Propagation and amplification of incoherent pulses in dispersive and nonlinear media

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We develop a statistical model to study all the relevant phenomena that can give rise to an anomalous intensity saturation in the propagation of incoherent pulses in a laser amplifier. The interplay among diffraction, self-focusing, group-velocity dispersion, gain narrowing, and gain saturation is investigated. Changes in the temporal and spatial characteristics of the pulses are shown.
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1. INTRODUCTION

The propagation of incoherent light has been studied extensively. Incoherent light with a short coherence time is of interest for coherent spectroscopy[1] but also for many other applications, such as tomography in random media,[2] and for use in smoothing techniques for uniform irradiation in plasma physics.[3] If propagation of incoherent light in linear media is now rather well understood, the evolution of the statistical properties of incoherent pulses in nonlinear media has been insufficiently examined. Although the literature contains papers about time-incoherent light in nonlinear media[4,5] and time–space-incoherent light in nonlinear media,[6–8] none of them deals with the regime that is obtained by use of time–space-incoherent pulses in large-scale laser amplifiers. Indeed, the results of Refs. 6–8 rely on the assumption that the fourth-order correlation functions still obey the rules of Gaussian processes. We shall see that this is not an adequate hypothesis in the experimental conditions that correspond to high-power laser chains and that the main effect of nonlinearity is to break the Gaussian property. The present study was stimulated by the initial observation of an anomalous intensity saturation effect in the amplification of intense incoherent pulses in a large Nd:glass power chain.[9] In what follows, we develop a statistical theory of propagation and amplification of incoherent pulses to predict the probable intensity saturation that could occur on future high-power laser chains designed for inertial confinement fusion. Indeed, a high level of irradiation uniformity is required for both direct and indirect drive.[3] This criterion can be reached by implementation of active smoothing methods, such as induced spatial incoherence with echelons,[10] smoothing by spectral dispersion,[11] and smoothing by multimode optical fibers.[12] All these methods involve the illumination on the target with an intensity that is a time-varying speckle pattern, so the time-integrated intensity averages toward a flat profile. This paper is a contribution to the study of optical smoothing for application to inertial confinement fusion. Some of the optical smoothing techniques involve intensity space–time modulations in the amplifiers. Because the amplification is planned to produce high power, we cannot neglect the nonlinear effects.

We consider here that the space–time modulations of the pulse can be divided into a slowly varying component, which constitutes the envelope of the beam, and fast varying modulations, which we assume obey Gaussian statistics. In particular, the instantaneous spatial intensity distribution of such a pulse is called a speckle pattern.[13] These statistical characteristics are usual and correspond to the pulses generated by the smoothing techniques that we named above.[14] We study the propagation of incoherent pulses and take into account all the phenomena that we believe are relevant: diffraction, self-phase modulation, self-focusing, filamentation, group-velocity dispersion (GVD), gain narrowing, and gain saturation. We shall see that the interplay among these effects gives rise to nontrivial results. In Section 2 we study the propagation of an incoherent pulse in a medium without gain, in order to review the different relevant mechanisms. Section 3 is devoted to a study of the amplification of an incoherent beam when the energy depletion of the host medium can be considered negligible. Finally, in Section 4 we consider the effects of the decay of the inverted population.

2. PROPAGATION IN NONLINEAR AND NONAMPLIFYING MEDIA

A. Formulation

Propagation of light pulses in the presence of Kerr nonlinearity has been studied extensively since the recent development of high-power laser chains. In particular, self-focusing of incoherent laser pulses is of great current importance because this phenomenon is a limiting factor for the new lasers projected for inertial confinement fusion. Indeed, they are planned to deliver pulses with energies of the order of 2 MJ and powers of the order of 1 PW focusing upon spherical targets consisting of deuterium–tritium to initiate fusion reactions. The self-focusing dynamics results from the competition between the nonlinear effects on the one hand and transverse diffraction and GVD of the wave on the other hand. Without dispersion, spatial self-focusing can develop if the nonlinear effects prevail over the natural linear diffraction. This mechanism may involve the self-contraction of the beam whose mean-square radius will tend to zero, which means that it collapses into a point singularity. This phenomenon is referred to in what follows as self-focusing, to be distinguished from filamentation, which consists in the growth of small but hot spots whose maximal intensities are many times larger than the mean intensity. Such singular dynamics may be responsible for damage to optical components. We shall see that filamentation usually grows before self-focusing for intense incoherent pulses that present small-scale intensity fluctuations.

We consider the propagation of a broadband pulse with carrier frequency ω_0 and wave number k_0 in a weakly nonlinear dispersive medium. Within the paraxial approximation, the scalar field A satisfies the parabolic equation[15]

$$i\,\frac{\partial A}{\partial z} - \frac{1}{2k_0}\,\Delta_r A - \sigma\,\frac{\partial^2 A}{\partial t^2} - \frac{k_2}{2}\,|A|^2 A = 0, \qquad (1)$$

where z is the wave-propagation axis and Δ_r is the orthogonal Laplacian, $\partial^2/\partial x^2 + \partial^2/\partial y^2$, which describes the diffraction of the wave in the transverse plane. We denote by r the vector consisting of the transverse coordinates (x, y) and by t the local time in the moving-pulse time frame. $k_2 = k_0 n_{2E}/n_0$, where n_0 is the unperturbed value of the index of refraction and n_{2E} is the optical Kerr coefficient that governs the response of the medium to the wave amplitude. Note that the laser intensity is proportional to the intensity of the scalar field: $I_{\mathrm{laser}} = \epsilon c|A|^2/2$, and that the usual n_2 is related to n_{2E} through the identity $n_{2E}|A|^2/2 = n_2 I_{\mathrm{laser}}$. Typical values for laser glass are $n_0 \simeq 1.55$ and $n_2 \simeq 2.7 \times 10^{-7}\ \mathrm{cm}^2/\mathrm{GW}$. The GVD coefficient σ is related to the dispersion factor $\partial^2 k/\partial\omega^2|_{\omega_0}$ through the formula $\sigma := -k''(\omega_0)/2$. Following the dispersive properties of the medium, the coefficient σ can be either positive in the case of a so-called anomalous dispersive medium or negative in the opposite case of a normally dispersive medium. Here we consider mainly the latter case, because the value of σ for Nd-doped glass at 1.05 μm is $\sigma \simeq -3.5 \times 10^{-4}\ \mathrm{ps}^2\,\mathrm{cm}^{-1}$. We study the propagation of an incoherent pulse whose statistical characteristics are locally stationary. We consider an incident field A_0 whose temporal and spatial slowly varying envelopes are deterministic, with spatial radius R_0 (the beam radius) and temporal duration T_0 (the pulse duration). These scales are referred to in what follows as macroscopic scales. In the experimental conditions that correspond to high-power laser chains, R_0 is of the order of 10–20 cm and T_0 is of the order of several nanoseconds. The incident pulse also has fast-varying random fluctuations, with characteristic scales ρ_c (the correlation radius) in the spatial domain and τ_c (the coherence time) in the temporal domain. These scales are referred to in what follows as the microscopic scales. To fix the sizes of orders, we assume that the value of ρ_c is of the order of 2–20 mm and that τ_c is of the order of 1–5 ps. These random fluctuations are assumed to obey Gaussian statistics. Their distributions are then characterized in the plane $z = 0$ by the autocorrelation function of the initial field $A_0(r, t) := A(z = 0, r, t)$, defined by

$$C_{0,R,T}(\rho, \tau) = \langle A_0(r_1, t_1) A_0^*(r_2, t_2)\rangle, \qquad (2)$$

with $R = (r_1 + r_2)/2$, $T = (t_1 + t_2)/2$, $\rho = r_1 - r_2$, and $\tau = t_1 - t_2$. We assume that the the autocorrelation function has a locally Gaussian shape, so the expression of $C_{0,R,T}$ can be taken to be equal to

$$C_{0,R,T}(\rho, \tau) = I_0(R, T)\exp-\left(\frac{|\rho|^2}{\rho_c^2} + \frac{\tau^2}{\tau_c^2}\right). \qquad (3)$$

$I_0 := I_0(0, 0)$ is the mean intensity of the scalar field at the center of the beam, and $R \mapsto I_0(R, T)$ $[T \mapsto I_0(R, T)]$ describes the spatial (temporal) shape of the slowly varying envelope of the intensity. To simplify, we assume that $I_0(R, T)$ is of the form

$$I_0(R, T) = I_0\ \exp-\left(\frac{|R|^p}{R_0^p} + \frac{T^q}{T_0^q}\right), \qquad (4)$$

where p and q are even integers. If p (or q) is equal to 2, the slowly varying envelope of the intensity has a Gaussian shape, and if p (or q) is larger than 2 the envelope has a hyper-Gaussian shape.

On the one hand we focus in this paper on the evolution of the autocorrelation function:

$$C_{z,R,T}(\rho, \tau) = \langle A(z, r_1, t_1) A^*(z, r_2, t_2)\rangle, \qquad (5)$$

where $r_1 = R + \rho/2$, $r_2 = R - \rho/2$, $t_1 = T + \tau/2$, and $t_2 = T - \tau/2$. This function is relevant because it provides the beam radius and the pulse duration in particular and also information about the statistical distribution of the microscopic hot spots. On the other hand we are interested in the amplitude of the intensity fluctuations, which can be measured by the contrast. In case of a pattern whose random fluctuations are statistically globally stationary (i.e., with a flat mean profile), the contrast is defined as the ratio of the standard deviation to the mean value of the intensity. Here we have to take care of the slowly varying envelope. As a consequence we adopt a more general definition of the contrast $c(z)$, which is the following:

$$c^2(z) = \frac{\iint |A|^4(z,\ r,\ t)\mathrm{d}^2r\mathrm{d}t - \iint C_{z,R,T}^2(0,0)\mathrm{d}^2R\mathrm{d}T}{\iint C_{z,R,T}^2(0,0)\mathrm{d}^2R\mathrm{d}T}. \quad (6)$$

In Eq. (6), $[\iint |A|^4(z,\ r,\ t)\mathrm{d}^2r\mathrm{d}t]^{1/2}$ is the L^2 norm of the intensity of the scalar field and $[\iint C_{z,R,T}^2(0,0)\mathrm{d}^2R\mathrm{d}T]^{1/2}$ is the L^2 norm of the slowly varying envelope of the intensity of the scalar field. An equivalent expression for the contrast, which we find by using the local spatial ergodicity of the field, is

$$c^2(z) = \frac{\iint \langle |A|^4\rangle(R,\ T) - \langle |A|^2\rangle^2(R,\ T)\mathrm{d}^2R\mathrm{d}T}{\iint \langle |A|^2\rangle^2(R,\ T)\mathrm{d}^2R\mathrm{d}T}.$$

As expected, in the plane $z = 0$ the contrast is $c(z=0) = 1$.

B. Conserved Quantities

The nonlinear Schrödinger equation has been studied extensively for many years.[16] Equation (1) is a Lagrangian system, which possesses conserved quantities. In particular, the square L^2 norm of the field, i.e., the total energy of the beam defined by

$$E := \iint |A|^2(z,\ r,\ t)\mathrm{d}^2r\mathrm{d}t, \quad (7)$$

is constant, and its value is imposed by the incident pulse:

$$E_0 = 4\pi R_0^2 T_0 \frac{I_0}{pq}\Gamma\left(\frac{2}{p}\right)\Gamma\left(\frac{1}{q}\right), \quad (8)$$

where Γ is the gamma function defined by the Euler integral $\Gamma(u) = \int_0^\infty s^{u-1}e^{-s}\mathrm{d}s$. The Hamiltonian integral H, defined by

$$H := \iint \frac{1}{2k_0}|\nabla_r A|^2 + \sigma\left|\frac{\partial A}{\partial t}\right|^2 - \frac{k_2}{4}|A|^4\mathrm{d}^2r\mathrm{d}t, \quad (9)$$

is another constant of motion for Eq. (1), and its value is given by

$$H_0 = \left(\frac{2}{k_0\rho_c^2} + \frac{2\sigma}{\tau_c^2} - \frac{k_2 I_0}{2^{2+\frac{2}{p}+\frac{1}{q}}}\right)E_0. \quad (10)$$

The values of E and H are computed in the plane $z = 0$ and are valid in the approximation $R_0 \gg \rho_c$ and $T_0 \gg \tau_c$. In the experimental conditions that we simulate here, the mean intensity is so high that we usually have $H_0 < 0$. We also introduce the evolution integral V, defined by

$$V(z) := \iint \left(2k_0|r|^2 + \frac{t^2}{\sigma}\right)|A|^2\mathrm{d}^2r\mathrm{d}t. \quad (11)$$

This quantity is convenient for use in determining whether a given initial wave will collapse into a point singularity within a finite propagation length, a point where the amplitude blows up. $V(z)$ is not preserved, but it satisfies the virial identity[17]

$$\frac{\partial^2 V}{\partial z^2} = 8H - k_2\iint |A|^4\mathrm{d}^2r\mathrm{d}t. \quad (12)$$

Inasmuch as the Hamiltonian H is constant, the negativity of the Hamiltonian appears to be a sufficient condition for collapse. Unfortunately these kinds of approaches can provide us only with pieces of information about the macroscopic shape of the beam but not on the rapid fluctuations and the statistics of the microscopic hot spots.

C. Evolution of an Isolated Hot Spot

In this section we estimate the microscopic behavior of the pulse by using a simple method. We actually carry out a heuristic study of the propagation of a single hot spot. This study is based on the following approximations: First, we consider that the hot spot does not interact with its neighboring hot spots. Second, we assume that the initial envelope of the hot spot is Gaussian. The second approximation is partially justified when we deal with a field with Gaussian statistics, with mean intensity I_0 and correlation function $\exp(-|\rho|^2/\rho_c^2 - \tau^2/\tau_c^2)$. Indeed, under such conditions the scalar envelope of the field of a hot spot is roughly a Gaussian peak with spatial width $\rho_{\mathrm{FWHM}} = 2\sqrt{\ln(2)}\rho_c$ and temporal width $\tau_{\mathrm{FWHM}} = 2\sqrt{\ln(2)}\tau_c$,[18] where FWHM is the full width at half-maximum. We therefore consider in this section an isolated spot with maximum $\sqrt{I_1}$:

$$A(z=0,r,\ t) = I_1^{1/2}\exp-\left(\frac{|r|^2}{\rho_c^2} + \frac{t^2}{\tau_c^2}\right) \quad (13)$$

whose propagation is governed by the Schrödinger equation, Eq. (1). The solution is classically[16] sought in the following self-similar form, which is valid up to the collapse point:

$$A(z,\ r,\ t) = I(z)^{1/2}\exp-\left[\frac{|r|^2}{\rho(z)^2} + \frac{t^2}{\tau(z)^2}\right] \times \exp i[\phi_0(z) + \phi_1(z)|r|^2 + \phi_2(z)t^2]. \quad (14)$$

Injecting this ansatz into Eq. (1), we find that the maximal intensity and the spatial and temporal radii of the field satisfy the following system of differential equations:

$$\frac{\partial^2\rho(z)}{\partial z^2} = \frac{4}{k_0^2\rho^3(z)} - \frac{k_2 I_1\tau_c\rho_c^2}{2k_0\tau(z)\rho^3(z)}, \quad (15a)$$

$$\frac{\partial^2\tau(z)}{\partial z^2} = \frac{16\sigma^2}{\tau^3(z)} - \frac{\sigma k_2 I_1\tau_c\rho_c^2}{\tau(z)^2\rho^2(z)}, \quad (15b)$$

$$I(z) = I_1\frac{\rho_c^2\tau_c}{\rho^2(z)\tau(z)}. \quad (15c)$$

When $\sigma < 0$, the temporal width of the spot always increases during the propagation. The evolution of the spatial transverse width results from the competition between linear diffraction and nonlinear self-focusing. However, we deduce from the last term of Eq. (15a) that the GVD interferes with self-focusing. In particular, the cumulative effect of GVD and diffraction may counterbalance self-focusing, whereas diffraction alone would not.

Actually the situation is rather complicated. We can indeed see several kinds of behavior according to the value of the maximal power $P_1 = I_1 \pi \rho_c^2/2$ of the hot spot and the importance of the GVD:

1. If P_1 is low, less than $P_{\mathrm{cr}} = 4\pi/(k_0 k_2)$, then diffraction dominates self-focusing. The spot spreads out in time and space.
2. If P_1 is larger than the critical value P_{cr}, then the beam spatially self-contracts and spreads out in time. This is of course the most exciting configuration to study.
   a. If the GVD is weak, $|\sigma| k_0 \rho_c^2 \leq \tau_c^2$, then the evolution of the maximal intensity is imposed by self-focusing and increases.
   b. If the effects of GVD are more important, $|\sigma| k_0 \rho_c^2 > \tau_c^2$, then the evolution of the maximal intensity is first imposed by time spreading and decreases. But this picture can be reversed. Indeed, if $\tau(z)$ slowly increases; then self-focusing, which tends to develop faster and faster, finally prevails. The temporal width $\tau(z)$ should grow quickly enough that the inflexion point $\tau_{\mathrm{inf}} = \tau_c P_1/P_{\mathrm{cr}}$ of Eq. (15a) is reached at the point z_{inf}, which lies before the point z_{foc} where self-focusing makes the wave form a point singularity. If $z_{\mathrm{inf}} < z_{\mathrm{foc}}$, then the pulse will never collapse and will spread out in time and space.

D. Statistical Approach

On the one hand the global analysis in Subsection 2.B gives indications about the evolution of the macroscopic radii of the pulse but fails to describe the deformation of hot spots and the probability of filamentation. On the other hand the local study of hot spots in Subsection 2.C is not rigorous, as the hypothesis of noninteraction of the hot spots is obviously rough. To get an accurate description of the evolution of the statistics of the hot spots we study here the evolution of the autocorrelation function, whose expression in the plane $z = 0$ is given by Eq. (3). Unfortunately it is not possible to get a closed-form equation to govern the evolution of the autocorrelation function, because nonlinearity makes the nth-order correlation depend on the $(n + 2)$nd-order correlation. The authors of Refs. 6–8 neglected GVD and considered the case when the characteristic distance over which the nonlinear interaction between the field and the fluctuations of $k_2|A|^2/2$ takes place is much greater than the size of the region of longitudinal field correlation $z_c = k_0 \rho_c^2$, which is determined by the length of diffraction spreading of a characteristic inhomogeneity in the cross section of an incoherent beam. In such conditions the statistics are only slightly affected by the nonlinear effect, so the fourth-order moments can be deduced from the second-order moments according to the simple rules that are valid for Gaussian processes[19] and a closed-form equation for the correlation function can be obtained. In the experimental conditions that correspond to high-power laser chains, the approximation discussed in Refs. 6–8 is not valid. On the one hand the intensity of the pulse is so high that the expected nonlinear term $k_2\langle|A_0|^2\rangle z/2$ is important and reaches values of the order of 1. On the other hand the microscopic scales are large enough that $k_0\rho_c^2$ and $\tau_c^2/|\sigma|$ are larger than the propagation length z. If we consider the most stringent conditions, $\rho_c = 2$ mm and $\tau_c = 1$ ps, then $k_0\rho_c^2 \simeq 25$ m and $\tau_c^2/|\sigma| \simeq 30$ m. Because the total glass thickness never exceeds a few meters, we can deal with these terms by using a perturbation theory. But the nonlinear terms cannot be neglected, and they actually prevail during the propagation. We shall see that nonlinearity destroys the Gaussian property of the statistics of the field. At the lowest order the wave acquires a nonlinear phase term $\phi_{\mathrm{NL}} = -k_2|A_0|^2 z/2$ by self-phase modulation, so the correlation function at the lowest order can be computed:

$$C_{z,R,T}(\rho, \tau) = \frac{C_{0,R,T}(\rho, \tau)}{\left\{1 + B_0^2(z, R, T)\left[1 - \exp\left(-\frac{2\tau^2}{\tau_c^2} - \frac{2|\rho|^2}{\rho_c^2}\right)\right]\right\}^2}, \tag{16}$$

where $B_0(z, R, T) = k_2 I_0(R, T) z/2$ is the mean cumulative B integral. The L^2 norms of the derivatives of the field can easily be deduced from the second derivatives of the autocorrelation function:

$$\int\int |\nabla_r A|^2 \mathrm{d}^2 r \mathrm{d}t = -\int\int \Delta_\rho C_{z,R,T}(0, 0) \mathrm{d}^2 R \mathrm{d}T, \tag{17a}$$

$$\int\int \left|\frac{\partial A}{\partial t}\right|^2 \mathrm{d}^2 r \mathrm{d}t = -\int\int \frac{\partial^2 C_{z,R,T}}{\partial \tau^2}(0, 0) \mathrm{d}^2 R \mathrm{d}T. \tag{17b}$$

From Eq. (16) we deduce that the evolution of the L^2 norm of the transverse gradient and the time derivative of the field are governed by

$$\int\int |\nabla_r A|^2 \mathrm{d}^2 r \mathrm{d}t = [1 + 4(3^{-2/p-1/q}) B_0^2(z)] \frac{4}{\rho_c^2} E_0, \tag{18a}$$

$$\int\int \left|\frac{\partial A}{\partial t}\right|^2 \mathrm{d}^2 r \mathrm{d}t = [1 + 4(3^{-2/p-1/q}) B_0^2(z)] \frac{2}{\tau_c^2} E_0, \tag{18b}$$

where $B_0(z) = B_0(z, 0, 0) = k_2 I_0 z/2$ is the mean cumulative B integral at the center of the beam. From the conservation of the Hamiltonian integral [Eq. (9)], the L^4 norm of the field can be computed, so we get that the contrast of the pulse defined by Eq. (6) is given by

$$c(z)^2 = 1 + 16(2/3)^{2/p+1/q} B_0(z)\left[\frac{z}{k_0\rho_c^2} + \frac{\sigma z}{\tau_c^2}\right]. \tag{19}$$

We conclude that the phase distribution is affected when $B_0(z)$ reaches values of order 1 but that the intensity distribution is affected by filamentation or GVD or both when the adimensional parameter $16B_0(z)\ [z/(k_0\rho_c^2) + \sigma z/\tau_c^2]$ reaches values of order 1. It is interesting to note that from the expansion of the autocorrelation function at order 0 with respect to the small parameters $z/(k_0\rho_c^2)$ and $\sigma z/\tau_c^2$ we have gotten the expansion at order 1 of the contrast.

3. NONLINEAR AMPLIFICATION

A. Formulation

We consider in this section the propagation of a spatially and temporally broadband incoherent pulse (carrier wave number k_0) in an amplifier. We assume that the amplifying medium corresponds to a homogeneously broadened two-level system embedded in a host medium. This host is characterized by a Kerr constant n_2, whereas the two-level system is characterized by a dephasing time T_2 and an excited-state lifetime that we assume to be infinite because it is much longer than the pulse duration. We take a pulse tuned to maximum amplification wavelength; electric field A and polarization P of the host material satisfy the coupled equations[15]

$$i\,\frac{\partial A}{\partial z} - \frac{1}{2k_0}\,\Delta_r A - \sigma\,\frac{\partial^2 A}{\partial t^2} - \frac{k_2}{2}\,|A|^2 A = \frac{P}{2}, \qquad (20a)$$

$$T_2\,\frac{\partial P}{\partial t} + P = i\gamma A, \qquad (20b)$$

where γ is the inverted population expressed in gain per length unit. In this section we remain with the case in which depletion of the stored energy can be neglected, so γ is kept constant. The effects of the gain saturation are studied in Section 4. We study the propagation of an incoherent beam whose macroscopic and microscopic descriptions were given in Subsection 2.A. We also study the evolution of the autocorrelation function, the contrast, and also the amplification of the energy of the pulse, defined as the square L^2 norm of the field [Eq. (7)].

B. Self-Phase Modulation That Is Due to Kerr Nonlinearity

Self-phase modulation gives rise to spectral broadening, as can be shown in the case when both diffraction and GVD are neglected. This phenomenon was considered in Ref. 5; as a consequence we merely state the main features. Assuming that the gain spectrum is flat ($T_2 = 0$), we have $P = i\gamma A$. The pulse is then amplified with the intensity small-signal gain γz and acquires a nonlinear phase term: $\phi_{\mathrm{NL}} = -k_2|A_0|^2(e^{\gamma z} - 1)/(2\gamma)$, giving rise to spectral broadening. We find a closed-form expression for the autocorrelation function:

$$C_{z,R,T}(\rho,\ \tau) = \frac{e^{\gamma z} C_{0,R,T}(\rho,\ \tau)}{\left\{1 + B_\gamma^2(z,\ R,\ T)\left[1 - \exp\left(-\frac{2\tau^2}{\tau_c^2} - \frac{2|\rho|^2}{\rho_c^2}\right)\right]\right\}^2}, \qquad (21)$$

which we recall is the Fourier transform of the spectrum, where $B_\gamma(z,\ R,\ T) = B_\gamma(z) I_0(R,\ T)/I_0$ and

$$B_\gamma(z) = \frac{k_2 I_0}{2}\,\frac{e^{\gamma z} - 1}{\gamma} \qquad (22)$$

is identified as the mean cumulative B integral at the center of the beam.

C. Interplay Between Amplification and Filamentation

From now on we take into account diffraction, time dispersion, and Kerr nonlinearity. We assume here a flat gain spectrum $T_2 = 0$, so scalar field A satisfies Eq. (20a) with $P = i\gamma A$. The total energy of the beam is therefore amplified with small-signal gain γz:

$$E(z) = E_0 e^{\gamma z}. \qquad (23)$$

To study the contrast we use the same approach as in Subsection 2.D. The expansion of the autocorrelation function with respect to the small parameters $z/(k_0\rho_c^2)$ and $\sigma z/\tau_c^2$ at the lowest order is given by Eq. (21). From Eqs. (21) and (17) we deduce that the L^2 norms of the derivatives of the field are given by

$$\iint |\nabla_r A|^2 \mathrm{d}^2 r\,\mathrm{d}t = [1 + 4(3^{-2/p-1/q}) B_\gamma^2(z)]\,\frac{4}{\rho r_c^2}\,E_0 e^{\gamma z}, \qquad (24a)$$

$$\iint \left|\frac{\partial A}{\partial t}\right|^2 \mathrm{d}^2 r\,\mathrm{d}t = [1 + 4(3^{-2/p-1/q}) B_\gamma^2(z)]\,\frac{2}{\tau_c^2}\,E_0 e^{\gamma z}. \qquad (24b)$$

The Hamiltonian integral H defined by Eq. (9) is no longer a conserved quantity but satisfies the evolution equation

$$\frac{\partial H}{\partial z} = -\frac{\gamma k_2}{4}\iint |A|^4 \mathrm{d}^2 r\,\mathrm{d}t + \gamma H.$$

It then appears that the L^4 norm to the power 4 of the field, which is expected to be amplified according to the rate $e^{2\gamma z}$, does not obey this ideal law. This dynamics is revealed by the expression of the contrast, which exhibits a corrective term that can be rewritten as a function of the mean cumulative B integral for high gain, $e^{\gamma z} \gg 1$:

$$c(z)^2 = 1 + 2\eta_1(z), \qquad (25)$$

$$\eta_1(z) = 16(2/3)^{2/p+1/q} B_\gamma(z)\left(\frac{1}{\gamma k_0 \rho_c^2} + \frac{\sigma}{\gamma \tau_c^2}\right). \qquad (26)$$

This expansion is valid when the corrective term is smaller than 1. The competition between filamentation and GVD is responsible for the perturbed term in Eq. (25). GVD in the case of a normal medium (i.e., $\sigma < 0$) counterbalances the filamentation of the beam if $k_0|\sigma|\rho_c^2 \geq \tau_c^2$. Under such conditions time spreading is faster than transverse spatial collapse, so the intensity at the top of the spots tends to decrease. As a numerical application, we may think of a broadband pulse with bandwidth (FWHM) 1.0 nm that corresponds to a coherence time $\tau_c = 1.5$ ps, which also presents microscopic spatial modulations of size $\rho_c = 5$–10 mm. Let us assume that this pulse propagates in an amplifier laser chain with gain $\gamma = 20\ \mathrm{cm}^{-1}$ and that it gives rise to a cumulative B integral of 1–4. It can then be verified that the corrective term $\eta_1(z)$ of Eq. (25) is negligible. However, with smaller speckle spots, larger cumulative B integrals, or both, filamentation and growth of the intensity fluctuations could become sizable.

D. Amplification with Gain-Narrowing Effects

In this subsection we take into account gain-narrowing effects, so the electric field and the polarization satisfy coupled equations (20a) and (20b), but we assume that the spectrum of the initial incoherent light is much narrower than the gain spectrum, which also reads as $T_2 \ll \tau_c$. Within this approximation, all the wavelengths contained in the spectrum of the incident pulse are expected to be amplified according to the optimal small-signal gain γz. In fact such is not always the case because of spectral broadening, as we find by an asymptotic analysis with respect to the small parameter T_2/τ_c. We find that the total field energy E at position z for important gain (i.e., $e^{\gamma z} \gg 1$) is given by

$$E(z) = E_0 e^{\gamma z}\{1 - 2(T_2/\tau_c)^2 \gamma z - 4(1/3)^{2/p+1/q}(T_2/\tau_c)^2 B_\gamma^2(z)[1 + \eta_2(z)]\}, \tag{27a}$$

$$\eta_2(z) = (4/3)^{2/p+1/q} B_\gamma(z)\left[\frac{72}{3k_0\gamma\rho_c^2} + \frac{176\sigma}{3\gamma\tau_c^2}\right]. \tag{27b}$$

The second term in braces in Eq. (27a) is the usual gain-narrowing effect; the third term in braces is the new corrective term that is specific to the mutual action of self-phase modulation and finite dephasing time. The explanation of this intensity loss saturation is the following: New wavelengths created by the self-phase-modulation mechanisms are scattered in the tails of the gain profile and therefore are less amplified than expected. This also implies that intensity saturation originates essentially from spectral broadening, the relevant parameter being the B-integral value given by Eq. (22). These statements were confirmed both theoretically and experimentally in Ref. 5, where the spatial fluctuations of the beam and the associated phenomena (diffraction, self-focusing, filamentation) were neglected. It was then found that the amplification was governed by Eq. (27a) with $\eta_2(z) \equiv 0$. When we take into account all phenomena, we find that correction $\eta_2(z)$ has the form of Eq. (27b). This correction is obtained by modifications of the microscopic statistical characteristics of the field in the transverse plane and over the time axis as a result of amplification. As Fig. 1 shows, the hot spots spatially collapse and temporally spread out, and this behavior affects the intensity fluctuations.

We can go deeper into the analysis. The striking observation is that the third coupled term in braces in Eq. (27a) is due to a nonlinear phenomenon and depends on the amplified intensity. As a consequence the hot spots are affected more by the corresponding phenomenon and consequently less amplified, as we show by studying the contrast of the amplified pulse:

$$c(z)^2 = 1 + 2\eta_1(z) - 32(1/2)^{2/p+1/q} B_\gamma^2(z)(T_2/\tau_c)^2. \tag{28}$$

It then appears that the mechanism behind the third term in braces in Eq. (27a) also gives rise to a reduction of the contrast. This nonlinear intensity saturation involves beam smoothing, which competes with filamentation.

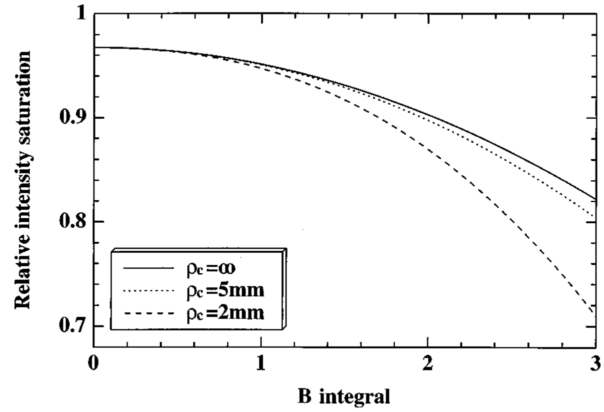

Fig. 1. Ratio of the output energy $E(z)$ (computed by inclusion of the gain-narrowing effects) over the expected optimal output $E_0 e^{\gamma z}$ as a function of the value of the B integral. The numerical values could correspond to the final section of a high-power Nd-doped glass laser. We have taken $\gamma = 20\ \text{cm}^{-1}$, $\gamma z = 4$, $\lambda_0 = 1.05\ \mu\text{m}$, and $T_2 = 100$ fs. The incident beam has a flat macroscopic profile (i.e., $p = q = \infty$), with coherence time $\tau_c = 1.6$ ps, which corresponds to a spectrum of full bandwidth extent (FWHM) 1.2 nm. We considered spatial modulations with radii of 1 cm, 5 mm, and 2 mm.

4. AMPLIFICATION WITH GAIN SATURATION

A. Main Equation

In this section we study the influence of the gain saturation that is due to the decay of the population inversion and the interplay with temporal and spatial incoherence. The gain medium is assumed to be homogeneously broadened. We choose to express the population inversion in gain per unit length, so it is governed by the equation

$$\frac{\partial\gamma}{\partial t} + \frac{\gamma}{T_\gamma} = \frac{i}{2f_s}(A^*P - AP^*), \tag{29}$$

where f_s is the saturation fluence, which depends on the gain medium and is proportional to the line-center frequency. T_γ is the decay time of the upper energy level, which is of the order of several hundred microseconds; it is therefore many orders of magnitude larger than the pulse duration, and the corresponding term γ/T_γ in Eq. (29) can be neglected. Assuming also for the sake of simplicity that the gain spectrum is flat, $T_2 = 0$, we can simplify Eq. (29), and its solution admits of a closed-form expression:

$$\gamma(z, r, t) = \gamma_0 \exp -\frac{1}{f_s}\left[\int_{-\infty}^{t} |A|^2(z, r, s)\mathrm{d}s\right]. \tag{30}$$

This solution is substituted into the evolution equation of the field, so we get

$$i\frac{\partial A}{\partial z} + \frac{1}{2k_0}\Delta_r A + \sigma\frac{\partial^2 A}{\partial t^2} + \frac{k_2}{2}|A|^2 A = \frac{i\gamma_0}{2}A \exp -\frac{1}{f_s}\left[\int_{-\infty}^{t}|A|^2(z, r, s)\mathrm{d}s\right]. \tag{31}$$

B. Enhancement of Gain Saturation by Incoherence

We denote by $E_s = \pi R_0^2 f_s$ the saturation energy of a cylindrical amplifier of radius R_0 per unit length. In the following paragraphs we compare gain saturation corresponding to pulses with different microscopic time–space modulations. All pulses have the same macroscopic temporal envelopes, which are uniform over the pulse duration T_0, and the same macroscopic spatial envelopes, which are assumed to be circular disks with radius R_0. The input energy is therefore $E_0 = \pi R_0^2 T_0 I_0$ for all the following configurations, where I_0 is the mean intensity inside the time–space envelope. In this simplified approach we neglect the Kerr nonlinearity so we can write simple closed-form expressions for the output energy. We study the correct nonlinear problem in Subsection 4.C.

1. *Uniform Intensity Profile*

Let us consider first a beam with duration T_0 whose spatial profile is a circular disk with radius R_0. The intensity profile is assumed to be uniform inside this envelope and to be equal to I_0. Solving Eq. (31), we find that the output energy is[20]

$$E_{\text{unif}}(z) = E_s \ln\{1 + [\exp(E_0/E_s) - 1]\exp(\gamma_0 z)\}, \tag{32}$$

which is plotted in Fig. 2. Gain saturation becomes noticeable when the ratio of the maximal energy output $E_0 \exp(\gamma_0 z)$ over E_s becomes of the order of 1. When this ratio is smaller than 1, Eq. (32) can be expanded as

$$E_{\text{unif}}(z) \simeq E_0 e^{\gamma_0 z}\left[1 - \frac{E_0 \exp(\gamma_0 z)}{2E_s}\right]. \tag{33}$$

2. *Speckle Pattern*

We consider here the case of a pulse that comprises n statistically independent speckle patterns with mean intensity I_0 that follow one another during pulse duration T_0. $n = 1$ $(n > 1)$ corresponds to a steady state in a time (time-incoherent) speckle pattern. The contrast of the time-integrated pattern is $c = 1/\sqrt{n}$; the output energy can be computed from Eq. (31):

$$E(z) = E_s \int_0^\infty \ln\left\{1 + \left[\exp\left(\frac{uE_0}{E_s}\right) - 1\right]\exp(\gamma_0 z)\right\} p_n(u)\mathrm{d}u, \tag{34}$$

where $p_n(u) = n^n u^{n-1} \exp(-u/n)/(n-1)!$ is the probability density of the normalized intensity of the time-integrated pattern. $E(z)$ is plotted in Fig. 2 and can be expanded for small gain saturation as

$$E(z) \simeq E_0 \exp(\gamma_0 z)\left[1 - (1 + c^2)\frac{E_0 \exp(\gamma_0 z)}{2E_s}\right]. \tag{35}$$

It then appears that energy loss first grows $1 + c^2 = 1 + 1/n$ times faster than in the case of a spatially uniform beam. In the regime that corresponds to high depletion, i.e., when the ratio $E_0 \exp(\gamma_0 z)/E_s$ is larger than 1, the difference between the output energy for a the time-integrated speckle pattern and that of a clean beam [expression (33)] with similar energy is roughly $(C + \ln n - \Sigma_{j=1}^{n-1} j^{-1})E_s$, where $C \simeq 0.577$ is Euler's constant. In the case of a static speckle pattern $(n = 1)$ this difference is reduced to CE_s. If the time-integrated contrast is 33% $(n = 10)$, then the asymptotic energy loss with respect to a clean beam is $\sim 0.051\, E_s$ (see Fig. 3).

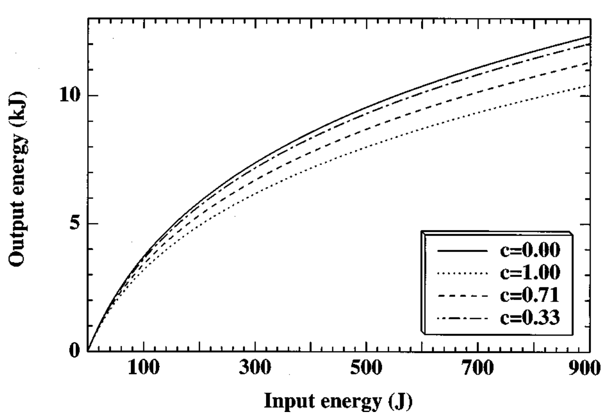

Fig. 2. Output energy as function of input energy for four input intensity profiles. The solid curve corresponds to a flat profile with contrast c as shown, and the dotted curve to a static speckle pattern. The two other curves represent the amplification of time-varying speckle patterns consisting of two independent speckle patterns (dashed curve) and ten independent speckle patterns (dotted–dashed curve). The small-signal gain is taken to be equal to $\gamma z = 4$, and the saturation energy to $E_s = 5000$ J.

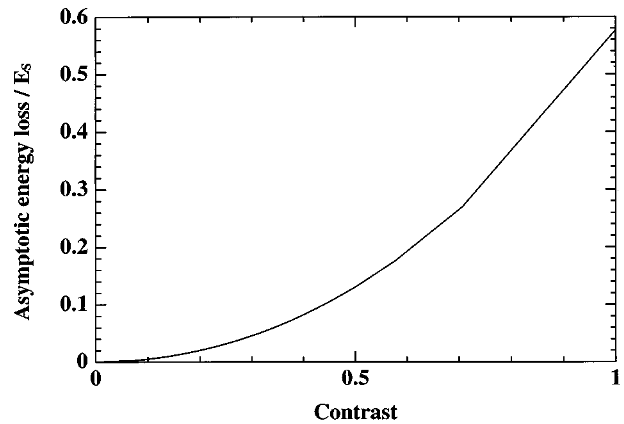

Fig. 3. Energy loss owing to the small-scale intensity fluctuations in the regime that correspond to high depletion as a function of the contrast of the time-integrated pattern. When the intensity profile is flat, the contrast is zero and the energy output is taken as a reference.

C. Amplification in Kerr Media with Gain Saturation

We now consider the amplification of a time–space incoherent pulse that satisfies Eq. (31). To show clearly the role of time incoherence, we do not assume in this subsection that the coherence time τ_c is much shorter than the duration T_0 of the pulse. More exactly, we assume that the incident pulse consists of the succession of N independent speckle patterns of elementary duration τ_c, with $N\tau_c = T_0$. The following results hold true for any integer N: Each speckle pattern obeys Gaussian statistics with the field correlation

$$\langle A_0(R - \rho/2)A_0^*(R + \rho/2)\rangle = I_0 \exp\left(-\frac{|R|^p}{R_0^p} - \frac{|\rho|^2}{\rho_c^2}\right). \tag{36}$$

We consider in what follows the case in which the gain saturation is not expected to be large, i.e., $E_0 \exp(\gamma_0 z) < E_s$. Neglecting GVD, we can then perform an asymptotic expansion with respect to the ratio $E_0 \exp(\gamma_0 z)/E_s$ and find that the output energy is given by

$$E(z) = E_0 \exp(\gamma_0 z) \times \left(1 - \frac{E_0 \exp(\gamma_0 z)}{2E_s}\left\{1 + \frac{\tau_c}{T_0}[1 + \eta_3(z)]\right\}\right), \tag{37a}$$

$$\eta_3(z) = \frac{32(2/3)^{2/p} B_{\gamma_0}(z)}{k_0 \gamma_0 \rho_c^2}. \tag{37b}$$

Regarding Eq. (37a), it appears that spatial incoherence enhances the effects of gain saturation by a factor of 2 in the case of a time-independent speckle pattern ($\tau_c = T_0$) or by even more than 2 if filamentation increases the contrast and the factor η_3. However, as is shown also by expression (35), time incoherence can cancel this enhancement. In the limit $\tau_c \ll T_0$ we simply find that the energy saturation is the same as for a uniform beam. Furthermore, the interplay among incoherence, gain saturation, and filamentation gives rise to a modification of the statistical distribution of the intensity fluctuations. We can expand the expression of the contrast of the amplified pulse with respect to the small parameter $E_0 \exp(\gamma_0 z)/E_s$ and find that

$$c(z)^2 = 1 + \eta_3(z) - (2/3)^{2/p}\frac{E_0 \exp(\gamma_0 z)}{2E_s} \times \left\{(9/8)^{2/p}\eta_3(z) + \frac{\tau_c}{T_0}[4 + 6(9/8)^{2/p}\eta_3(z)]\right\}, \tag{38}$$

where $\eta_3(z)$ is defined by Eq. (37b). This equation shows that gain saturation competes with filamentation and can succeed in reducing the contrast and smoothing the beam. Indeed, gain saturation is more important in the places where the amplifier corresponds to the hot spots of the beam. These hot spots are then less amplified. However, time incoherence can average the filling of the amplifier to a flat profile and partially cancel this effect (see Fig. 4).

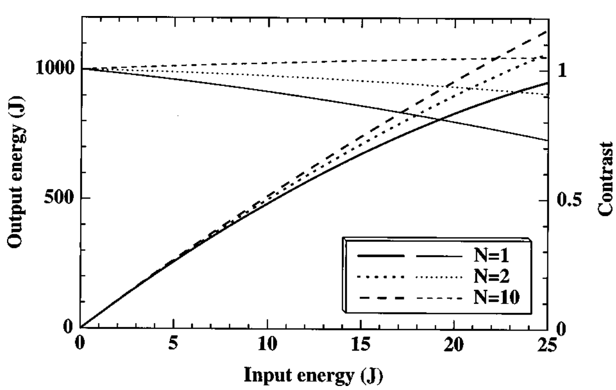

Fig. 4. Output energy and contrast as functions of input energy for several input pulses. The incident pulses consist of the succession of N independent speckle patterns. We assume that the spatial macroscopic envelope is uniform (i.e., $p = \infty$) and that $\eta_3(z) = 0.2$ for $E_0 = 25$ J. The small-signal gain is taken to be equal to $\gamma z = 4$, and the saturation energy to $E_s = 5000$ J.

5. CONCLUSION

We have reviewed most of the problems related to propagation and amplification of nonlinear and incoherent pulses. The general feature that appears throughout this paper is that the competition between nonlinearity and incoherence gives rise to an enhancement of the different saturation phenomena. This result is essentially due to the basic relation $\langle I^2\rangle = 2\langle I\rangle^2$, which holds true for pulses with Gaussian statistics. We then showed that many nonlinear mechanisms can involve an anomalous amplification reduction. Although the interplay and feedback effects are complicated, they exhibit general and common characteristics.

The nonlinear phenomena that are responsible for amplification saturation all involve a decrease of the contrast by reduction of the maximal intensities of the hot spots. As a consequence the microscopic fluctuations and the saturation associated with spatial incoherence decrease, whereas the induced nonlinear beam smoothing competes with filamentation.

Furthermore, the effects of space and time incoherence may accumulate or else cancel. We have seen that spatial incoherence enhances the intensity saturation involved by the interplay between spectral broadening that is due to self-phase modulation and gain-narrowing effects. However, we have also shown that time incoherence tends to lower and cancel the gain saturation that results from the spatial fluctuations. In this complicated picture the cumulative B integral seems to remain a key parameter. Indeed, the interplay among incoherence, nonlinearity, and gain narrowing effects, which appears to be one of the most limiting phenomena, actually holds when the cumulative B integral reaches values of the order of 1.

As a concluding remark, we underline that the maximal value of the intensity of the pulse and the role of the corresponding hot spot do not appear in the statistical approach. This is not surprising because the hottest spot actually carries a negligible part of the total energy. However, the maximal value of the intensity is worth considering because the hottest spot can involve damage to the optical components or can give rise to the growth of some laser-plasma instabilities. When a study of this effect is carried out and associated with the results deduced from the statistical study, it will provide an accurate description of relevant phenomena.

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