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Statistics of the hottest spot of speckle patterns generated by smoothing techniques

J. GARNIER

Centre de Mathématiques Appliquées, Centre National de la Recherche Scientifique, Unité Mixte de Recherche 7641, Ecole Polytechnique, 91128 Palaiseau Cedex, France

C. GOUÉDARD

Commissariat à l'Energie Atomique, Centre d'Etudes de Limeil-Valenton, 94195 Villeneuve St. Georges Cedex, France

and A. MIGUS

Laboratoire pour l'Utilisation des Lasers Intenses, Centre National de la Recherche Scientifique, Unité Mixte de Recherche 7605, Ecole Polytechnique, 91128 Palaiseau Cedex, France

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Abstract. This paper is concerned with the statistical distribution of the maximal hot spots of speckle patterns such as those generated by optical smoothing methods designed for inertial confinement fusion. It is proved that the maximal intensity at the first order is proportional to the logarithm of the ratio of the pulse volume over the mean hot spot volume. Nevertheless the complete description of the maximal intensity exhibits a quite important variance. Different ways for reducing either the maximal fluence or the maximal intensity are investigated, which are based upon time incoherence or polarization smoothing.

1. Introduction

Incoherent light has become a subject of great interest for many applications such as coherent spectroscopy [1], tomography in random media [2] or smoothing techniques for uniform irradiation in plasma physics [3]. This paper is a contribution to the study of optical smoothing for application to inertial confinement fusion (ICF), which requires a high level of irradiation uniformity for both direct and indirect drive. This criterion can be reached by implementing active smoothing methods, such as induced spatial incoherence (ISI) with echelons [4], smoothing by spectral dispersion (SSD) [5], smoothing by multi-mode optical fibre (SOF) [6], and polarization smoothing (PS) [7]. All these methods, except for the last, involve illuminating the target with an intensity which is a time varying speckle pattern, so that the time integrated intensity averages towards a flat profile.

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We aim, in this paper, to study the statistical distribution of the maximal intensity of a time–space incoherent pulse. This aspect of incoherent pulses was not addressed in our previous works [8, 9] which dealt with a statistical approach to the amplification of incoherent pulses. Indeed the energy contained in the hottest spot of a speckle pattern is very negligible compared with the total energy of the pulse, so that the role of the hottest spot was then found negligible in terms of the amplification efficiency. However it is relevant to estimate the intensity maxima in the focal spot because the hot spots play a primary role in that they can give rise to the growth of laser-plasma instability such as filamentation, stimulated Brillouin scattering, and stimulated Raman scattering [10]. This is especially the case for the most intense hot spot [11]. More important, some of the optical smoothing techniques involve intensity space–time modulations in the amplifiers and other optical components (SOF). Therefore the maximal value of the intensity in the laser chain is worth considering because the hottest spot is the one which will just induce damage to the optical components. This study is thus clearly triggered by ICF concerns. Nevertheless it should be mentioned that one can apply the following results to more general situations where the field obeys Gaussian statistics, which is the case as soon as it consists of the overlap of many independent modes by the Central Limit Theorem [12].

Studies exist in the literature about maxima of the electric field intensity of a laser speckle pattern. Weinrib and Halperin in [13] derive expressions for the densities of maxima, minima and saddle points of two-dimensional speckle patterns. Rose and DuBois in [14] consider the general statistics of hot spots of a speckle pattern in ICF configurations. All previous work are concerned with the statistical properties of the *local* maxima of the intensity distribution. Our work is original in that we focus on the precise study of the complete statistical distribution of the maximal hot spot, i.e. the *global* maximum of the intensity (or fluence), which is a critical point in terms of laser-plasma instability in the focal spot and in terms of damage to the optical components of the laser chain. We shall assume in this paper that the space–time modulations of the pulse obey Gaussian statistics. In particular the instantaneous spatial intensity distribution of such a pulse is a speckle pattern [12]. These statistical characteristics are actually very usual, and correspond to the pulses generated by the smoothing techniques that we quote here [8, 14].

In section 2 the maximal intensity and fluence distribution of a spatially and temporally incoherent pulse is studied in some reference plane, while section 3 is devoted to the maximal intensity distribution of a monochromatic and spatially incoherent pulse. In section 4 we address the particular case of polarization smoothing, which differs from the other methods in that it instantaneously smoothes the hot spots. Finally, section 5 is devoted to the maximal intensity of time-dependent and time-independent propagating speckle patterns in three spatial dimensions.

2. Statistics of the maximal intensity of broadband pulses

2.1. *Statistics of partially coherent pulses*

We consider an electromagnetic field A whose temporal and spatial slowly varying envelopes are deterministic, with spatial radius R_0 (the so-called beam radius) and temporal duration T_0 (the so-called pulse duration). These scales will be referred to in the following as the macroscopic scales. In the experimental conditions corresponding to high power laser chains such as the French Laser MegaJoule [15] or the US National Ignition Facility [16], R_0 is of the order of 20 cm in the laser chain, and 250 μm in the focal spot. T_0 is of the order of several nanoseconds. The incident pulse also has fast varying random fluctuations, with characteristic scales ρ_c (the so-called correlation radius) in the spatial domain and τ_c (the so-called coherence time) in the temporal domain. These scales will be referred to in the following as the microscopic scales. In order to fix the sizes of orders, we may assume that the value of ρ_c is of the order of 2–20 mm in the laser chain, and 2–20 μm in the focal spot. The coherence time τ_c is of the order of 1- 10 ps. These random fluctuations are assumed to obey Gaussian statistics. Their distributions are then characterized by the autocorrelation function defined by:

$$C_{R,T}(\rho,\tau) = \langle A(r_1,t_1)A^*(r_2,t_2), \tag{1}$$

with $R=(r_1+r_2)/2$, $T=(t_1+t_2)/2$, $\rho=r_1-r_2$, and $\tau=t_1-t_2$. Throughout this paper we consider an incoherent pulse with mean intensity I_0 whose macroscopic envelope is flat, with beam radius R_0 and pulse duration T_0. Its volume is therefore

$$V_0 := \pi R_0^2 T_0$$

and the autocorrelation function reads as:

$$C_{R,T}(\rho,\tau) = I_0 f\left(\frac{\rho}{\rho_c},\frac{\tau}{\tau_c}\right) \tag{2}$$

for $|R|\le R_0$, $|T|\le T_0/2$, where $f(\tilde\rho,\tilde\tau)$ is the normalized shape of C which is assumed to fulfil some mathematical technical conditions.

(1) $(\tilde\rho,\tilde\tau)\mapsto f(\tilde\rho,\tilde\tau)$ is 4 times differentiable.
(2) There exist positive constants c_1, c_2, δ such that for any $|\tilde\rho|^2+\tilde\tau^2<\delta$:

$$c_1(|\tilde\rho|^2+\tilde\tau^2) \le 1-f(\tilde\rho,\tilde\tau) \le c_2(|\tilde\rho|^2+\tilde\tau^2).$$

(3) The correlation function decays fast enough so that:

$$\int_{\mathbb{R}^3} |f(\tilde\rho,\tilde\tau)|^2\,\mathrm{d}^2\tilde\rho\,\mathrm{d}\tilde\tau < \infty,$$

where $\mathbb{R}^3$ is the real space of dimension 3.

These three conditions give some requirements of regularity and integrability for the normalized correlation function f, so that we will be allowed to apply results which can be found in the mathematical literature. In the numerical applications, we will take f to have Gaussian shape $f(\tilde\rho,\tilde\tau)=\exp(-|\tilde\rho|^2-\tilde\tau^2)$. In the following, V_c will be referred to as the volume of a hot spot:

$$V_c := \pi\rho_c^2\tau_c.$$

The scalar field can be expressed in terms of its real and imaginary parts: $A(r,t)=A_R(r,t)+iA_I(r,t)$. A_R and A_I are independent and identically distributed zero-mean real Gaussian processes, with autocorrelation function:

$$\langle A_R(r,t)A_R(r',t')\rangle = \tfrac{1}{2}C(r-r',t-t'). \tag{3}$$

The intensity obeys an exponential distribution with mean intensity I_0, which readily implies that the fraction area of the beam contained in regions where the intensity is higher than αI_0 is $\exp(-\alpha)$. However this elementary result does not take into account the fact that there is only a finite number of hot spots in the pattern. Consequently it does not indicate the right way to determine the statistics of the maximal hot spot, which obviously should depend on the ratio V_0/V_c. Indeed the maximal intensity is expected to be all the larger since there are many hot spots.

It is also necessary to describe what are the underlying meanings of the statistical expectation $\langle . \rangle$ and probability $\mathbb{P}(.)$ in our framework. Indeed one speckle pattern is associated with one value of the maximal intensity. This value depends on the realization of the speckle pattern, which, in the context of a high power laser chain, means that a different speckle pattern is observed at each laser shot. Therefore, we cannot use any ergodicity principle which means that the statistical distribution we speak of throughout the paper is the distribution with respect to the possible realizations of the speckle patterns.

2.2. *Maximum of the instantaneous intensity distribution*

Here we deal with the statistics of the maximal instantaneous intensity of the time–space incoherent pulse. This study is based upon the following two statements.

(1) Let us consider the local maxima in the space–time domain of the real-valued field A_R^2 and denote by $I_{\max,1}^{A_R} \geq \cdots \geq I_{\max,n}^{A_R}$ the values of the n hottest local maxima of A_R^2. In the asymptotic framework $V_c \ll V_0$, the statistical distribution of $I_{\max,n}^{A_R}$ can be expressed in terms of independent random variables Z_j, $j = 1, \ldots, n$ with exponential densities $p(z) = \exp(-z)\mathbb{1}_{z\geq 0}$ and means 1:

$$I_{\max,n}^{A_R} = I_0(\Gamma_V - \ln(Z_1 + \cdots + Z_n)), \tag{4}$$

$$\Gamma_V = \ln\left(\frac{D_V V_0}{\pi V_c}\right) + \ln_2\left(\frac{D_V V_0}{\pi V_c}\right), \tag{5}$$

where $\ln_2(x) := \ln(\ln(x))$ and D_V is the squared root of the determinant of the 3×3 matrix Λ^V of the second derivatives of f:

$$\Lambda_{i,j}^V = -\left.\frac{\partial^2 f}{\partial x_i \partial x_j}\right|_{\mathbf{x}=0}, \quad \text{where } \mathbf{x} = (\tilde{\rho}, \tilde{\tau}).$$

If C has Gaussian shape, then we have $D_V = 2^{3/2}$.

(2) The most intense local maximum of the intensity distribution corresponds to a strong local maximum of A_R (or A_I), the contribution of A_I (or A_R) being much smaller, following a Gaussian random variable with variance $I_0/2$.

These statements are proved in appendices A and B respectively. Since the real and imaginary parts of the field are identically distributed, the first point also holds true for the values of the n hottest local maxima $I_{\max,1}^{A_I} \geq \cdots \geq I_{\max,n}^{A_I}$ of the square of the imaginary part A_I^2 of the field. The second point actually proposes an ansatz to deduce the statistical distribution of the maximal value of the intensity $A_R^2 + A_I^2$ from that of the real part A_R. Indeed $I_{\max} := \max_{V_0} |A(r,t)|^2$ will be smaller than

u^2 if and only if $I^{A_R}_{\max,n} + A^2_{I,n}$ are smaller than u^2 for every n, and $I^{A_I}_{\max,n} + A^2_{R,n}$ are smaller than u^2 for every n, where $A_{R,n}$ and $A_{I,n}$ are independent, real-valued, zero-mean, Gaussian random variables with variance $I_0/2$. Since the fields A_R and A_I obey the same distribution and are independent, this reads as:

$$\mathbb{P}(I_{\max} \leq I) = \mathbb{P}\left(\forall n,\, A^2_{I,n} + I^{A_R}_{\max,n} \leq I\right)^2. \tag{6}$$

Substituting equation (4) into equation (6) then yields after some algebra that the maximal intensity $I_{\max}$ can be expressed as the sum of a deterministic term $I_0 \Gamma_V$ of order $I_0 \ln(V_0/V_c)$ and of a random term $-I_0 \ln Z$ of order I_0:

$$I_{\max} = I_0(\Gamma_V - \ln Z). \tag{7}$$

The random variable Z is independent of V_0 and V_c. It is very difficult and in fact not necessary to solve the infinite-dimensional system which would provide the exact expression of the distribution of Z, but would require one to take into account the joint statistical distribution of all local maxima of A_R and A_I. By considering only the n_0 hottest spots of A_R and A_I, we can substitute for equation (6) the following relation:

$$\mathbb{P}(I_{\max} \leq I) \simeq \mathbb{P}\left(\forall n \leq n_0, |A_{I,n}|^2 + I^{A_R}_{\max,n} \leq I\right)^2.$$

We adopt the choice $n_0 = 2$, which is a compromise between the accuracy (which increases with n_0) and the simplicity (which decreases with n_0) of the corresponding expression of the statistical distribution of Z. We then find that the so-called repartition function of Z can be expressed to a good approximation by ($z \geq 0$):

$$\mathbb{P}(Z \geq z) = \left[\frac{4}{\pi^{1/2}} \int_0^\infty \exp\left(-z \exp(x^2) - x^2\right)\left(\operatorname{erf}(x) + z \int_0^x \operatorname{erf}(y) \exp(y^2)\, y\, \mathrm{d}y\right) \mathrm{d}x\right]^2, \tag{8}$$

where $\operatorname{erf}(x) = 2/\pi^{1/2} \int_0^x \exp(-a^2)\, \mathrm{d}a$. In particular, the mean value of the random part of $I_{\max}$ is $-\langle \ln Z \rangle \simeq 1.92$, and its variance is $\langle (\ln Z - \langle \ln Z \rangle)^2 \rangle \simeq 1.99$. The tails of the random variable $-\ln Z$ can be estimated from (8). The left tail, corresponding to the probabilities of exceptional low values, decays very fast:

$$\mathbb{P}(-\ln Z \leq -I) \simeq \exp\left(-I - 2\exp(I)\right) \text{ for } I \gg 1,$$

while the right tail, which corresponds to exceptional high values, decays as:

$$\mathbb{P}(-\ln Z \geq I) \simeq \frac{4}{\pi^{1/2}} I^{1/2} \exp(-I) \text{ for } I \gg 1.$$

The mean value of the maximal intensity is plotted in figure 1 for different microscopic volumes. It appears that the maximal intensity depends on the coherence time in that it increases when the coherence time decreases and it can reach very high values. However the maximal energy contained in each microscopic cell decreases due to the small coherence time, so that the maximal fluence is actually low for small coherence times (see also section 2.3).

It should be also mentioned that one of the most discussed methods to generate the space–time incoherent pulses which are designed for ICF applications is SSD using one sinusoidal modulator [5]. In such conditions, the field in the focal plane can be efficiently approximated by a pulse with Gaussian statistics only over a time interval whose duration is equal to the modulation period.

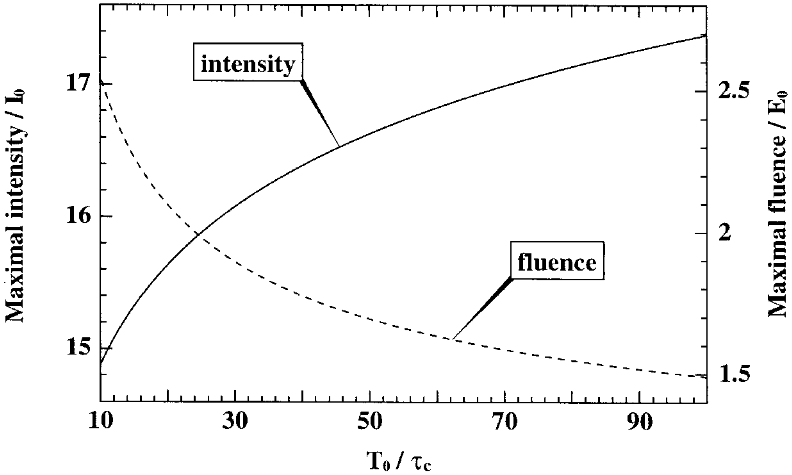

Figure 1. Theoretical mean values of the maximal intensity (solid line) and of the maximal fluence (dashed lines) of a pulse with Gaussian correlation function, beam radius $R_0 = 20\,\text{cm}$, and correlation radius $\rho_c = 3\,\text{mm}$ (these values are equivalent to $R_0 = 250\,\mu\text{m}$ and $\rho_c = 2.4\,\mu\text{m}$). The maximal intensity and fluence are plotted versus the ratio T_0/τ_c.

Therefore T_0 in all the results of this paper should then be understood as the modulation period. Note also that other methods such as SOF or the use of at least two sinusoidal modulators with incommensurate frequencies break this periodic effect.

2.3. *Statistics of the maximal fluence*

We still consider a time-incoherent speckle pattern and look for the statistics of the maximal hot spot of the *time-integrated* pattern. In other words we aim at exhibiting the effect of temporal smoothing on the maximal hot spot statistics. We still develop our study in the asymptotic framework $\rho_c \ll R_0$ and $\tau_c \ll T_0$. In such conditions, the contrast defined as the ratio of the normalized variance of the fluence over the mean fluence is:

$$c^2 = \frac{\tau_c}{T_0} F(0, \infty), \tag{9}$$

$$F(\tilde{\rho}, \tilde{t}) = \int_{-\tilde{t}/2}^{\tilde{t}/2} |f(\tilde{\rho}, \tilde{\tau})|^2 \, d\tilde{\tau}, \tag{10}$$

which means that the time-integrated pattern consists of the overlap of $N = c^{-2}$ statistically independent speckle patterns. In particular, if f has Gaussian shape, then $F(0, \infty) = (\pi/2)^{1/2}$. More precisely, we get by applying the central limit theorem that the fluence $E(r)$ can be expressed as:

$$E(r) = E_0(1 + ce(r)),$$

where $E_0 = I_0 T_0$ is the mean fluence of the time-integrated pattern, and $e(r)$ is a real zero-mean Gaussian field whose covariance is equal to:

$$\left\langle e\left(r+\frac{\rho}{2}\right)e\left(r-\frac{\rho}{2}\right)\right\rangle = \frac{F(\rho/\rho_c, \infty)}{F(0, \infty)}.$$

The maximal fluence statistics is then straightforward to establish by using the same approach as in appendix A which deals with the real part of the field. We find that the maximal fluence $E_{max} := \max_{S_0} E(r)$ can be represented as:

$$E_{max} = E_0 + E_0 c\Gamma_F^{1/2}\left(1 - \frac{\ln Z_1}{\Gamma_F}\right), \tag{11}$$

$$\Gamma_F = 2\ln\left(\frac{D_F}{2(2\pi)^{1/2}}\frac{S_0}{S_c}\right) + \ln_2\left(\frac{D_F}{2(2\pi)^{1/2}}\frac{S_0}{S_c}\right) + \ln 2, \tag{12}$$

where Z_1 is a random variable with an exponential probability density and mean 1 and D_F is the squared root of the determinant of the 2×2 matrix Λ^F of the second derivatives of $F(\tilde{\rho}, \infty)/F(0, \infty)$. It thus appears that the maximal fluence is rather close to the mean fluence due to time incoherence, and that the relative difference is of the order of $c\Gamma_F^{1/2}$ (see figure 1).

2.4. *Influence of the integration time*

Section 2.2 deals with the maximal instantaneous intensity of a time–space incoherent pulse, while section 2.3 is devoted to the maximal fluence, which is the intensity time integrated over the whole pulse duration. These configurations correspond to limit cases of a more general problem which takes into account the influence of the integration time t_i which is the time duration with respect to which the intensity is integrated, so that the relevant field intensity is:

$$I^{t_i}(r, t) := \frac{1}{t_i}\int_{t-t_i/2}^{t+t_i/2} |A|^2(r, s)\, \mathrm{d}s.$$

We aim at exhibiting the maximal value of the intensity I^{t_i} as a function of t_i. Note that in the limit $t_i \ll \tau_c$, we get back the instantaneous intensity, while if $t_i \geq T_0$ then I^{t_i} is simply proportional to the fluence of the field. We shall show that there exists three regimes. For small t_i ($t_i \leq T_1$), the distribution of the time-integrated intensity is close to the distribution of the instantaneous intensity up to a multiplicative corrective term. For long t_i ($t_i \geq T_2$), it is close to the fluence distribution up to another corrective term. Finally there exists between these two regimes an intermediate regime ($T_1 < t_i < T_2$).

2.4.1. *Short integration time*

Part of the following results will be based upon the statement due to Adler [17, Lemma 6-7-3] and rewritten in [14]: The field around a local maximum (r_0, t_0) is non-random and obeys:

$$A(r, t) \simeq A(r_0, t_0) f\left(\frac{r - r_0}{\rho_c}, \frac{t - t_0}{\tau_c}\right). \tag{13}$$

In particular, in the neighbourhood of the global maximum of the instantaneous intensity distribution whose position is denoted by (r_0, t_0), the intensity is:

$$I(r_0, t) \simeq I_{max} \times |f(0, (t - t_0)/\tau_c)|^2.$$

This statement holds true whenever the shape $I_{\max}|f(0,(t-t_0)/\tau_c)|^2$ prevails over the background of order I_0, which means for any $|t-t_0| \le T_1/2$, where T_1 is such that†:

$$|f(0, T_1/(2\tau_c))|^2 = I_0/I_{\max} . \tag{14}$$

In particular, in the case of a Gaussian correlation function, we have $T_1 \simeq \tau_c(2\ln \Gamma_V)^{1/2}$. If t_i is shorter than T_1, then the time integrated intensity at the point (t_0, r_0) is $I^{t_i}(t_0, y_0) = I_{\max}F(0, t_i/\tau_c)/(t_i/\tau_c)$ and $I^{t_i}_{\max}$ corresponds to this point. We can then state the following statement.

If $t_i \le T_1$:

$$I^{t_i}_{\max} = I_0(\Gamma_V - \ln Z)\frac{F(0, t_i/\tau_c)}{t_i/\tau_c}, \tag{15}$$

where Z is the random variable characterized by equation (8).

2.4.2. *Moderate integration time*

If t_i is longer than T_1, then the value of $I^{t_i}(r_0, t_0)$ is the sum (divided by t_i) of the maximal instantaneous hot spot time integrated over $[-T_1/2, T_1/2]$ and of the intensity background with mean I_0 time integrated over $[-t_i/2, -T_1/2] \cup [T_1/2, t_i/2]$:

If $T_1 \le t_i \le T_2$:

$$I^{t_i}_{\max} = I_0\left(1 + \frac{\tau_c}{t_i}F(0,\infty)(\Gamma_V - \ln Z) - \frac{T_1}{t_i}\right). \tag{16}$$

Note that, when t_i becomes longer and longer, the contribution of the maximal instantaneous hot spot becomes smaller and smaller. At some critical time T_2, the maximal time-integrated intensity does not correspond anymore to the buried maximal instantaneous hot spot (r_0, t_0), but will switch to some other local maximum of the time-integrated intensity. This will induce the third regime examined in the next paragraph.

2.4.3. *Long integration time*

After T_2, the time-integrated intensity distribution consists of a sequence of T_0/t_i independent patterns with contrast:

$$c(t_i)^2 = F(0,\infty)\frac{\tau_c}{t_i}.$$

The intensity distribution of the jth pattern can be written as $I_0(1 + c(t_i)e_j(r))$, where e_j has Gaussian statistics with correlation function $F(\rho/\rho_c, \infty)/F(0,\infty)$. The maximal intensity of the jth pattern can be estimated in the same way as the maximal fluence in section 2.3: $I_0(1 + c(t_i)\Gamma_F^{1/2}(1 - \ln Z_j/\Gamma_F)$, where the Z_j's are independent random variables with exponential densities and means 1. The maximal intensity of the integrated pulse is the maximum of these values. Computing ($N = T_0/t_i$):

† For general configurations f may not be monotonically decreasing. A rigorous definition of T_1 is therefore the infimum of the positive times which satisfy equation (14).

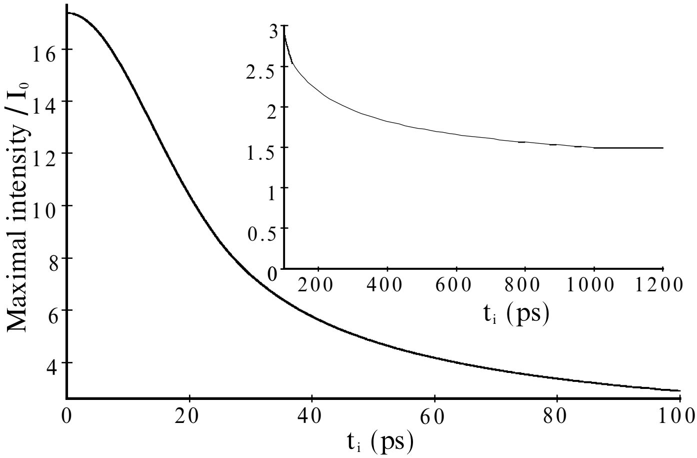

Figure 2. Theoretical mean value of the maximal intensity versus the integration time. We have assumed that the correlation function has Gaussian shape and we have adopted the values $R_0 = 20$ cm, $\rho_c = 3$ mm, $T_0 = 1$ ns and $\tau_c = 10$ ps. In such conditions $T_1 \simeq 24$ ps and $T_2 \simeq 150$ ps. In the main figure the integration time goes from 0 to 100 ps, and in the insert it goes from 100 ps to 1.2 ns.

$$\mathbb{P}\left(\max_{j=1,\dots,N}(-\ln Z_j) \leq a\right) = \exp(-N\exp(-a)),$$

which is equal to $\exp(-z)$ if $a = \ln N - \ln z$, we get that:

If $T_2 \leq t_i \leq T_0$:

$$I_{\max}^{t_i} = I_0\left(1 + c(t_i)\Gamma_F^{1/2}\left(1 + \frac{\ln(T_0/t_i)}{\Gamma_F} - \frac{\ln \bar{Z}_1}{\Gamma_F}\right)\right), \quad (17)$$

where $\bar{Z}_1$ is a random variable with exponential density and mean 1.

The transition between the second and the third regimes occurs at some time T_2 which is random, but can be estimated by comparing the typical values given by formulae (16) and (17):

$$\langle T_2 \rangle \simeq \Gamma_V \tau_c.$$

Note also that, if $t_i \geq T_0$, then we have $I_{\max}^{t_i} = I_{\max}^{T_0}$.

Figure 2 plots the expected maximal intensity of a time–space incoherent pulse with the Gaussian correlation function in terms of the integration time t_i. It gives an illustration of the different regimes pointed out above when t_i goes from 0 to T_0. One can first observe the rapid decrease (15), whose rate depends on the precise shape of the correlation function. The second and third regimes depend on the correlation function only through the parameters Γ_V, Γ_F and $F(0,\infty)$. The regime (16) has a decay rate τ_c/t_i, while (17) has a slower rate of about $(\tau_c/t_i)^{1/2}$.

3. Statistics of the maximal intensity of monochromatic pulses

Let us now consider the case when the pulse is steady state in time. This configuration is simpler than the one considered in section 2 in that the instantaneous intensity distribution coincides with the time-integrated intensity distribution. If we denote by $S_0 = \pi R_0^2$ the area of the beam and by $S_c = \pi\rho_c^2$ the area of

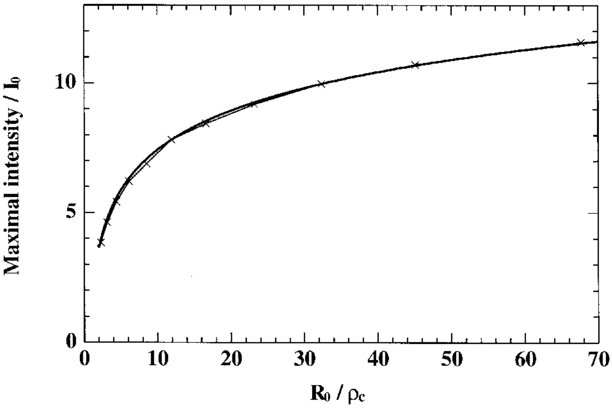

Figure 3. Mean value of the maximal intensity of the pulse expressed in multiples of the mean intensity I_0. The pulse has a flat macroscopic envelope with beam radius R_0 and is steady state in time. Its correlation function has Gaussian shape. The thick line corresponds to the theoretical values. The crosses represent the mean values of the maximal intensity averaged over 100 realizations for 10 different values of the correlation radius ρ_c.

a speckle spot, then by the very same method as in subsection 2.2 we find that for $S_0 \gg S_c$ the maximal intensity $I_{max} := \max_{S_0} |A(r)|^2$ of a two-dimensional speckle pattern is governed by:

$$I_{max} = I_0(r_S - \ln Z), \tag{18}$$

$$r_S = \ln\left(\frac{D_S}{\pi^{1/2}}\frac{S_0}{S_c}\right) + \tfrac{1}{2}\ln_2\left(\frac{D_S}{\pi^{1/2}}\frac{S_0}{S_c}\right), \tag{19}$$

where D_S is the squared root of the determinant of the 2×2 matrix Λ^S of the second derivatives of f: $\Lambda^S_{i,j} = -\partial^2 f / \partial\tilde{\rho}_i \partial\tilde{\rho}_j(0)$. In particular, if f has Gaussian shape, then $D_S = 2$. The statistical distribution of Z is described by equation (8).

In order to test the asymptotic formulae (7) and (18) we have carried out numerical simulations in the two-dimensional case (i. e. the time-independent speckle pattern). The routine to simulate a Gaussian field is given in the appendix C. Figure 3 compares the theoretically and numerically determined mean values of the maximal intensities of a two-dimensional speckle pattern with Gaussian statistics and the Gaussian correlation function. It shows good agreement between theory and simulations for large values of $S_0/S_c = R_0^2/\rho_c^2$, but also for rather small values of this ratio, which was unexpected. It appears that we can have a maximal hot spot with an intensity 10 times larger than the mean intensity for $R_0 \geq 32\rho_c$. In figure 4 we compare the probability density functions of the maximal intensity for fixed values of R_0 and ρ_c. It demonstrates that the above theory efficiently predicts not only the mean value, but also the whole statistical distribution of the maximal intensity of a speckle pattern.

4. Orthogonally polarized fields

Polarization smoothing (PS) consists of overlapping two orthogonally polarized and statistically independent speckle patterns A_1 and A_2, so that it instantaneously

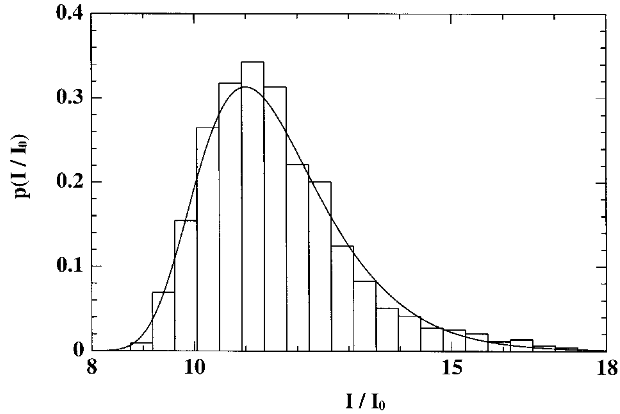

Figure 4. Theoretical probability density of the maximal intensity of a two-dimensional steady state for a time speckle pattern (solid line). The correlation function is assumed to have Gaussian shape. The histogram has been numerically determined by computing 2000 different realizations. We have adopted the values $R_0 = 20\,\mathrm{cm}$ and $\rho_c = 3\,\mathrm{mm}$.

smoothes the spatial beam structure [7]. The intensity statistics $|A|^2 = A_{1,R}^2 + A_{1,I}^2 + A_{2,R}^2 + A_{2,I}^2$ obeys the so-called chi-square distribution with four degrees of freedom and mean intensity I_0, whose probability density is $p(I) = 4I/I_0^2 \exp(-2I/I_0)$. The contrast of the instantaneous pattern is then $c = 1/2^{1/2}$. With very similar arguments as in section 2 and the corresponding ansatz '*the most intense local maximum of the intensity distribution corresponds to a strong local maximum of $A_{1,R}$ (or $A_{1,I}$, or $A_{2,R}$, or $A_{2,I}$), the contributions of the three other real fields being much smaller, following independent Gaussian random variables with variance $I_0/4$*', we find that the maximal intensity $I_{\max} := \max_{V_0} |A|^2(r,t)$ of a broadband orthogonally polarized field with coherence time τ_c and correlation radius ρ_c obeys the following distribution:

$$I_{\max} = \frac{I_0}{2}\left(r_V - \ln Z_{ps}\right), \tag{20}$$

where Z_{ps} is a random variable independent of V_0 and $V_c = \pi\rho_c^2\tau_c$ and r_V is given by (5). With the same conditions the maximal intensity $I_{\max} := \max_{S_0} |A|^2(r)$ of a monochromatic ($\tau_c = \infty$) orthogonally polarized field with correlation radius ρ_c is:

$$I_{\max} = \frac{I_0}{2}\left(r_S - \ln Z_{ps}\right), \tag{21}$$

where $S_c = \pi\rho_c^2$ and r_S is given by (19). The repartition function of the random variable Z_{ps} can be expressed to a good approximation by ($z \geq 0$):

$$\mathbb{P}(Z_{ps} \geq z) = \left(\frac{4}{\pi^{1/2}}\int_0^{\infty} \exp\left(-z\exp(x^2) - x^2\right) x^2\,\mathrm{d}x\right)^4. \tag{22}$$

In particular, the mean value of the random part of $I_{\max}$ is $-\langle \ln Z_{ps}\rangle \simeq 3.96$, and its variance is $\langle (\ln Z_{ps} - \langle \ln Z_{ps}\rangle)^2\rangle \simeq 1.85$. The left tail is approximately:

$$\mathbb{P}(-\ln Z_{ps} \leq -I) \simeq \exp\left(-6I - 4\exp(I)\right) \text{ for } I \gg 1,$$

while the right tail decays as:

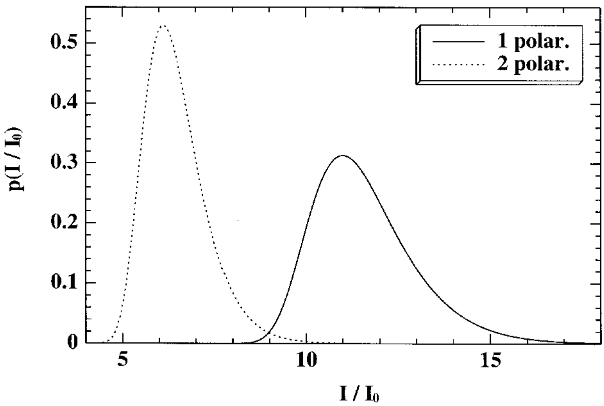

Figure 5. Comparison of the theoretical probability densities of the maximal intensities of two-dimensional steady state in time patterns. The solid line represents a linearly polarized speckle pattern and the dashed line stands for the overlap of two orthogonally polarized and statistically independent speckle patterns with the same statistics and Gaussian correlation function. We have adopted the values $R_0 = 20\,\mathrm{cm}$ and $\rho_c = 3\,\mathrm{mm}$.

$$\mathbb{P}\left(-\ln Z_{ps} \geq I\right) \simeq \frac{16}{3\pi^{1/2}} I^{3/2} \exp\left(-I\right) \text{ for } I \gg 1.$$

It thus appears that the intensity of the maximal hot spot is much lower in the PS configuration than in the standard configuration by a factor close to 2, as demonstrated by comparing equation (7) with equation (20) (broadband pulses) and equation (18) with equation (21) (monochromatic pulses). This reduction is also shown by figure 5. Furthermore, if we combine temporal smoothing and polarization smoothing, then both effects cumulate so that the maximal fluence is given by equation (11) with a further reduction by a factor $1/2^{1/2}$ of the second term in the right-hand member.

5. 3D speckle pattern and propagation

In the above sections we have studied the statistics of the hot spots of time-dependent or time-independent speckle patterns in some reference plane. These results will be accurate if we can neglect in practice the modifications of the hot spots along the propagation axis (say z). The Rayleigh distance of a speckle spot is

$$z_c := \frac{k_0 \rho_c^2}{2},$$

where k_0 is the central wavenumber of the pulse. We shall take into account z-propagation effects if z_c is smaller than the typical depth of the considered system. On the one hand, z_c is of the order of several tens of metres in the laser chain, where $\rho_c \geq 2\,\mathrm{mm}$. This distance is much larger than the width of any optical component of the chain, amplifier, mirror, and so on. Consequently it is consistent to study the maxima in some plane and to neglect z-propagation effects. On the other hand, in the focal plane the correlation radius ρ_c is of the order of a few microns, and the corresponding Rayleigh distance z_c is then of the order of some tens of microns. In the indirect drive scheme, for which the French Laser MegaJoule [15] and the US National Ignition Facility [16] are designed, the pulse propagates in a hohlraum from the window to the inner gold wall through an underdense plasma. z_c is consequently much smaller than the typical size of the hohlraum (a few hundreds of microns), so a full four-dimensional model $xyzt$ has to be taken into account. Nevertheless the choice of the propagation model in z is difficult, because a lot of phenomena are expected to play some role, especially stimulated Brillouin and Raman scattering. In this study we shall consider a very simple free propagation model and assume that the equation which governs the propagation of field A is linear:

$$\mathrm{i}\frac{\partial A}{\partial z} + \frac{1}{2k_0}\Delta_r A = 0. \tag{23}$$

This model is surely valid while the plasma density fluctuations can be neglected. The field $A(x, y, z, t)$ then has Gaussian statistics with zero mean and a generalized correlation function (GCF):

$$C_{R,T,Z}(\rho, \tau, \zeta) = \langle A(r_1, t_1, z_1)A^*(r_2, t_2, z_2)\rangle, \tag{24}$$

with $R = (r_1 + r_2)/2$, $T = (t_1 + t_2)/2$, $Z = (z_1 + z_2)/2$ $\rho = r_1 - r_2$, $\tau = t_1 - t_2$, and $\zeta = z_1 - z_2$. We also assume that the z length of the domain is smaller than the Rayleigh distance of the envelope of the field (which is roughly $k_0\rho_c R_0$), so that we can neglect the Z dependence of the GCF. This hypothesis is fulfilled in the experimental conditions corresponding to ICF configurations. For $\zeta = 0$, the GCF coincides with the correlation function (2). From equation (23) we can compute the partial derivatives of the generalized correlation function:

$$\frac{\partial C}{\partial \zeta} = \frac{\mathrm{i}}{2k_0}\Delta_\rho C$$

and

$$\frac{\partial^2 C}{\partial \zeta^2} = -\frac{1}{4k_0^2}\Delta_\rho\Delta_\rho C.$$

We can use the same theory as in the previous sections to derive the statistical properties of the hot spots of the field A. Nevertheless we must take care to satisfy the technical conditions under which we can apply the mathematical results for the maxima of the real and imaginary parts of the field. This causes one to regard instead of A the field $\tilde{A}(x, y, z, t) := \exp(-\mathrm{i}\alpha z)A(x, y, z, t)$, where $\alpha := \Delta_{\tilde{\rho}} f/(4z_c)$. This insures in particular that all first derivatives of the GCF of $\tilde{A}$ vanish at 0. Obviously this does not affect the statistical properties of the maximum of the intensity distribution. The only difference is that the squared root of the determinant of the 4×4 matrix of the second derivatives of the normalized GCF of $\tilde{A}$ is:

$$\tilde{D}_V = D_V \times \lambda_z, \quad \lambda_z = \tfrac{1}{4}\left[\Delta_{\tilde{\rho}}\Delta_{\tilde{\rho}} f - (\Delta_{\tilde{\rho}} f)^2\right]^{1/2}.$$

If f has Gaussian shape, then we have $\lambda_z = 1$. We can now state the result. Denoting the disk with centre at 0 and radius R_0 by $D(0, R_0)$, the maximal intensity of the field A in the domain $D(0, R_0) \times [0, T_0] \times [0, z_0]$ is:

$$I_{\max} = I_0(\tilde{r}_V - \ln Z), \tag{25}$$

$$\tilde{r}_V = \ln\left(\frac{\tilde{D}_V V_0 z_0}{\pi^{3/2} V_c z_c}\right) + \frac{3}{2}\ln_2\left(\frac{\tilde{D}_V V_0 z_0}{\pi^{3/2} V_c z_c}\right), \tag{26}$$

where Z is a random variable whose statistical distribution is described by equation (8), $V_0 = \pi R_0^2 T_0$ and $V_c = \pi\rho_c^2\tau_c$.

In the same way, for the case of a time-independent speckle pattern (i.e. monochromatic or $\tau_c = \infty$), the maximal intensity of field A in the domain $D(0, R_0) \times [0, z_0]$ is:

$$I_{\max} = I_0(\tilde{r}_S - \ln Z), \tag{27}$$

$$\tilde{r}_S = \ln\left(\frac{\tilde{D}_S S_0 z_0}{\pi S_c z_c}\right) + \ln_2\left(\frac{\tilde{D}_S S_0 z_0}{\pi S_c z_c}\right), \tag{28}$$

where $\tilde{D}_S = D_S \times \lambda_z$, $S_0 = \pi R_0^2$ and $S_c = \pi\rho_c^2$. By comparing equation (7) with equation (25) on the one hand and equation (18) with equation (27) on the other, one can see that the maximal intensity increases with the ratio z_0/z_c roughly as $\ln(z_0/z_c)$. This was expected, since the speckle patterns in different planes separated by z_c are almost independent.

6. Conclusion

In this paper we have described the statistical distribution of the maximal intensity of a speckle pattern in terms of its macroscopic and microscopic characteristic parameters. Inspection of formulae (5), (19) and (12) yields that the only parameter related to the distribution of the field is the determinant of the matrix of the second order partial derivatives of the autocorrelation function evaluated at 0. It appears that the mean value of the maximal intensity $\langle I_{\max}\rangle$ varies logarithmically as a function of the ratio of the pulse volume over the mean hot spot volume. However the variance of the maximal intensity is rather important, and especially the right tail of the histogram which corresponds to very high values of the intensity. This indicates that it is probable that one of many experiments will give rise to an anomalous high peak which will exceed the expected value $\langle I_{\max}\rangle$.

Another important parameter when one takes into account that the response time of the medium is non-zero is the ratio of the response time over the coherence time of the incoherent pulse. The response time plays in our framework the role of an integration time with respect to which the intensity distribution is integrated. When the ratio is smaller than one, then the results demonstrated for the instantaneous intensity hold true. However if this ratio becomes of order 1 or even larger, then the maximal value of the integrated intensity gets smaller and smaller.

The roles of time incoherence and space incoherence are therefore not symmetric. Space incoherence enhances the probability to observe a very high peak. But if time incoherence also increases the maximal instantaneous intensity of the pulse, it decreases the maximal fluence. Furthermore, if the coherence time is so short that it reaches values of the order of the response time of the medium, then the maximal value of the effective intensity distribution (that is to say the intensity distribution integrated over the response time) is drastically reduced.

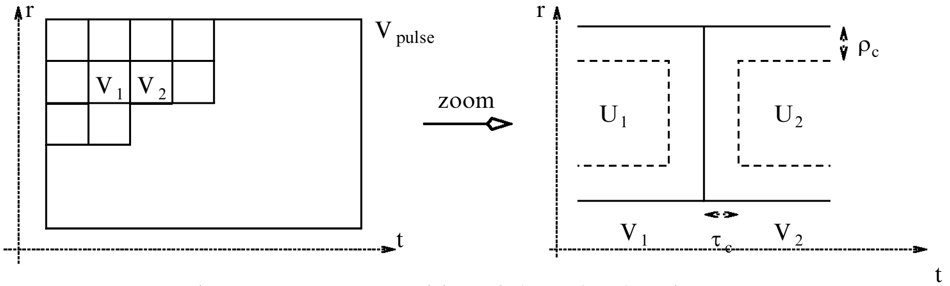

Figure 6. Decomposition of the pulse domain V_{pulse}

To sum up, if one deals with a broadband pulse so that the coherence time (which is proportional to the inverse of the spectrum bandwidth) is shorter than the response time of the medium, then any smoothing technique based on time incoherence (ISI, SSD, or SOF) can succeed in reducing the maximum of the effective intensity distribution. But if the pulse is bandwidth limited below this value, then polarization smoothing is the only way to reduce the maximal value of the instantaneous intensity distribution.

Acknowledgments

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Appendix A

Maxima of real-valued fields with Gaussian statistics

This appendix deals with the statistics of the maxima of the real part of a time–space incoherent pulse. In the following we denote by M_u the volume density of local maxima of A_R above the level $u \gg I_0^{1/2}$, and p_u^V stands for the probability that the maximum of A_R^2 over a domain V is larger than u^2. The pulse domain V_{pulse} is a time–space cylinder with spatial radius R_0, time duration T_0, and volume $V_0 = \pi R_0^2 T_0$. Then Adler shows that [17, Theorem 6-3-1]:

$$\langle M_u \rangle = \frac{D_V u^2}{2\pi V_c I_0} \exp\left(-\frac{u^2}{I_0}\right)\left(1 + O\left(\frac{I_0^{1/2}}{u}\right)\right). \tag{A1}$$

Since the statistical distribution of A_R is symmetric with respect to 0, the mean volume density of local maxima of A_R^2 above level u^2 is simply $2\langle M_u \rangle$. We develop in the following an asymptotic analysis with respect to the large parameter V_0/V_c. Consider u such that $2\langle M_u \rangle V_0$ is of order 1.

Divide the pulse domain V_{pulse} into $N \gg 1$ elementary domains V_i, $i = 1, \ldots, N$, of identical volume (see figure 6). N is chosen so that $V_c \ll V_0/N \ll V_0$. Then remove from each V_i a boundary of thickness ρ_c in the spatial direction and τ_c in the time direction. The fields over the resulting domains U_i are statistically independent. Besides, for each i the volume of the boundary $V_i \backslash U_i$ is of order:

$$\mathrm{Vol}\left(V_i \backslash U_i\right) \simeq V_c^{1/3}\left(V_0/N\right)^{2/3}. \tag{A 2}$$

Since $V_c \ll V_0/N$, this shows that we have $\mathrm{Vol}\left(U_i\right) \simeq V_0/N$. Thus the mean number $\langle K_u^{U_i}\rangle$ of local maxima above level u^2 in each domain U_i is:

$$\langle K_u^{U_i}\rangle = \frac{2\langle M_u\rangle V_0}{N}$$

up to terms of order $2\langle M_u\rangle V_c^{1/3}\left(V_0/N\right)^{2/3} \ll 2\langle M_u\rangle V_0/N$. Since $\langle K_u^{U_i}\rangle$ is of order $N^{-1} \ll 1$, this implies that the probability that a domain U_i contains a maximum above level u^2 is of order N^{-1}, and the probability that a domain U_i contains more than two maxima above level u^2 is of order $N^{-2} \ll N^{-1}$. This yields that the mean number $\langle K_u^{U_i}\rangle$ is equal to the probability $p_u^{U_i}$ to a very good approximation. Consequently, for every $i = 1, \dots, N$:

$$p_u^{U_i} = \frac{2\langle M_u\rangle V_0}{N} \tag{A 3}$$

up to terms of order N^{-2}. Equation (A 2) also establishes that the total volume of the boundaries $\bigcup_i V_i \backslash U_i$ represents a negligible contribution to the total pulse domain:

$$\mathrm{Vol}\left(\bigcup_{i=1}^{N} V_i \backslash U_i\right) \simeq N^{1/3} V_c^{1/3}\left(V_0\right)^{2/3} \ll V_0.$$

Thus the probability that the maximum of A_R^2 over the domain V_{pulse} is smaller than u^2 is:

$$1 - p_u^{V_{\mathrm{pulse}}} \simeq \mathbb{P}\left(\forall i, \max_{U_i} A_R(r,t)^2 \le u^2\right). \tag{A 4}$$

From the independence of the fields over the domains U_i and from equation (A 3) we get that:

$$1 - p_u^{V_{\mathrm{pulse}}} = \prod_{i=1}^{N}\left(1 - p_u^{U_i}\right) = \left(1 - \frac{2\langle M_u\rangle V_0}{N}\right)^N.$$

It is well know that $\left(1 - \alpha/N\right)^N \simeq \exp\left(-\alpha\right)$ for large N. From equation (A 1) we can then deduce that the maximal value $I_{\mathrm{max},1}^{A_R} := \max_{V_{\mathrm{pulse}}} A_R\left(r,t\right)^2$ obeys:

$$\mathbb{P}\left(I_{\mathrm{max},1}^{A_R} \le u^2\right) = \exp\left[-\frac{D_V V_0 u^2}{\pi V_c I_0}\exp\left(-\frac{u^2}{I_0}\right)\right].$$

We can then express the maximal intensity $I_{\mathrm{max},1}^{A_R}$ in terms of a random variable Z_1 with an exponential probability density $p_1\left(z\right) = \exp\left(-z\right)\mathbb{1}_{z\ge 0}$:

$$I_{\mathrm{max},1}^{A_R} = I_0\left(\Gamma_V - \ln Z_1\right), \tag{A 5}$$

where Γ_V is given by (5). The representation (A 4) can also be deduced from Theorem 6-9-4 [17] whose proof relies on the same basic estimates as ours. However this result by itself is insufficient because the precise expression of the statistical distribution of the maximal intensity of the field $A_R^2 + A_I^2$ requires the statistical distributions of the hottest spots $I_{\mathrm{max},1}^{A_R} \ge \cdots \ge I_{\mathrm{max},n}^{A_R}$ for n large enough. Furthermore our arguments can be easily repeated to derive the statistical distribution of the nth hottest spot $I_{\mathrm{max},n}^{A_R}$. Indeed the probability that $I_{\mathrm{max},n}^{A_R}$ is smaller than u^2 is equal to the probability that there is at most $n-1$ domains U_i which contain a maximum above level u^2. As a consequence:

$$\mathbb{P}\left(I^{A_R}_{\max,n} \le u^2\right) = \sum_{i=0}^{n-1} C_N^i p_u^{U_1 i}\left(1-p_u^{U_1}\right)^{N-i}.$$

Using the fact that $C_N^i \simeq N^i/i!$ for $N \gg 1$ and equation (A 3) yields:

$$\mathbb{P}\left(I^{A_R}_{\max,n} \le u^2\right) = \sum_{i=0}^{n-1} \frac{\left(2\langle M_u\rangle V_0\right)^i}{i!} \exp\left(-2\langle M_u\rangle V_0\right).$$

It is then straightforward to identify the statistical distribution of $I^{A_R}_{\max,n}$ as (4).

Appendix B

Ansatz

The above results rely on ansatz (6) which consists of assuming that the maximal intensity of the scalar field A is obtained at a point corresponding to a strong maximum of A_R (or A_I) and a value of A_I (or A_R) which obeys the standard Gaussian distribution. This ansatz can actually be justified. We shall show that points (r,t) where both A_R and A_I reach high values do not bring a noticeable contribution to the maximal intensity of the total field $A_R + iA_I$. Once again, this result is based on the independence of the processes A_R and A_I. Indeed the real Gaussian process A_R can be expanded around a local maximum (r_0, t_0) as (13), so that the volume of points (r,t) around a local maximum (r_0,t_0) at height I_1 of $|A_R|^2$ which are above the level $(1-\delta)I_1$ is:

$$v^{A_R}(\delta) \simeq \frac{4V_c\delta^{3/2}}{3D_V}$$

for $\delta \ll 1$. Let us fix $I_1, I_2 \gg I_0$. If $\bar{I}_1 \ge I_1$ and $\bar{I}_2 \ge I_2$, then the probability that there exists a local maximum of A_R between $[\bar{I}_1, \bar{I}_1 + \delta\bar{I}_1)$ and a local maximum of A_I between $[\bar{I}_2, \bar{I}_2 + \delta\bar{I}_2)$ inside the elementary domain U_i is:

$$q(\bar{I}_1)q(\bar{I}_2)\delta\bar{I}_1\delta\bar{I}_2, \quad q(\bar{I}) = \frac{2V_0}{N}\left|\frac{\mathrm{d}\langle M^{A_R}_{\bar{I}^{1/2}}\rangle}{\mathrm{d}\bar{I}}\right|.$$

Furthermore, if such an event happens, then the probability that the volume around the local maximum of A_R above level I_1 intersects the volume around the local maximum of A_I above level I_2 is:

$$\bar{p}^{U_i}_{I_1,I_2}(\bar{I}_1,\bar{I}_2) = \frac{v^{A_R}(1-I_1/\bar{I}_1) + v^{A_I}(1-I_2/\bar{I}_2)}{V_0/N}.$$

Combining these results implies that the probability that some point $(r,t) \in U_i$ both satisfies $A_R(r,t)^2 \ge I_1$ and $A_I(r,t)^2 \ge I_2$ is:

$$p^{U_i}_{I_1,I_2} = \int_{I_1}^{\infty} \mathrm{d}\bar{I}_1 \int_{I_2}^{\infty} \mathrm{d}\bar{I}_2 q(\bar{I}_1)q(\bar{I}_2)\bar{p}^{U_i}_{I_1,I_2}(\bar{I}_1,\bar{I}_2).$$

Since the fields in the elementary domains are independent, the probability that some point $(r,t) \in V_{\text{pulse}}$ both satisfies $A_R(r,t)^2 \ge I_1$ and $A_I(r,t)^2 \ge I_2$ can be expressed as:

$$p_{I_1,I_2}^{V_{\mathrm{pulse}}} = 1 - \prod_{i=1}^{N} \left(1 - p_{I_1,I_2}^{U_i}\right).$$

Combining with equation (A 1) we compute this expression:

$$p_{I_1,I_2}^{V_{\mathrm{pulse}}} = 1 - \exp\left[-\frac{D_V}{\pi^{3/2}} \frac{V_0}{V_{\mathrm{c}}} \left(\frac{I}{I_0}\right)^{1/2} \exp\left(-\frac{I}{I_0}\right) \xi\left(\frac{I_1}{I_2}\right)\right],$$

where $I = I_1 + I_2$ and $\xi(\alpha) = (\alpha^{3/4} + \alpha^{-3/4})/(\alpha^{1/2} + \alpha^{-1/2})^{1/2}$. Assuming the total intensity I is fixed, one can check that the above probability increases as either $\alpha \to 0$ or $\alpha \to \infty$. These limit cases correspond to the situations when the maximal value of $|A|^2 = A_{\mathrm{R}}^2 + A_{\mathrm{I}}^2$ is reached in a point (r, t) corresponding to a strong maximum of either A_{R}^2 or A_{I}^2 respectively, and a relatively low value of either A_{I}^2 or A_{R}^2 respectively. This statement justifies the adopted ansatz and consequently equation (6). In fact it is due to the fact that the superposition of high values of both A_{R} and A_{I} does not contribute to the generation of a very high maximum of the total field $A = A_{\mathrm{R}} + \mathrm{i}A_{\mathrm{I}}$, because the probability of such an overlap is too small.

Appendix C

Routine to simulate a complex-valued field with Gaussian statistics

We aim at simulating a stationary complex-valued zero-mean two-dimensional Gaussian field A over a square $[0, D] \times [0, D]$. The correlation function of the field is denoted by C. First note that the Fourier transform of C is non-negative real valued. Indeed the Fourier transform of the correlation function of a stationary Gaussian field is proportional to its power spectral density which is obviously non-negative [18]. The physical computation domain is $D = Nh$ square with elementary step h. Consider a $N \times N$ array $X_{j,k}$ of independent, complex valued, zero-mean and Gaussian random variables with $\langle |X_{j,k}|^2 \rangle = 1$. This array is a discrete Gaussian white noise, that is filtered in three steps. First apply the discrete Fourier transform to the array:

$$\hat{X}_{u,v} = \sum_{j,k=0}^{N-1} X_{j,k} \exp\left(\mathrm{i}2\pi(ju + kv)/N\right).$$

Then multiply it by $\hat{C}^{1/2}$, where $\hat{C}$ is the discrete Fourier transform of the correlation function C:

$$\hat{C}_{u,v} = \sum_{j,k=0}^{N-1} C(jh, kh) \exp\left(\mathrm{i}2\pi(ju + kv)/N\right), \tag{A 6}$$

$$\tilde{X}_{u,v} = \hat{X}_{u,v} \hat{C}_{u,v}^{1/2}. \tag{A 7}$$

Finally apply the discrete inverse Fourier transform:

$$\bar{X}_{j,k} = \frac{1}{N^2} \sum_{u,v=0}^{N-1} \tilde{X}_{u,v} \exp\left(-\mathrm{i}2\pi(ju + kv)/N\right).$$

The resulting $N \times N$ array $\bar{X}$ is precisely a realization of a Gaussian field with correlation function C:

$$\bar{X}_{j,k} = A(jh, kh), \quad j, k \in \{0, \ldots, N-1\}.$$

Indeed the three steps of the filter are linear operations which do not alter the Gaussian property of the initial field X. Since a Gaussian field is fully characterized by its mean and covariance, the proof will be complete if we establish that $\bar{X}$ possesses the desired correlation function. If we denote by $\bar{C}$ the inverse Fourier transform of $\hat{C}^{1/2}$, then $\bar{X}$ is simply the convoluted array $X \star \bar{C}$†:

$$\bar{X}_{j,k} = \frac{1}{N^2} \sum_{j',k'=0}^{N-1} X_{\lceil j-j' \rceil, \lceil k-k' \rceil} \bar{C}_{j',k'}.$$

Since the $X_{i,j}$ are independent from each other and zero-mean, straightforward calculations yield:

$$\langle \bar{X}_{i_1,j_1} \bar{X}^*_{i_2,j_2} \rangle = (\bar{C} \star \bar{C}^*)_{\lceil i_1-i_2 \rceil, \lceil j_1-j_2 \rceil}.$$

In this expression the exponent * holds for complex conjugation while the sign $\star$ holds for the discrete convolution. The convoluted array $\bar{C} \star \bar{C}^*$ is the inverse Fourier transform of the multiplied array $\hat{C}^{1/2} \cdot \hat{C}^{*\,1/2} = |\hat{C}|$. Since $\hat{C}$ is non-negative real-valued, we have $|\hat{C}| = \hat{C}$ so that the array $\bar{C} \star \bar{C}^*$ is exactly C.

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† If j is an integer $(j \in \mathbb{Z})$ then there exists a unique couple of integers $(j_1, j_2) \in \mathbb{Z} \times [0, N)$ such that $j = j_1 N + j_2$, and we denote j_2 by $\lceil j \rceil$.

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