GENERATION OF A PURE PHASE-MODULATED PULSE BY CASCADING EFFECT : A THEORETICAL APPROACH

L. Videau and C. Rouyer
Commissariat à l'Energie Atomique,
Centre d'Études Scientifiques et Techniques d'Aquitaine,
33114 Le Barp Cedex, France , phone: 33 557 04 4352, fax: 33 557 04 5315
J.Garnier
Centre de Mathématiques Appliquées, Unite Mixte de Recherche 7641, Centre National de la Recherche Scientifique, Ecole Polytechnique, 91128 Palaiseau Cedex, France
A.Migus
Laboratoire pour l'Utilisation des Lasers Intenses, Unite Mixte de Recherche 7605
Centre National de la Recherche Scientifique et Commissariat à l'Energie Atomique
Ecole Polytechnique, 91128 Palaiseau Cedex, France

ABSTRACT

We propose new smoothing techniques involving nonlinear cascaded processes. The scheme is based on the transfer of incoherent amplitude modulations of a pump beam to the phase of a monochromatic plane wave signal. For that, we can use nonlinear cascaded processes which create crossed phase modulation without efficient energy transfer. With this technique, we should be able to produce a temporal or/and spatial incoherent phase modulation without using electro-optic devices. We describe the mechanisms and the different schemes we propose such as a random temporal phase modulator or a temporally varying random phase plate. We present theoretical results by developing analytical calculations of the nonlinear phase. Then, we analyse the incoherent phase modulation with the correlation functions formalism. We present results which take into account temporal limitations such as the Group Velocity Dispersion (GVD) or the Group Velocity Walk-off (GVW) and spatial limitations such as the Diffraction and the Spatial Walk-off. Finally, we present an all-optical sinusoidal phase modulator setup.

Keywords: Smoothing techniques, Statistical laser fields, Frequency conversion, Nonlinear processes, All-Optical devices

1. INTRODUCTION

The development of high power laser chains has become an extensively studied object, especially for inertial confinement fusion. For the French Laser MegaJoule (LMJ) project[1] and the U.S. National Ignition Facility (NIF) project,[2] a major issue is the uniformity of the illumination of the target[3] . The direct drive scheme requires contrast level less than 5 percent per beam which can be obtained only with two dimensional optical smoothing techniques[4] such as the Smoothing by Optical Fiber (SOF)[5] or the Two Dimensional Smoothing by Spectral Dispersion (2D-SSD).[6] All smoothing techniques need large broadband spectrum in order to create different incoherent speckle pattern on the target. Generally, one creates the broadband spectrum in the front end of the laser while the large focal spot is generated by a Phase Plate near the focusing system. Here, we propose and analyze new smoothing techniques involving the cascading effect. The major issue of this paper is to propose the transfer of incoherent amplitude modulations of a pump beam to the phase of a monochromatic plane wave signal. Then, spatio-temporal phase modulations induce independent speckle patterns in the focal plane of the focusing system. The transfer mechanism is based on well-known nonlinear cascaded processes which create crossed phase modulation without efficient energy transfer[7 8 9] . With this technique, we should be able to produce a temporal or/and spatial incoherent phase modulation. We explain this idea in the Section 2.

E-mail : videau@bordeaux.cea.fr

Growth, Fabrication, Devices, and Applications of Laser and Nonlinear Materials, Jeffrey W. Pierce, Kathleen I. Schaffers, Editors, Proceedings of SPIE Vol. 4268 (2001) © 2001 SPIE · 0277-786X/01/$15.00

In Section 3 we develop theoretical results by developing analytical calculations of the nonlinear phase, and we present some comparisons with numerical results. In Section 4, we present the correlation function formalism we use for the description of incoherent modulations. Then, we take into account in Section 5 some limitations of the mechanism such as the Group Velocity Dispersion (GVD), the Group Velocity Walk-off (GVW), the diffraction or the Angular Walk-off (AW). In particular, we show that the equation system is reduced to a linear equation for the signal wave. Finally, we present some particular results for a sinusoidal phase modulator setup in Section 6.

2. PRINCIPLE OF PHASE MODULATION BY SECOND ORDER NONLINEAR EFFECTS

We propose in the paper to create a random phase modulation using an incoherent amplitude modulated wave. The amplitude modulated wave will be called the pump wave and the phase modulated one the signal wave. The outline of the paper is to transfer amplitude modulations of the pump wave to the signal wave phase by using nonlinear cascading effects. By using a nonlinear crystal, both waves interact and can produce an harmonic wave. The mixing of harmonic wave and fundamental waves induces a nonlinear phase modulation on the signal wave especially for non phase-matching conditions[78] .

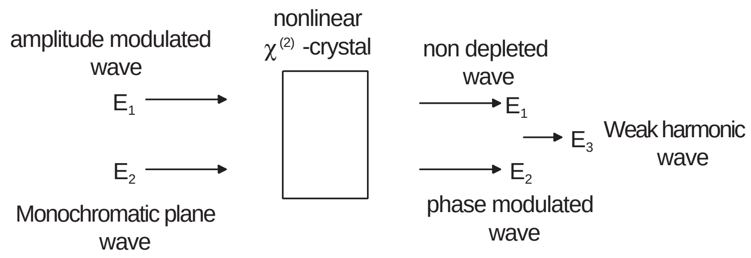

Figure 1. Schematic setup for the transfer of amplitude modulation to phase modulation.

As the pump depletion is negligible, we have an approximated result for the nonlinear phase due to the interaction for the signal wave[10] :

$$\phi_{NL}(x,y,t) = -\frac{2I_p(x,y,t)L}{\Delta k P_c} \tag{1}$$

where L is the crystal length, I_p the pump intensity, Δk the phase mismatch and

$$P_c = \frac{\epsilon_0 c n_s n_p n_h \lambda_s \lambda_p}{8\pi^2 d_{eff}^2} \tag{2}$$

where $n_{s,p,h}$ are respectively the index for the signal wave, the pump wave and the harmonic wave, λ_s is the signal wavelength and λ_p the pump wavelength, d_{eff} is the nonlinear coefficient of the crystal. The formula (Eq.1) is similar to a third order nonlinear phase characterized by a nonlinear coefficient $n_2 = 2/(k_0 \Delta k P_c)$. In this configuration, this coefficient could be adapted by changing the phase-mismatch Δk which is not possible for cross phase modulation using the Kerr effect. Then, at the output of the crystal the signal wave has a modulated phase proportional to the pump amplitude modulation while its amplitude is not changed. Thanks to this process, we can generate an Incoherent Phase Modulator in temporal domain, an Incoherent Phase Plate in spatial domain or a Time-varying Phase Plate in spatio-temporal domain. In fact, this method is the same as the cross-modulation, but it is an extension in spatial domain with a nonlinear technique which permits many configurations and it is certainly more efficient.

3. THEORETICAL RESULTS IN THE CASCADING CONFIGURATION

In Section 2, we presented a phase expression (Eq.1) which is easily obtained by neglecting the pump depletion and the cubic nonlinearity. Here, we propose more precise expressions for the intensity and the phase of the three waves at the output of the nonlinear crystal.

3.1. Expansion of analytical results

The system of frequency conversion consists of three coupled equations for the signal wave E_s, the pump wave E_p and the harmonic wave E_h. We have normalized the equations so that the intensity I_j is simply the square modulus of E_j. The electric field $\mathcal{E}_j$ is then given by

$$\mathcal{E}_j = \sqrt{\frac{1}{2n(\omega_j)c\epsilon_0}} E_j \exp i(k_j z - \omega_j t) + c.c. \tag{3}$$

First, we do not take into account spatial and temporal limitations and suppose that the waves verify the equations for monochromatic waves. Then, the normalized fields verify the following system of equations[11] :

$$\begin{aligned} \frac{\partial E_s}{\partial z} &= i\sqrt{\frac{\omega_s}{\omega_p}}\frac{1}{\sqrt{P_c}}E_p^* E_h \exp(i\Delta kz) \\ \frac{\partial E_p}{\partial z} &= i\sqrt{\frac{\omega_p}{\omega_s}}\frac{1}{\sqrt{P_c}}E_s^* E_h \exp(i\Delta kz) \\ \frac{\partial E_h}{\partial z} &= i\frac{\omega_h}{\sqrt{\omega_p\omega_s}}\frac{1}{\sqrt{P_c}}E_p E_s \exp(-i\Delta kz) \end{aligned} \tag{4}$$

ω_j are the frequencies of the waves, Δk is the phase mismatch and P_c is given by Eq.2 .
We can solve this system and obtain analytical results for the phase and amplitude of the three waves. We focus here on the case without initial harmonic wave and we expand expressions with respect to the parameter γ^{-1} where $\gamma = P_c\Delta k^2/I_t$, I_t being the sum of the three intensities.

$$I_{s,p}(z) = I_{s0,p0} - \frac{8I_{s0}I_{p0}}{I_t}\gamma^{-1}\left(1 - 8\gamma^{-1}\right)\sin^2\left(\frac{\Delta kz}{2}\Big(1 + 4\gamma^{-1} - 8\gamma^{-2}(1+\beta)\Big)\right) + O(\gamma^{-3}) \tag{5}$$

$$I_h(z) = \frac{16I_{s0}I_{p0}}{I_t}\gamma^{-1}\left(1 - 8\gamma^{-1}\right)\sin^2\left(\frac{\Delta kz}{2}\Big(1 + 4\gamma^{-1} - 8\gamma^{-2}(1+\beta)\Big)\right) + O(\gamma^{-3}) \tag{6}$$

$$\phi_{s,p}(z) = \frac{\Delta kz}{2}(h_{s,p}^{(1)}\gamma^{-1} + h_{s,p}^{(2)}\gamma^{-2} + h_{s,p}^{(3)}\gamma^{-3} + h_{s,p}^{(4)}\gamma^{-4}) + O(\gamma^{-5}) \tag{7}$$

where the $h_{s,p}^{(j)}$ functions are defined by the following relations

$$\begin{aligned} h_{s,p}^{(1)} &= -4N_{p0,s0}\frac{t_2(\phi)}{\phi} \\ h_{s,p}^{(2)} &= -8\Big((\beta - 4N_{p0,s0})\frac{t_2(\phi)}{\phi} + 2N_{p0,s0}^2\frac{t_4(\phi)}{\phi}\Big) \\ h_{s,p}^{(3)} &= -32\Big(2\big(N_{p0,s0}(4+\beta) - 2\beta\big)\frac{t_2(\phi)}{\phi} + N_{p0,s0}(\beta - 8N_{p0,s0})\frac{t_4(\phi)}{\phi} + 2N_{p0,s0}^3\frac{t_6(\phi)}{\phi}\Big) \end{aligned} \tag{8}$$

and $\phi(z)$ is given by

$$\phi(z) = z\sqrt{\frac{N_c I_t}{P_c}} = \frac{\Delta kz}{2}\Big(1 + \frac{4}{\gamma} - \frac{8}{\gamma^2}(1+\beta) + \frac{32}{\gamma^3}(1+3\beta) - \frac{160}{\gamma^4}(1+6\beta+\beta^2)\Big) \tag{9}$$

The parameter β is defined by

$$\beta = \frac{\omega_h}{\omega_p}\frac{\omega_h}{\omega_s}\frac{I_{p0}I_{s0}}{(I_{p0}+I_{s0})^2} \tag{10}$$

and

$$N_{p0,s0} = \frac{\omega_h}{\omega_{p,s}}\frac{I_{p0,s0}}{I_{p0}+I_{s0}} \tag{11}$$

and $t_{2l}(\phi) = \frac{2l-1}{2l} t_{2(l-1)}(\phi) - \frac{1}{2l} \sin^{(2l-1)}(\phi)\cos(\phi)$ and $t_0(\phi) = \phi$, $\binom{a}{n} = \frac{a(a-1)...(a-n+1)}{1.2.3...n}$ and $\binom{a}{0} = 1$.

It is interesting to consider a useful expansion until the first order of the phase and intensity, assuming that the input phases are null. We put $E_j = \sqrt{I_j}\exp i\phi_j$ and we find

$$I_{s,p}(z) = I_{s0,p0} - \frac{8I_{s0}I_{p0}}{P_c\Delta k^2}\sin^2(\phi(z)) \tag{12}$$

$$\phi_{s,p}(z) = -\frac{2I_{p0,s0}}{P_c\Delta k^2}\left(\Delta kz - \sin(2\phi(z))\right) \tag{13}$$

In this case, we can expand the expression 9 to the first order : $\phi(z) = \frac{\Delta kz}{2}(1 + 4\gamma^{-1})$. For large value of γ and Δkz, the expression 13 is similar to the expression Eq.1.

3.2. Comparison between analytical and numerical results

With these formulae, we can retrieve numerical results when the parameter γ^{-1} is large enough. For instance, we compare numerical results with expansions in Fig.2. In this configuration, $\gamma = 50$ and a maximum nonlinear phase of 2 rad is generated.

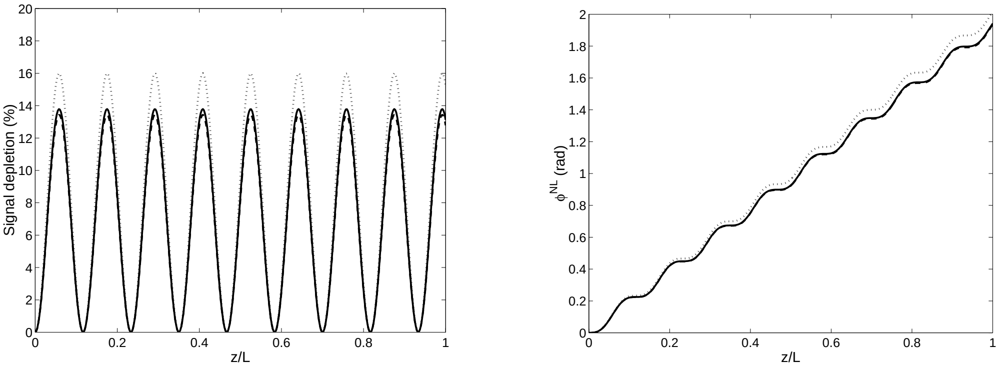

Figure 2. Nonlinear phase and intensity depletion as functions of the ratio z/L. The solid curves are numerical results, the dotted curves are obtained with a first order expansion and the dashed one with a second order expansion. $L = 1cm$, $I_{p0} = 50GW/cm^2$, $I_{s0} = 0.01GW/cm^2$, $P_c = 1GW$, $\Delta k = 50cm^{-1}$.

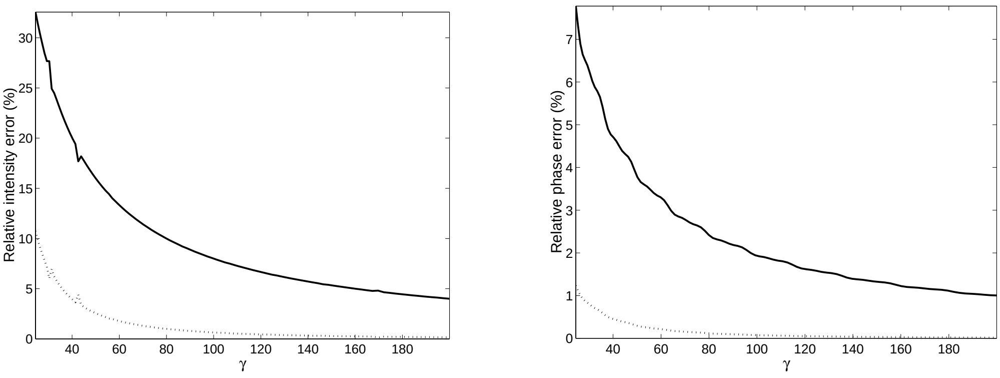

Figure 3. Relative error for the intensity depletion and the nonlinear phase between numerical and expansion results. The solid curves compare the numerical results with the first order expansion and the dotted curves compare the numerical results with the second order expansion. $L = 1cm$, $I_{p0} = 50GW/cm^2$, $I_{s0} = 0.01GW/cm^2$, $P_c = 1GW$, Δk varies from 35 to $100cm^{-1}$.

In Fig.2, the solid curves are the reference curves we obtained with a numerical code solving the equations system (4) while the dotted curves correspond to the first order calculation and the dashed curves to the second order one. The behavior of the phase is well-known with sinusoidal part in addition with a linear slope[7,10]. We show that the second order expansion of the phase and the amplitude predicts the signal wave behavior with a very good agreement, while there is a small difference when using only the first order expansion.

We have studied also the relative error between the expansion and the numerical results as a function of the parameter γ (Fig.3). We show that the second order expansion is a very good approximation even for values of γ not very large. For $\gamma = 40$, we have a relative intensity error less than 5 percent and a relative phase error less than 1 percent. The first order expansion is not so close to the numerical results but it is also a good approximation for the nonlinear phase since the phase relative error is less than 5 percent for $\gamma = 40$.

4. STATISTICAL FORMALISM

It is easier to use a statistical formalism in order to characterize incoherent waves. Since the formalism is similar for both spatial and temporal domains, we will describe the waves using only the temporal variable for having more simple expressions. All the results obtained for the temporal domain will be valid for the spatial one. In this way, we will suppose that the pump wave is a partially coherent pulse characterized by its correlation function $f_p(t) = I_p^{-1}\langle E_p(0)E_p^*(t)\rangle$, where I_p is the averaged intensity $\langle E_p E_p^*\rangle$. By the Wiener-Khintchine theorem,[12] we can deduce the spectral intensity $\widetilde{I_p}(\nu)$ of the pump from the Fourier transform of $f_p(t)$

$$\widetilde{I_p}(\nu) = \int_{-\infty}^{+\infty} f_p(t)\exp(-2i\pi\nu t)dt \tag{14}$$

For example, we may assume a Gaussian-type correlation function for the pump field

$$f_p(t) = \frac{1}{I_p}\langle E_p(0)E_p^*(t)\rangle = \exp(-t^2/2t_c^2) \tag{15}$$

$$\widetilde{I_p}(\nu) = \widetilde{I_p}\exp(-\nu^2/2\nu_c^2) \tag{16}$$

where t_c is called the coherence time and ν_c the spectral bandwidth equal to $(2\pi t_c)^{-1}$. By using Eq.1 for the nonlinear phase and assuming the input signal amplitude equal to 1, we find that the correlation function of the output signal wave is[13]

$$f_s(t) = \frac{1}{1 + B(L)^2[1 - f_p^2(t)]} \tag{17}$$

where B(L) is the positive averaged phase

$$B(L) = \frac{2I_pL}{\Delta k P_c} \tag{18}$$

With a simple Fourier transform, we have also the spectral intensity of the signal wave which has two parts, a dirac function and a continuous function with respective weights $\alpha = (1 + B(L)^2)^{-1}$ and $(1 - \alpha)$.

$$\widetilde{I_s}(\nu) = \int_{-\infty}^{+\infty} f_s(t)\exp(-2i\pi\nu t)dt = \alpha\delta(\nu) + (1-\alpha)\widetilde{I_s^c}(\nu) \tag{19}$$

with

$$\widetilde{I_s^c}(\nu) = \int_{-\infty}^{+\infty} \frac{f_p^2(t)}{1 + B(L)^2(1 - f_p^2(t))}\exp(-2i\pi\nu t)dt \tag{20}$$

We have plotted in Fig.4 the correlation functions and the spectral intensity of the signal wave for $B = 1$ and $B = 2$. We have also plotted the input functions as references. We can see that the generated signal spectrum width is larger than the pump one even for a B equal to 1. Then, we have shown that we can generate a large broadband phase modulated wave by using cross phase-modulation with nonlinear second order effects.

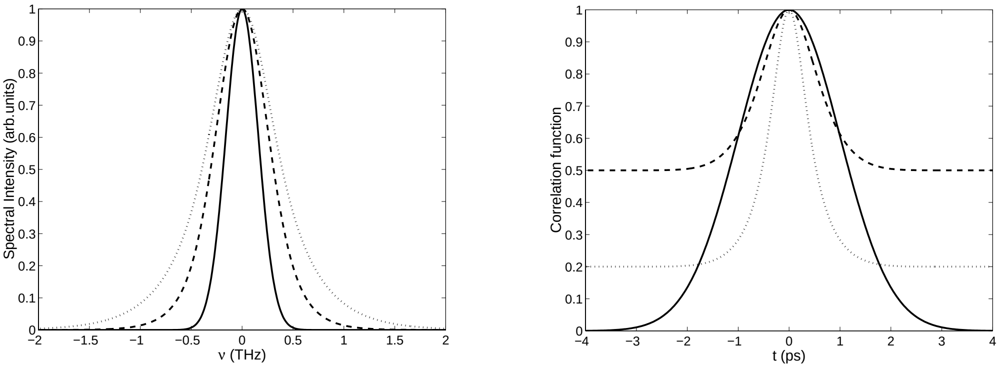

Figure 4. Spectral function (a) and correlation intensity (b) for the signal wave and the pump wave. The solid curves correspond to the pump functions, the dashed ones to the signal wave for $B = 1$ and the dotted ones to the signal wave for $B = 2$. The pump correlation function is a Gaussian function with a coherence time t_c of 1 ps. We have removed the dirac function from the spectra.

5. TEMPORAL AND SPATIAL EFFECTS

In Section 4, we described the correlation function of the signal wave. We assumed that the frequency conversion equations for monochromatic waves were valid in our configurations. In fact, this is not true if the coherence time is in the order of several picoseconds or the beam size too small compared to the natural diffraction. So we now study the effects of temporal and spatial limitations on the incoherent phase imprint. We show the effect on the signal wave correlation function taking into account the simple first order expansion. First, we study the Group Velocity Walk-off (GVW) and the Group Velocity Dispersion (GVD), then we will show that results are similar for spatial effects.

5.1. Group Velocity Walk-off

The system of coupled equations (4) for monochromatic plane waves must be changed if we take into account the GVW in the following system.[14]

$$\begin{aligned}\frac{\partial E_s}{\partial z} &= -\Delta_s^v \frac{\partial E_s}{\partial t} + i\sqrt{\frac{\omega_s}{\omega_p}}\frac{1}{\sqrt{P_c}}E_p^* E_h \exp(i\Delta k z) \\ \frac{\partial E_p}{\partial z} &= i\sqrt{\frac{\omega_p}{\omega_s}}\frac{1}{\sqrt{P_c}}E_s^* E_h \exp(i\Delta k z) \\ \frac{\partial E_h}{\partial z} &= -\Delta_h^v \frac{\partial E_h}{\partial t} + i\frac{\omega_h}{\sqrt{\omega_p\omega_s}}\frac{1}{\sqrt{P_c}}E_p E_s \exp(-i\Delta k z)\end{aligned} \tag{21}$$

where $\Delta_s^v = (v_s^g)^{-1} - (v_p^g)^{-1}$ and $\Delta_h^v = (v_h^g)^{-1} - (v_p^g)^{-1}$ with $(v^g)^{-1} = \frac{\partial k}{\partial \omega}$. The equation system is written in the pump moving time pulse frame. For large value of γ, we find an effective equation for the signal wave while the pump wave is unperturbed.

$$\frac{\partial E_s(z,t)}{\partial z} = -\Delta_s^v \frac{\partial E_s(z,t)}{\partial t} - i\frac{2I_p(z,t)}{\Delta k P_c}E_s(z,t) \tag{22}$$

whose solution is $E_s(z,t) = \exp i\phi(z,t)$ with

$$\phi(z,t) = -\frac{2}{\Delta k P_c}\int_0^z I_p(t - \Delta_s^v z')dz' \tag{23}$$

The first order effect of GVW is only a phase average, the signal amplitude being equal to 1. We can note that if the GVW is null, Eq.23 is the same than Eq.1. Then, we can find an expanded expression for the signal correlation function with respect to $\Delta_s^v z t_c^{-1} \ll 1$:

$$f_s(t) = f_{s0}(t)\Big(1 + \frac{\Delta_s^{v2} z^2}{6t_c^2} B(z)^2 f_{s0}^2\big(g_1(t/t_c) + B(z)^2 g_2(t/tc)\big)\Big) + O\Big(\big(\frac{\Delta_s^v z}{t_c}\big)^3\Big) \quad (24)$$

where f_{s0} is the signal correlation function without GVW (Eq.17) and $\tilde{f}_p$ is the normalized correlation function of the pump field :

$$\begin{aligned} \tilde{f}_p(u) &= f_p(ut_c) \\ g_1(u) &= -\tilde{f}_p''(0) + \tilde{f}_p'^2(u) + \tilde{f}_p \tilde{f}_p''(u) \\ g_2(u) &= \tilde{f}_p''(0)(\tilde{f}_p^2(u) - 1) + \tilde{f}_p'^2(u) + \tilde{f}_p \tilde{f}_p''(u) + 3\tilde{f}_p^2 \tilde{f}_p'^2(u) - \tilde{f}_p^3 \tilde{f}_p''(u) \end{aligned}$$

The expression of the spectral intensity is the same as Eq.19, the parameter α being now:

$$\widetilde{I_s}(\nu) = \int_{-\infty}^{+\infty} f_s(t) \exp(-2i\pi\nu t) dt = \alpha\delta(\nu) + (1-\alpha)\widetilde{I_s^c}(\nu)$$
$$\alpha = \frac{1}{1+B(L)^2}\Big(1 - \frac{\Delta_s^{\nu 2} L^2}{6t_c^2} \frac{B(L)^2}{1+B(L)^2} \tilde{f}_p''(0)\Big) \quad (25)$$

Since $\tilde{f}_p''(0)$ is always negative ($\tilde{f}_p(0)$ is a maximum point), the continuous contribution to the signal spectral intensity is reduced compared to the Dirac function. So, the effect of the GVW is a spectral narrowing and a broadening of the correlation function. For example, we have studied this effect with the case $B = 2$ shown in Fig.4 by adding a nonzero group velocity difference.

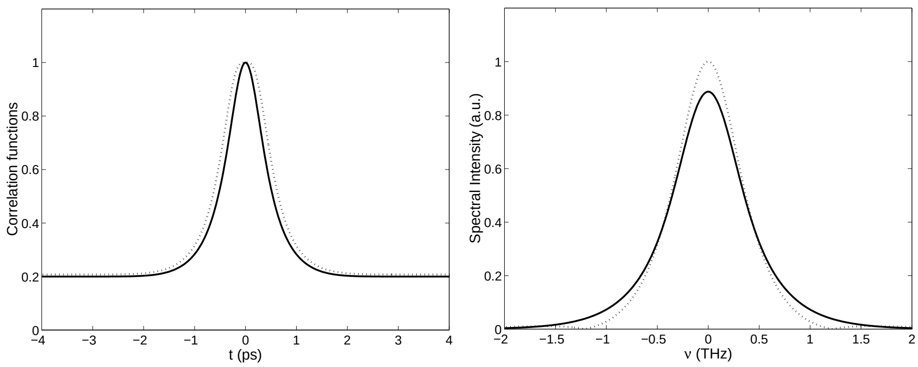

Figure 5. Correlation function (a) and spectral intensity (b) for the signal wave in the same configuration as Fig.4. The solid curves correspond to the case without group velocity difference effect, the dashed one to the case with $B^2\Delta_s^{\nu 2} z^2/6t_c^2 = 0.2$.

We have plotted in Fig.5 the case $B^2\Delta_s^{\nu 2} z^2/6t_c^2 = 0.2$. We can notice the broadening of the correlation function while the spectral intensity is narrower. In fact, the main perturbation in the spectrum is on the wings which decrease quickly. Indeed, the phase average due to group velocity difference in Eq.23 is more important for high frequencies than for low frequencies.

5.2. Group Velocity Dispersion effect

For short coherence time (a few picoseconds), another main limitation is the Group Velocity Dispersion (GVD). The system 4 then writes :

$$\frac{\partial E_s}{\partial z} = -i\sigma_s \frac{\partial^2 E_s}{\partial t^2} + i\sqrt{\frac{\omega_s}{\omega_p}} \frac{1}{\sqrt{P_c}} E_p^* E_h \exp(i\Delta k z)$$

$$\frac{\partial E_p}{\partial z} = -i\sigma_p \frac{\partial^2 E_p}{\partial t^2} + i\sqrt{\frac{\omega_p}{\omega_s}}\frac{1}{\sqrt{P_c}} E_s^* E_h \exp(i\Delta k z) \quad (26)$$
$$\frac{\partial E_h}{\partial z} = -i\sigma_h \frac{\partial^2 E_h}{\partial t^2} + i\frac{\omega_h}{\sqrt{\omega_p \omega_s}}\frac{1}{\sqrt{P_c}} E_p E_s \exp(-i\Delta k z)$$

where $\sigma_i = \frac{1}{2}k''(\omega_i)$. For large value of γ we find a new effective equation for the signal wave propagation :

$$\frac{\partial E_s(z,t)}{\partial z} = -i\sigma_s \frac{\partial^2 E_s(z,t)}{\partial t^2} - i\frac{2I_p(z,t)}{\Delta k P_c} E_s(z,t) \quad (27)$$

We can find an expanded solution for the signal correlation function with respect to the parameter $\mid \sigma_s z/t_c^2 \mid \ll 1$:

$$f_s(t) = f_{s0}(t)\Big(1 - \frac{2\sigma_s L}{3t_c^2} B(L)^3 \tilde{f}_p''(0)\big(1 - \tilde{f}_p(t/t_c)^2\big) f_{s0}(t)\Big) + O\Big(\big(\frac{\sigma_s L}{t_c^2}\big)^2\Big) \quad (28)$$

The weight α of the dirac function in the spectral intensity is then equal to

$$\alpha = \frac{1}{1+B(L)^2}\Big(1 - \frac{2\sigma_s L}{3t_c^2}\frac{B(L)^3}{1+B(L)^2}\tilde{f}_p''(0)\Big) \quad (29)$$

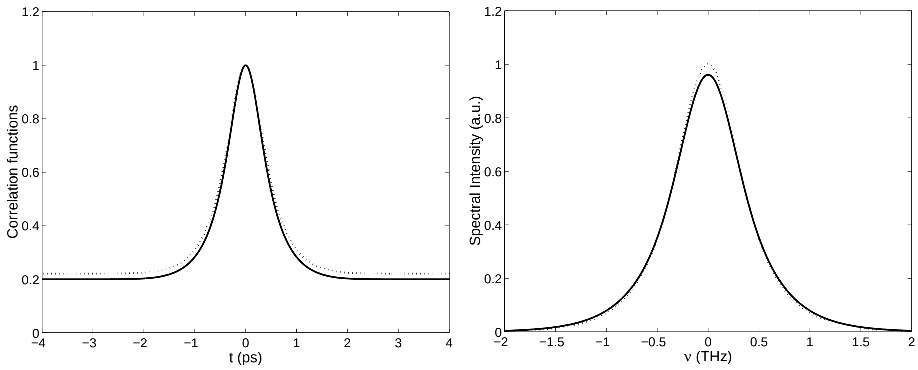

Figure 6. Correlation function (a) and spectral intensity (b) for the signal wave in the same configuration as Fig.4. The solid curves correspond to the case without group velocity dispersion effect, the dashed one to the case with $\mid \sigma_s L/t_c^2 \mid = 0.1$.

If $\sigma_s > 0$ (normally dispersive media), the correlation function is larger and the spectral intensity is narrower. On the other hand, when $\sigma_s < 0$ (anomalous dispersive media), the spectrum is broadened and the correlation function is narrowed. These effects are enhanced by the B parameter because of the factor $B^3/(1+B^2)$ which is growing. Furthermore, the GVD produces amplitude modulations in addition of the phase modulations. Since the correlation function at point $t = 0$ is unchanged, the averaged intensity is the same. These amplitude modulations are charactrized by the contrast $C_s(L)$ of the signal wave defined by the identity :

$$C_s^2(L) = \frac{\langle \mid E_s \mid^4 \rangle - \langle \mid E_s \mid^2 \rangle^2}{\langle \mid E_s \mid^2 \rangle^2} \quad (30)$$

We find the following expansion for this parameter :

$$C_s(L) = B(L)\frac{\mid \sigma_s \mid L}{t_c^2}\sqrt{\frac{3}{2}\tilde{f}_p''(0)^2 + 2\tilde{f}^{(4)}(0)} + O\Big(\big(\frac{\sigma_s L}{t_c^2}\big)^2\Big) \quad (31)$$

We have plotted in Fig.6 the case $| \sigma_s L/t_c^2 | = 0.1$. We can notice that the effect is very small for both the correlation function and the spectral intensity. In fact, the main perturbation concerns the amplitude modulations which appear due to the GVD. The contraste value is then equal to 54 percent. So the main effect of GVD is the creation of amplitude modulation on the signal wave.

5.3. Spatial limitations

In fact, all previous results could be used in this Section for the spatial limitations that could appear like the angular walk-off and the diffraction. The angular walk-off is similar to the Group Velocity Walk-off and the diffraction to the Group Velocity Dispersion. In this Subsection, we just have to define the spatial correlation function $f_p(x,y)$:

$$f_p(x) = \frac{1}{I_p}\langle E_p(0,0)E_p^*(x,y)\rangle = \exp(-(x^2+y^2)/2r_c^2) \tag{32}$$

where r_c is so-called the correlation radius.

5.3.1. Angular walk-off effect

Taking into account the angular walk-off, system (4) writes

$$\begin{aligned} \frac{\partial E_s}{\partial z} &= -\Delta_s^x \frac{\partial E_s}{\partial x} + i\sqrt{\frac{\omega_s}{\omega_p}}\frac{1}{\sqrt{P_c}}E_p^* E_h \exp(i\Delta kz) \\ \frac{\partial E_p}{\partial z} &= i\sqrt{\frac{\omega_p}{\omega_s}}\frac{1}{\sqrt{P_c}}E_s^* E_h \exp(i\Delta kz) \\ \frac{\partial E_h}{\partial z} &= -\Delta_h^x \frac{\partial E_h}{\partial x} + i\frac{\omega_h}{\sqrt{\omega_p\omega_s}}\frac{1}{\sqrt{P_c}}E_p E_s \exp(-i\Delta kz) \end{aligned} \tag{33}$$

where $\Delta_s^x = \tan(\alpha_p) - \tan(\alpha_s)$,$\Delta_h^x = \tan(\alpha_p) - \tan(\alpha_h)$ and α_i is the angular departure for the corresponding wave between the Poynting vector direction and the wave vector direction. System (33) is the same as system (21) if we replace t for x and Δ_i^v for Δ_i^x. Then the results derived in Subsection 5.1 can be applied in the spatial domain to the spatial correlation function and the spatial spectral intensity.

5.3.2. Diffraction effect

When the beam divergence is so large that the corresponding Rayleigh distance is smaller than the crystal thickness, we have to take into account the diffraction inside the nonlinear crystal. Then, system (4) writes :

$$\begin{aligned} \frac{\partial E_s}{\partial z} &= -i\sigma_s^{dif}\Big(\frac{\partial^2 E_s}{\partial x^2} + \frac{\partial^2 E_s}{\partial y^2}\Big) + i\sqrt{\frac{\omega_s}{\omega_p}}\frac{1}{\sqrt{P_c}}E_p^* E_h \exp(i\Delta kz) \\ \frac{\partial E_p}{\partial z} &= -i\sigma_p^{dif}\Big(\frac{\partial^2 E_p}{\partial x^2} + \frac{\partial^2 E_p}{\partial y^2}\Big) + i\sqrt{\frac{\omega_p}{\omega_s}}\frac{1}{\sqrt{P_c}}E_s^* E_h \exp(i\Delta kz) \\ \frac{\partial E_h}{\partial z} &= -i\sigma_h^{dif}\Big(\frac{\partial^2 E_h}{\partial x^2} + \frac{\partial^2 E_h}{\partial y^2}\Big) + i\frac{\omega_h}{\sqrt{\omega_p\omega_s}}\frac{1}{\sqrt{P_c}}E_p E_s \exp(-i\Delta kz) \end{aligned} \tag{34}$$

where $\sigma_i^{dif} = 1/(2k_i\cos^2\alpha_i)$. System (34) is then similar to system (26) and the results obtained in Subsection 5.2 can be applied to the spatial domain. We only have to use the spatial correlation function and to substitute σ_i for the corresponding parameter σ_i^{dif}, (∂t^2) for $(\partial x^2 + \partial y^2)$, (∂t^4) for $(\partial x^4 + 2\partial x^2 y^2 + \partial y^4)$ and t_c for r_c.

6. SINUSOIDAL PHASE MODULATOR SETUP

In Sections 2 and 5, we assumed that the pump wave was partially coherent. Here, we develop some results for a pump wave with a sinusoidal intensity behavior. The pump field is given by :

$$E_p(t) = \sqrt{2I_p}\cos(2\pi\nu_m t) \tag{35}$$

so that the time-averaged intensity is I_p, ν_m being the frequency modulation. Assuming that we can use the relation 1, the output signal wave is then a sinusoidal phase modulation, the modulation depth being equal to the factor $B_0(L)$ given by the relation 18. The frequency modulation of the phase is $2\nu_m$:

$$E_s(t) = \exp i\big(B_0(L)\cos(4\pi\nu_m t)\big) \tag{36}$$

We can estimate the effect of the GVW using the relation 23. In fact, the integration is now analytical because the pump intensity has a simple sinusoidal behavior. We find that the signal phase is always sinusoidal with the same frequency but with a different modulation depth $B(L)$

$$B(L) = B_0(L)\mathrm{sinc}(2\pi\nu_m\Delta_s^v L) \tag{37}$$

where $\mathrm{sinc}(r) = \sin(r)/r$.

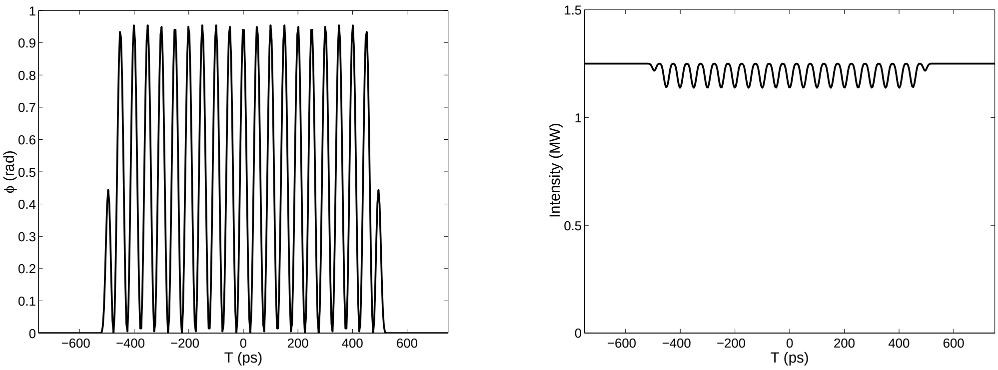

Figure 7. Nonlinear phase (a) and normalized signal intensity (b) as a function of time (in ps). The pump intensity is sinusoidal with $\nu_m = 10GHz$ and $I_p = 2.5GW/cm^2$. The crystal is supposed to be a 2cm KDP crystal in a type II configuration. $L = 2cm$, $P_c = 1GW$, $\Delta k = 19.3cm^{-1}$, $\sigma_s = -0.03ps^2/m$.

This relation is an analytical result without expansion. If the GVW is too high, the depth modulation is then strongly reduced. So, we have to limit the crystal depth to the value $0.1(\nu_m\Delta_s^v)^{-1}$ in order to obtain more than 90 percent of the maximum value of $B_0(L)$. Finally, we find the expression of the amplitude modulations due to the GVD of the signal wave :

$$C_s(L) = 4\pi^2 B(L) \mid \sigma_s \mid L\nu_m^2 + O\Big(\big(4\pi^2\sigma_s L\nu_m^2\big)^2\Big) \tag{38}$$

Here $C_s(L)$ is defined as Eq.30 where $\langle\rangle$ stands for an averaging over a time period. We have plotted Fig.7 an example of a sinusoidal phase modulator setup using a 2 cm Type II KDP crystal. The pump intensity is sinusoidal with a frequency modulation of 10 GHz and an averaged intensity of $2.5GW/cm^2$. We suppose here that the phase mismatch in the crystal is $\Delta k = 19.3cm^{-1}$ and $P_c = 1GW$. Using the relations 12 and 13 we find that the maximum nonlinear phase ϕ_{NL} is 1.03 rad and the total amplitude modulation $\alpha = 2(I_{max} - I_{min})/(I_{max} + I_{min})$ is 0.11 . The numerical results in Fig.7 give $\phi_{NL} = 0.95rad$ and $\alpha = 0.094$ which are very close to the analytical results. We omit in this configuration the spatial dimension. We have taken into account the effects of the GVW and the GVD, but they appear here negligible. No optical damage is expected in nanosecond regime for the KDP crystal. We have also estimated the cubic nonlinear phase shift which is less than 0.1 rad for the maximal intensity.

7. CONCLUSION

We study in this paper the creation of spatio-temporal phase modulated wave using second order cascading nonlinear effects. First, we develop some analytical calculations in order to find expansion formulae for the signal phase and amplitude with respect to a parameter γ. We also estimate the relative error for the first and second order expansions. In fact, we show that for large value of γ we can use the simple first order expansion. We apply those results to the generation of incoherent phase modulation on a signal wave. We describe the mechanism with a correlation function formalism. Then, we take into account some spatial or temporal limitations and show that we can use for each effect a new effective equation for the signal wave propagation. Using the effective equation we have found some results concerning the correlation function. Finally, we propose a realistic experimental setup using a KDP crystal. We show that it would be possible to create a sinusoidal phase modulation with a depth of 1 rad with a frequency modulation of 10 GHz.

REFERENCES

1. M.Andre, "Megajoule solid state laser for ICF applications",Technical Committee Meeting on Drivers and Ignition Facilities for Inertial Fusion, *eds Proceedings of the International Atomic Energy Agency*, pp.77-78, CEA/DAM Publications, Limeil-Valenton, France, 1995.
2. J.E. Rothenberg, "A comparison of beam smoothing for direct drive inertial confinement fusion" *J.Opt.Soc.Am.B* **14**, pp.1664-1671, 1997.
3. R.H. Lehmberg and S.P. Obenschain, "Use of induced spatial incoherency for uniform illumination" *Optics Comm.* **46**, pp.27-31, 1983.
4. J. Garnier, L. Videau, C. Gouedard, and A. Migus, "Statistical analysis for beam smoothing and some applications" *J.Opt.Soc.Am.A* **14**, pp.1928-1937, 1997.
5. D. Veron, G. Thiell, and C. Gouedard, "Optical smoothing of the high power laser Phebus Nd-glass laser using the multimode optical fiber technique" *Optics Comm.* **97**, pp. 259-271, 1993.
6. J.E. Rothenberg, "Two-dimensional smoothing by spectral dispersion for direct drive inertial confinement fusion", *Solid State Lasers for Applications to ICF*, M. Andre and H. Powell, Proc. SPIE **2633**, pp.634-644 , 1995.
7. G. Stegeman, M. Shiek-Bahae, E. Van Stryland and G. Assanto, "Large nonlinear phase shifts in second-order nonlinear-optical processes" *Opt.Lett.* **18**, pp.13-15, 1993.
8. D.C. Hutchings, J.S. Aitchison and C.N. Ironside, "All-optical switching based on nondegenerate phase shifts from a cascaded second-order nonlinearity" *Opt.Lett.* **18**, pp793-795 ,1993.
9. I.Buchvarov, S.Saltiel, Ch.Iglev and K.Koynov, "Intensity dependent change of the polarization state as a result of non-linear phase shift in type II frequency doubling crystals" *Opt.Comm.* **141**, pp.173-179, 1997.
10. R. DeSalvo, D.J. Hagan, G. Stegeman, and E.W. Van Stryland, "Self-focusing and self-defocusing by cascaded second-order effects in KTP" *Opt.Lett.* **18**, pp.574-576, 1993.
11. A. Yariv, *Quantum Electronics*, Wiley, New York, 1988).
12. D. Middleton, *Introduction to Statistical Communication Theory*, McGraw-Hill, New-York, 1960.
13. J.T. Manassah, "Self-phase modulation of incoherent light revisited" *Opt.Lett.* **16**, pp.1439-1441, 1991.
14. J.T. Manassah, "Phase modulation in second-order nonlinear-optical processes" *Phys.Rev.* **A42**, pp.4085-4101, 1990.