CONTEMPORARY MATHEMATICS

548

Mathematical and Statistical Methods for Imaging

NIMS Thematic Workshop Mathematical and Statistical Methods for Imaging August 10–13, 2010 Inha University Incheon, Korea

> Habib Ammari Josselin Garnier Hyeonbae Kang Knut Sølna Editors



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2010 Mathematics Subject Classification. Primary 35R30, 35A20, 35B40, 44A12, 78M05, 78M25, 78A45.

Library of Congress Cataloging-in-Publication Data

NIMS Thematic Workshop (2010 : Inch'on, Korea), Mathematical and statistical methods for imaging: NIMS Thematic Workshop, August 10–13, 2010, Inha University, Incheon, Korea / Habib Ammari...[et al.], editors.

p. cm. – (Contemporary mathematics ; v. 548) Includes bibliographical references. ISBN 978-0-8218-5289-7 (alk. paper)

1. Electromagnetic theory—Mathematics—Congresses. 2. Ordinary differential equations—Congresses. I. Ammari, Habib. II. Title.

 $\substack{ QC670.N56 & 2010 \\ 621.36'70151-dc22 }$

2011011086

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Preface

During the months of July and August 2010, a thematic program on *mathematical and statistical methods for imaging* was held at Inha University, Korea. As a part of the program, a three-day international conference was organized, at which prominent experts in the field were invited. The lectures they delivered covered a variety of hot topics of current research on imaging. Recent advances in imaging are certainly consequences of innovative mathematical approaches to fundamental issues such as detectability, resolution, and stability, as well as of a strong interest in potential applications. These mathematical approaches include multi-scale analytical and computational techniques, statistical methods, random matrix theory, and signal theory.

A multi-scale approach plays a key role in imaging. It leads to effective and robust reconstruction algorithms in many imaging problems since it allows us to overcome the severe ill-posedness character of image reconstruction. The mathematical tools involved come from a wide range of areas of pure and applied mathematics ranging from potential theory to PDEs, to scattering theory, to complex analysis, to numerical methods. At the same time, a lot of effort has been devoted to design new and efficient approaches for retrieving information from random media. These approaches promise to allow anomaly wave imaging in the presence of both medium and measurement noises. Moreover, the recent use of random matrix theory for defect imaging has added a new dimension to the field.

This volume provides a forum for a deeper and more unified understanding of the field of imaging and for combining analytical and statistical tools in imaging. It offers the reader a good overview of current research and direction for further pursuit. Challenging problems are addressed from analytical, numerical, as well as statistical perspectives. The objective of the volume is fourfold: (i) To analytically investigate the robustness, with respect to incomplete data, measurement, and medium noises of the recently developed multi-scale approaches; (ii) To establish hypothesis testing and resolution analysis, particularly for anomaly detection;

(iii) To design new efficient imaging techniques; (iv) To take into account the effects of anisotropy, dissipation, or attenuation in imaging.

The tremendous success of the workshop was only possible due to the enthusiastic participation of wonderful speakers and authors of this volume. We are thankful to all of them. We also acknowledge with gratitude the generous support from NIMS (National Institute for Mathematical Sciences) during the thematic program. We would also like to thank the host institution—Inha University.

Habib Ammari, Josselin Garnier, Hyeonbae Kang, and Knut Sølna

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This volume contains the proceedings of the NIMS Thematic Workshop on Mathematical and Statistical Methods for Imaging, which was held from August 10–13, 2010, at Inha University, Incheon, Korea.

The goal of this volume is to give the reader a deep and unified understanding of the field of imaging and of the analytical and statistical tools used in imaging. It offers a good overview of the current status of the field and of directions for further research. Challenging problems are addressed from analytical, numerical, and statistical perspectives. The articles are devoted to four main areas: analytical investigation of robustness; hypothesis testing and resolution analysis, particularly for anomaly detection; new efficient imaging techniques; and the effects of anisotropy, dissipation, or attenuation in imaging.



