Modulational instability of electromagnetic waves in randomly perturbed fibers

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As is well known, the interplay between optical Kerr effect and chromatic dispersion leads to the phenomenon of modulational instability (MI) of light waves [1]. Such instability, also called Benjamin-Feir instability, occurs in different physical environments : plasmas, fluids, solid-state lattices, electrical circuits and nonlinear optics. MI leads to the breakup of a cw or quasi-cw beam into a train of ultrashort pulses and it can be used to generate a train of soliton-like pulses [2]. MI also sets a fundamental nonlinear limiting factor in the transmission of dense wavelength-division multiplexed signals in long-distance fiber links.

All these results were obtained by deterministic models. In realistic fiber transmission links, the chromatic dispersion, nonlinearity and birefringence are not constant but can fluctuate stochastically around a constant value. The evolution of the polarized fields in randomly birefringent fibers is ruled by a modified vector nonlinear Schrödinger system [3]:

$$iu_z + i\Delta(z)u_t + \beta(z)u_{tt} + \gamma(z)\left(|u|^2 + \alpha\,|v|^2\right)u = 0, \tag{1}$$

$$iv_z - i\Delta(z)v_t + \beta(z)v_{tt} + \gamma(z)\left(|v|^2 + \alpha\,|u|^2\right)v = 0. \tag{2}$$

The group velocity dispersion β (resp. the group velocity mismatch Δ and the nonlinear coefficient γ) is assumed to be sum of a constant term β_0 (resp. Δ_0, γ_0), and zero-mean random fluctuations, which may be represented by a white Gaussian-distributed noise. The average group velocity dispersion coefficient β_0 is equal to 1 or -1, for the anomalous and normal dispersion regime, respectively. Note also that $\alpha = 1$ and $\Delta_0 = 0$ corresponds to the Manakov model [4].

The standard linear stability analysis of MI consists in perturbing the stationary solution of the nonlinear Schrödinger equations [1]. In our notation, we write a perturbed plane wave as:

$$u(z,t) = (A + u_1(z,t))e^{i\gamma_0\left(A^2+\alpha B^2\right)z}, \quad v(z,t) = (B + v_1(z,t))e^{i\gamma_0\left(B^2+\alpha A^2\right)z}. \tag{3}$$

One obtains a linear system of equations for u_1 and v_1 ; using the complex representation, $u_1 = \bar{c}+i\bar{d}$, $v_1 = \bar{e} + i\bar{f}$ and performing the Fourier transform $c = \int \bar{c}e^{-i\omega t}dt$, we obtain a differential system $\frac{dq}{dz} = Q(z)q$ with $q = (c, d, e, f)$. This ODE system describes the evolution of the amplitude of the perturbation along the fiber. The eigenvalues of the associated matrix Q give the MI gain. Unlike the deterministic case, the perturbation matrix is no longer constant but varies randomly with distance. So a stochastic approach is required to exhibit the statistical properties of the random MI gain and spectrum. Qualitatively we have found that the MI gain spectrum spreads out to high frequencies with increasing fluctuations, both in the case of anomalous and normal dispersion, and in situations that are modulationally stable in the deterministic case. The aim of the talk will be to present precise and quantitative results, which may have interesting implications to the stability of fiber transmissions, in particular when dispersion compensation techniques are employed.

References

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