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Mathematical and Statistical Methods for Multistatic Imaging

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Introduction

In multistatic imaging one uses waves to probe for information about an unknown medium. These waves can be acoustic, elastic, or electromagnetic. They can be at zero-frequency and consequently modeled by the conductivity equation or at nonzero-frequency and hence modeled by the Helmholtz equation. They are generated by an array of transmitters and recorded by an array of receivers (transducers in acoustics, seismographs in geophysics, or antennas in electromagnetics).

Multistatic imaging usually involves two steps. The first step is experimental. It consists in recording the waves generated by sources on an array of receivers. The second step is numerical. It consists in processing the recorded data in order to estimate some relevant features of the medium (source or reflector locations and shapes).

This book covers recent mathematical, numerical, and statistical approaches for multistatic imaging of targets with waves at single or multiple frequencies. The waves are generated by point sources on a transmitter array and measured on a receiver array. For the sake of simplicity, we consider coincident transmitter and receiver arrays. There are two interesting problems: one is finding small targets and the other is reconstructing shape deformations of an extended target. A target is called small when its characteristic size times the operating frequency is less than one while it is called extended when this factor is much larger than one. In both situations, we are interested in imaging small perturbations with respect to known situations.

Our approach is based on an asymptotic analysis of the measured data in terms of the size of the unknown targets or the order of magnitude of the shape deformation. The asymptotic analysis plays a key role in characterizing all the information about the small target or the shape deformations of an extended target that can be stably reconstructed from the measured data. It provides robust and accurate reconstruction of the location and some geometric features of the small targets as well as small changes of the shape of an extended target, even with moderately noisy data.

When we are dealing with target imaging problems, a fundamental problem is to have a shape representation well suited for solving the inverse problem. Parametric representations do not do a good job. In fact, the data have highly nonlinear dependence with respect to parametric representations. However, high-order polarization tensors can be stably reconstructed from the data by solving a least-squares problem. Therefore, they provide a well-suited representation of the target. Moreover, they capture both the high-frequency information in its shape and its topology. The high-order polarization tensors generalize the (classical) magnetic and electric polarization tensors associated to a small target with a given set of electromagnetic parameters. For an arbitrary shape, we can find an equivalent ellipse (or ellipsoid) with the same first-order polarization tensor. Using high-order polarization tensors we can not only recover finer details of the shape of a given target but also separate its electromagnetic parameters from its volume. In this book, we derive an optimization procedure to reconstruct a target from its high-order polarization tensors. We also present a dictionary matching technique based on new invariants for the generalized polarization tensors. For extended targets, a concept equivalent to the polarization tensor can be introduced and direct algorithms can be designed for reconstructing small shape changes.

The main applications that we have in mind are medical imaging (such as microwave and electrical impedance breast cancer detections), airport security screening, geophysical exploration, and nondestructive testing. For such applications, the general purpose of multistatic imaging is, from imperfect information (rough forward models, limited and noisy data), to estimate parts of the unknown structure that is of interest. In this book we consider, in the presence of noise, the detection and localization of sources, reflectors, and the reconstruction of small inclusions and shape deformations.

An important problem in multistatic imaging is to quantify and understand the trade-offs between data size, computational complexity, signal-to-noise ratio, and resolution. For instance, in geophysics, very large amount of data are collected and the computational complexity of the imaging algorithm is a limiting factor. In this book we carefully address the trade-off between resolution and stability when the data are noisy. We provide imaging algorithms and analyze their resolution and stability with respect to noise in the measurements. Resolution analysis is to estimate the size of the finest detail that can be reconstructed from the data while stability analysis is to quantify the localization error in the presence of noise. The noise models discussed in this book are measurement and medium (or clutter) noises. They affect the stability and resolution of the imaging functionals in very different ways.

The book is organized as follows. Chapter 1 reviews some of the fundamental mathematical and statistical concepts that are key to understanding imaging principles. Chapter 2 collects some preliminary results regarding layer potentials. This chapter offers a comprehensive treatment of the subject of integral equations and provides key identities for solving imaging problems. Chapter 3 covers the method of small volume expansions. It provides the leading-order term in the asymptotic expansion of the solution to the conductivity or Helmholtz equation with respect to the size of a small inclusion. In Chap. 4 we introduce the concept of high-order polarization tensors (also called generalized polarization tensors) associated with a conductivity inclusion and present their main properties. Chapter 5 is devoted to the frequency-dependent polarization tensors associated with an electromagnetic (or acoustic) inclusion. We introduce the notion of scattering coefficients and prove some of their properties. Scattering coefficients can be obtained from far-field measurements and are the Fourier coefficients of the scattering amplitude. Chapter 6 deals with the structure of the multistatic response matrix. It introduces a Hadamard technique for noise reduction and provides statistical distributions of significant singular values of the multistatic response matrices associated with point reflectors and inclusions. In Chap. 7, using multipolar expansions for the solutions to the conductivity and Helmholtz equations, we analyze the structure of the corresponding multistatic response matrices. Based on the method of small volume expansions we derive in Chap. 8 localization techniques for inclusions in the continuum approximation that take advantage of the smallness of the inclusions. Direct algorithms for imaging small conductivity and electromagnetic inclusions are introduced and their stability with respect to medium and measurement noises as well as their resolution is investigated. Chapter 9 outlines detection and localization techniques from noisy multistatic measurements. The results of Chap. 6 on the statistical properties of the multistatic response matrix in the presence of noise are used to design detection tests. The detection test is to decide whether a point reflector is present or not. An extension of Berens' modeling for point reflector detection is given. Chapter 10 deals with the reconstruction of the generalized polarization tensors from multistatic response measurements. A stability analysis for the reconstruction in the presence of measurement noise which quantifies the ill-posedness of the reconstruction problem is provided. Chapter 11 is devoted to target identification from multistatic data using generalized polarization tensors. It provides a fast and efficient procedure for target identification in multistatic imaging based on matching on a dictionary of precomputed generalized polarization tensors. The approach is based on the use of invariants for the generalized polarization tensors. Chapter 11 also applies an extended Kalman filter to track both the location and the orientation of a mobile target from multistatic measurements. Chapters 12–14 discuss multistatic imaging techniques for extended targets. We start with inverse source problems and introduce time reversal techniques. Then we focus on reconstructing shape changes of an extended target. We introduce several algorithms and analyze their resolution and stability for the linearized reconstruction problem. Finally, we describe optimal control approaches for solving the nonlinear problem.

Chapters 15 and 16 present results on electromagnetic invisibility. Electromagnetic invisibility is to make a target invisible for electromagnetic probing. Many schemes are under active current investigations. These include active cloaking, transmission line cloaking, interior cloaking, and exterior cloaking. Chapter 15 focusses on interior cloaking while Chap. 16 is devoted to exterior cloaking.

The main tool to obtain interior cloaking, where the target to be cloaked is inside the cloaking structure, is to use a change of variables scheme (also called transformation optics). The change of variables-based cloaking method uses a singular transformation to boost the material property so that it makes a cloaking region look like a point to outside measurements. However, this transformation induces the singularity of material constants in the transversal direction (also in the tangential direction in two dimensions), which causes difficulty both in the theory and in the applications. To overcome this weakness, the so-called near-cloaking is naturally considered, which is a regularization or an approximation of singular cloaking. Instead of the singular transformation, one can use a regular one to push forward the material constant in the conductivity equation, in which a small ball is blown up to the cloaking region. The aim of Chap. 15 is to discuss recent advances in near-cloaking. We first provide a method of constructing effective near-cloaking structures for the conductivity problem. These new structures are such that their first generalized polarization tensors vanish. We show that this in particular significantly enhances the cloaking effect. Then we extend this method to scattering problems. We construct very effective near-cloaking structures for the scattering problem at a fixed frequency. These new structures are, before using the transformation optics, layered structures and are designed so that their first scattering coefficients approximately vanish. Inside the cloaking region, any target has near-zero scattering cross section for a band of frequencies. As for the conductivity problem, we analytically show that this new construction significantly enhances the cloaking effect for the Helmholtz equation. Chapter 16 aims to give a mathematical justification of exterior cloaking due to anomalous localized resonance. We consider the dielectric problem with a source term in a structure with a layer of plasmonic material. The real part of the permittivity inside the plasmonic layer is negative. In the case of concentric disk structure, we show that for any source supported outside a critical radius cloaking does not take place, and for sources located inside the critical radius satisfying certain conditions cloaking does take place as the loss parameter inside the metamaterial layer goes to zero.

The last part of this book provides the reader with practical implementations and performance evaluations of the described imaging methods and techniques.

Chapter 17 provides MATLAB codes for the main algorithms described in this book. Chapter 18 presents numerical illustrations using these codes in order to highlight the performance and show the limitations of our numerical approaches for multistatic imaging.

The bibliography provides a list of relevant references. It is by no means comprehensive. However, it should provide the reader with some useful guidance in searching for further details on the main ideas and approaches discussed in this book.

The book has grown out of lecture notes for a summer school on mathematical and statistical methods in imaging at the Institute of Computational Mathematics of Chinese Academy of Sciences in Beijing. It was also taught through a series of intensive lectures at the Korean Advanced Institute of Science and Technology. We are very grateful to the organizers of these two events. Some of the material in this book is from our wonderful collaborations with Elie Bretin, Thomas Boulier, Giulio Ciraolo, Pierre Garapon, Vincent Jugnon, Hyundae Lee, Graeme Milton, Abdul Wahab, Sanghyeon Yu, and Habib Zribi. We feel indebted to all of them. We would also like to acknowledge the support of the European Research Council Project MULTIMOD and of Korean ministry of education, science, and technology through grant NRF 2010-0017532. M. Lim was supported by TJ Park Junior Faculty Fellowship.

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