Giant collective incoherent shock waves in strongly nonlinear turbulent flows

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Abstract: Contrary to conventional coherent shocks, we show theoretically and experimentally that nonlocal turbulent flows lead to the emergence of large-scale incoherent shock waves, which constitute a collective phenomenon of the incoherent field as a whole.
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1. Introduction

Shock waves have been thoroughly investigated during the last century in many different branches of physics. In conservative (Hamiltonian) systems the shock singularity is regularized by weak wave dispersion, thus leading to the formation of a rapidly and regular oscillating structure, usually termed in the literature dispersive shock wave (DSW), or undular bore in hydrodynamics [1-3]. Here, we show that this fundamental singular process of DSW formation can break down in a system of incoherent nonlinear waves. We consider the strong turbulent regime of a system of nonlocal nonlinear optical waves. We report theoretically and experimentally a characteristic transition: Strengthening the nonlocal character of the nonlinear response drives the system from a fully turbulent regime, featuring a sea of *coherent* small-scale DSWs (shocklets) towards the unexpected emergence of a giant collective *incoherent* shock wave [4]. Then contrary to conventional shocks that are inherently coherent deterministic entities, here the giant collective shock is a collective phenomenon of the incoherent field as a whole. Nonlinear optics experiments performed in a solution of graphene nano-flakes clearly highlight this remarkable transition [4].

2. Theoretical Model and Numerical simulations

The starting point is the two-dimensional nonlocal nonlinear Schrödinger equation (NLSE):

$$i\partial_z \psi = -\frac{\beta}{2}\nabla^2\psi + \gamma\psi \int U(|\vec{r}-\vec{r}\,'|)|\psi|^2(z,\vec{r}\,')d\vec{r}\,' \qquad (1)$$

where the dynamics occurs in the 2D transverse plane (x, y). The parameters $\beta = 1/k_L$ and γ refer to the linear (dispersive) and nonlinear coefficient, k_L being the laser wave number. We denote by σ the spatial extension of the nonlocal response function $U(r)$, i.e. the range of nonlocal interaction. Equation (1) conserves two important quantities during the propagation, the 'power' and the 'energy' (Hamiltonian) $\mathcal{H} = \mathcal{E}+\mathcal{U}$, where $\mathcal{E}(z)$ and $\mathcal{U}(z)$ denote the evolutions of the linear and nonlinear energy, respectively. The dynamics is ruled by the comparison of σ with the "healing length" $\Lambda = [\beta/(2\gamma\rho)]^{1/2}$, where ρ is the intensity. The healing length denotes the typical length scale for which linear and nonlinear effects are of the same order, e.g. the typical modulation instability period.

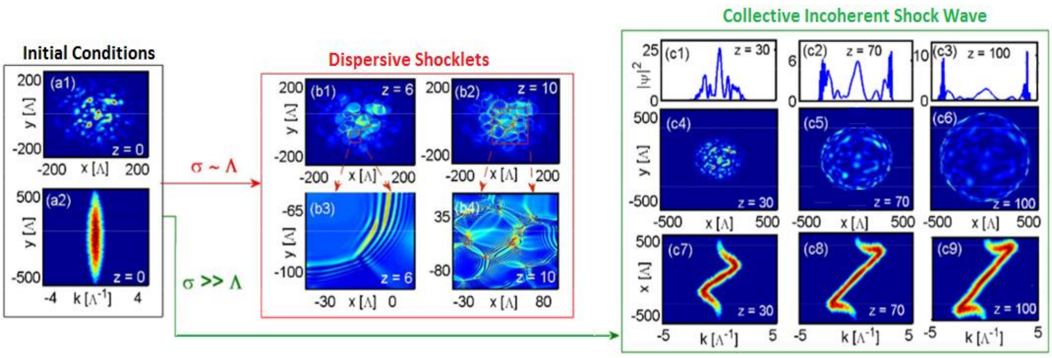

Fig. 1: Numerical simulations of Eq. (1) starting from the incoherent wave (a1) in the strong nonlinear regime, $\mathcal{E}_0 >> \mathcal{U}_0$; (a2) the spectrogram corresponding to this initial condition. (b) In the quasi-local regime $\sigma = 2\Lambda$ (short-range interaction), the incoherent beam develops a sea of dispersive shocklets (b1,b2), as evidenced by the formation of several DSWs [see zooms in (b3,b4)]. (c) Starting from the same initial condition (a1), in the highly nonlocal regime $\sigma = 100\Lambda$ (long-range interaction), the random wave as a whole develops a giant collective incoherent shock (c4-c6), whose corresponding intensity lineouts $y = 0$ (c1–c3) indicate an annular collapse behavior. (c7–c9) Corresponding spectrogram evolutions of the incoherent shock: The Z-shaped distortion reveals a dramatic coherence degradation on the annular boundaries of the beam. The propagation length in the sample, z, is in units of the nonlinear length, $L_0 = 1/(\gamma\rho)$.

The formation of shock waves is known to require a strong nonlinear interaction, i.e., $\mathcal{E}$ (z = 0) << $\mathcal{U}$ (z = 0). In this strong turbulence regime, we compare simulations performed in the short-range ($\sigma \sim \Lambda$), and long-range ($\sigma >> \Lambda$), interaction regimes. For $\sigma \sim \Lambda$, the incoherent beam evolves into a turbulent sea of coherent DSWs: Each individual speckle of the initial condition develops its own DSW, which subsequently interacts with the other DSWs, see Fig. 1(b). Conversely, in the long-range interaction regime $\sigma >> \Lambda$, the system exhibits a global collective behavior: It is the incoherent beam as a whole which develops a large scale shock singularity.

This phenomenon of incoherent shock wave can be described in detail in the framework of the long-range Vlasov formalism [5-6]. Specifically, we introduced singular solutions of the long-range Vlasov equation, which reduces the description to a hydrodynamic-like model for the evolutions of the intensity $N(r, z)$, and momentum $K(r, z)$, of the *random wave*. Solving this model by the method of the characteristics, reveals that the system exhibits a double singularity: a *shock* singularity for the momentum and an unexpected *collapse* singularity for the intensity on the annular boundary of the beam -- for details see [4]. An important result is that the formation of regular DSW oscillations are not necessary to regularize the incoherent shock singularity: The regularization occurs through an original process of dramatic coherence degradation on the boundary of the beam.

3. Experimental observations

We performed experiments that provide evidence of both regimes of dispersive shocklets (short-range), and large-scale incoherent shocks (long-range), by varying the range of nonlocality (see Fig. 2). The solution of graphene nano-flakes in the sample exhibits a thermal nonlocal nonlinearity [2,7-8], whose interaction range σ has been tuned by varying the concentration of nano-flakes. Figs. 2(a-c) report measured output intensity patterns of the optical beam and corresponding simulations, with σ = 590 μm. As the input power is increased, we observe an annular reshaping of the speckled beam, with high-frequency components piling up on the boundaries, whereas low-frequency components dominate the internal region of the beam (also see the corresponding y = 0 lineouts in Fig2. (a)). The corresponding spectrogram measurements in Fig. 2(d-e) provide a clear signature of the formation of the incoherent shock phenomenon.

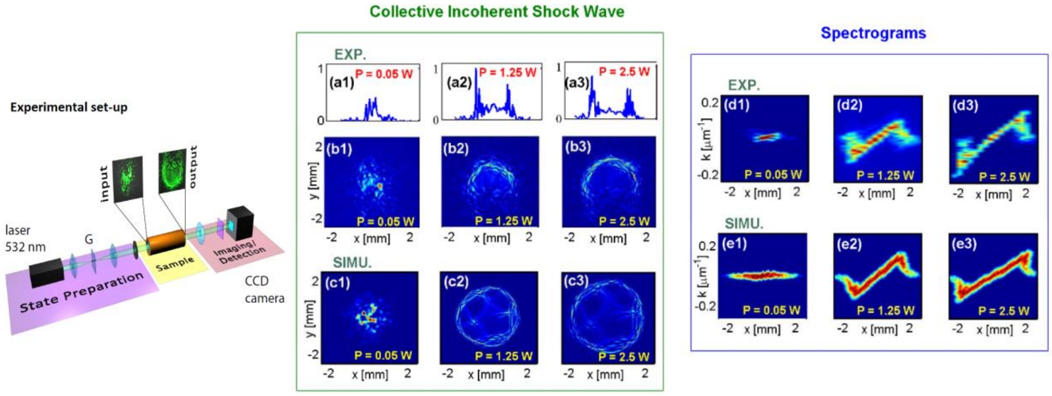

Fig. 2. Left: Experimental set-up. The sample consists of a cylindrical tube filled with a solution of methanol and grapheme nanoscale flakes. After the sample, the beam is imaged into a CCD camera. (a-c) Experimental observation of the giant incoherent shock wave (long-range regime): Beam profiles of the output intensity taken at low power P = 0,025W (b1), P = 1,25W (b2), P = 2,5W (b3); corresponding lineouts (at y = 0) (a1-a3). The asymmetry in the lower part of the beam is due to convection within the sample. (c) Numerical simulations of NLSE performed with the experimental parameters ($\lambda_{c0} \simeq 200\mu$m, $\Lambda = 4\mu$m, $\sigma = 590\mu$m). (d-e), Spectrogram signature of incoherent shocks: Experimental (d), and numerical (e), spectrograms retrieved from the lineouts y = 0. As the pump power increases, the spectrogram evidences the development of a shock singularity on the annular boundary of the incoherent beam.

We have made a second series of experiments in the short-range interaction regime to investigate the development of dispersive shocklets. According to the scaling $\sigma \sim 1/\alpha^{1/2}$ [2], we significantly increased the concentration of graphene nano-flakes so as to increase the absorption α, and thus reduce the nonlocal range of interaction by one order of magnitude ($\sigma = 60\mu$m). We observed the turbulent sea of DSWs featured by the formation of regular undular patterns, whose typical spatial period is found in good agreement with NLSE (1) simulations. We refer to Ref. [4] for more details on the experimental and theoretical results.

4. References

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