rm opt}$ [Fig. 2(b)]. We can thus state that, if $|\mathbf{k}| \ll k_{\rm opt}$, then the ablation process plays no role in the linear and WNL regimes. If $|\mathbf{k}|$ is of the order of $k_{\rm opt}$, then ablative effects have to be taken into account in the linear and WNL regimes.

Let us consider an initial multimode disturbance:

$$\eta(t = 0, \mathbf{y}) = \frac{1}{2\pi} \int \hat{\eta}_0(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{y}) d^2\mathbf{k}. \tag{7}$$

A second-order WNL analysis yields that the Fourier modes are

$$\begin{aligned} \hat{\eta}(t, \mathbf{k}) = {} & \hat{\eta}_1(\mathbf{k}) \exp[\gamma(\mathbf{k})t] \\ & + \frac{1}{2\pi} \int d^2\mathbf{k}_1 G(\mathbf{k}_1, \mathbf{k} - \mathbf{k}_1) \hat{\eta}_1(\mathbf{k}_1) \hat{\eta}_1(\mathbf{k} - \mathbf{k}_1) \\ & \times \exp[\gamma(\mathbf{k}_1)t + \gamma(\mathbf{k} - \mathbf{k}_1)t], \end{aligned} \tag{8}$$

where $\hat{\eta}_1(\mathbf{k}) = \hat{\eta}_0(\mathbf{k})/2$ and $G(\mathbf{k}_1, \mathbf{k}_2)$ is the sum-frequency generation (SFG) efficiency for the conversion $\mathbf{k}_1 + \mathbf{k}_2$. The expression of G allows a precise description of mode coupling in the multimode spectrum. The spectral gain narrowing enhances modes around $k_{\rm opt}$, so that these modes will be the first ones to reach the WNL regime in the case of small initial disturbance. Accordingly we focus our attention to the conversion configurations with wave numbers $|\mathbf{k}_1|$ and $|\mathbf{k}_2|$ close to $k_{\rm opt}$. The SFG efficiency is then of the form $G(\mathbf{k}_1, \mathbf{k}_2) = |\mathbf{k}_{12}| F_{12}(A, |\mathbf{k}_{12}|/k_{\rm opt})$, where $\mathbf{k}_{12} = \mathbf{k}_1 + \mathbf{k}_2$. For $A \simeq 1$, F_{12} becomes simple:

$$G(\mathbf{k}_1, \mathbf{k}_2) = \frac{|\mathbf{k}_{12}|}{2} \frac{(|\mathbf{k}_{12}|/k_{\rm opt})^2 - 4}{(|\mathbf{k}_{12}|/k_{\rm opt})^2 - 2|\mathbf{k}_{12}|/k_{\rm opt} + 4}. \tag{9}$$

Previous papers have developed mode-coupling formulas based on classical RT systems with surface tension, which is a system much simpler than the ablative RT system. In a second step these formulas are applied to ablative RT by substitution of the ablative linear growth rate, for instance, the Takabe formula [15]. This approach is interesting in that it gives a first simple estimate of the mode-coupling effects in ablative RT systems. It has been widely applied [18], especially for the design of ICF targets. Haan has proposed in [15] a mode-coupling formula that can be applied with an arbitrary dispersion relation $\gamma(\mathbf{k})$. Applying this formula with (1) for $A = 1$ we get

$$G_H(\mathbf{k}_1, \mathbf{k}_2) = \frac{|\mathbf{k}_{12}|}{4} \frac{(|\mathbf{k}_{12}|/k_{\rm opt})^2 + 4|\mathbf{k}_{12}|/k_{\rm opt} - 8}{(|\mathbf{k}_{12}|/k_{\rm opt})^2 - 2|\mathbf{k}_{12}|/k_{\rm opt} + 4}. \tag{10}$$

Comparing with the exact WNL expression (8) and (9), we can see that Haan's formula gives a right estimate for the low-frequency modulations but overestimates high-frequency conversion [Fig. 3(b)].

To conclude, the analytical formulas that we have derived show the influence of ablation on RT instability in the linear and WNL regimes. The WNL corrections in the ablative case are dramatically different from the ones that are obtained in a classical RT system with surface tension. This shows that the WNL regime strongly depends on the stabilization phenomena. Accordingly mode-coupling formulas derived for classical RT systems with surface tension are difficult to extrapolate to ablative RT configurations. Note finally that the expression of the thermal conductivity in terms of the density and temperature does not appear explicitly in the formulas, but it plays an important role in the determination of the self-consistent Atwood number [9]. We shall discuss this issue relevant for ICF in a forthcoming paper.

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